Study of Transverse Coupled Bunch Instabilities by Using Non-Linear Taylor Maps for the Advanced Light Source (ALS)*

M. Meddahi and J. Bengtsson
Lawrence Berkeley Laboratory, University of California
1 Cyclotron Road, Berkeley, CA 94720 USA

Abstract
To study coherent bunch instabilities for the third generation light source, ALS, it is insufficient to rely on a linear map with radiation damping for the lattice to obtain the correct dynamics. We have implemented a code for transverse bunch instabilities, using a 4-dimensional map for the lattice and wake field expansions for cavities and resistive wall effects. This model has been used to study the injection process and to determine the needed performance of the proposed feedback system for the storage ring.

I. INTRODUCTION

The effects of the cavity HOMs and the resistive wall on the beam dynamics have been studied for the ALS. A tracking code used at SLAC [1, 2] was generalized to the 4-dimensional transverse case. The model is based on a map for the ring and localized kicks in both planes due to cavities and resistive wall wake fields [3]. The dynamics of the beam at injection has also been studied.

II. CALCULATIONS ON TRANSVERSE COHERENT BEAM INSTABILITIES.

The linearized equation of motion perturbed by transverse dipole wake fields is [3, 4] (see the references for definition of the parameters) :

$$x_i'' + \left(\frac{\omega_{\beta 0}}{c}\right)^2 x_i = \frac{Q\,e}{E_0\,C} \sum_{j=1}^{N_b} \sum_{n=0}^{N_t} W_\perp\left(L_{ij} + n\,C\right) x_j\left(s - L_{ij} - n\,C\right)$$

Using the following ansatz we get the corresponding dispersion relation

$$x_i(s) = x_{0i} \exp\left(-\,i\,\frac{\omega_\beta}{c}\,s\right),$$

$$\Delta\omega = \omega_\beta - \omega_{\beta 0} = \frac{1}{2\,\omega_{\beta 0}\,x_{0i}} \sum_{j=1}^{N_b} \chi_{ij}^{\perp}\, x_{0j}\,,$$

$$\chi_{ij}^{\perp} \equiv \frac{c^2\,Q\,e}{E_0\,C} \exp\left(i\,L_{ij}\frac{\omega_{\beta 0}}{c}\right) \sum_{n=0}^{\infty} W_\perp\left(L_{ij} + n\,C\right) \exp\left(i\,n\frac{\omega_{\beta 0}}{c}C\right)$$

It follows that

$$\frac{1}{\tau} = \mathrm{Im}\left(\omega_\beta\right), \quad \Delta\omega = \mathrm{Re}\left(\omega_\beta\right) - \omega_{\beta 0}$$

where τ is the time constant. The eigenmodes are therefore determined by the linear eigenvalue problem

$$M\,\bar{x}_0 = \omega_\beta\,\bar{x}_0\,, \quad M_{ij} = \omega_{\beta 0}\,\delta_{ij} - \frac{1}{2\,\omega_{\beta 0}}\chi_{ij}^{\perp}$$

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, of the U.S. Department of Energy under Contract No. DE-AC-76SF00098.

For a given eigenmode the relative amplitude and phase of the initial conditions for each bunch are given by the corresponding (complex) eigenvector. In this case, all the bunches are oscillating coherently with the same frequency. Comparison of analytical results with numerical simulation (tracking) for this case can be found in section 4. For the general case, however, we have a superposition of eigenmodes.

For the non-displaced bunches (i.e. stored beam) we use the following ansatz and dispersion relation

$$x_i(s) = \frac{T_i}{c}\,s\,\exp\left(-\,i\,\frac{\omega_{\beta 0}}{c}\,s\right)$$

$$T_i = \left|\frac{1}{2\,\omega_{\beta 0}} \sum_{j=1}^{N_b} \chi_{ij}^{\perp}\, x_{0j}\right|$$

where T_i is the linear growth rate.

III. OVERVIEW OF THE ALS

The ALS is a 1.5 GeV Synchrotron Radiation Source based on a triple bend achromat lattice [5]. The injection system consists of a high intensity electron gun, a 50 MeV traveling wave linac, and a 1 Hz, 1.5 GeV booster synchrotron. The injection scenario requires the storage ring closed orbit to be deflected close to the end of the injection septum using four bump magnets capable of displacing the orbit up to 15 mm. A pulse of electrons is ejected from the booster and transported through the injection septa into the storage ring.
Assuming that the HOMs of both RF cavities (iris radius 0.035 m, length 0.28 m, frequency 499.764 MHz, harmonic number 328) has been damped by a factor of 30 giving an average transverse shunt impedance of 2 MΩ/m, we investigate the HOMs with a shunt impedance greater than this value to study the effect on the beam dynamics. Table 1 shows the HOMs used [6].

f (MHz)	Polarization	Q	Rsh (kΩ/m)
810.08	H	48000	14800
1121.77	V	7000	3700
1122.72	H	17000	9000
1189.85	H	18000	92
1529	H	1800	200
1801.61	V	2000	1100

Table 1.

The effect of the resistive wall is also addressed using a total impedance (at the first sideband of the DC component) of 355.013 kΩ/m in the horizontal and 883.259 kΩ/m in the vertical plane.

IV. ANALYSIS

In the following analysis 10 bunches were used: 4 bunches (1 to 4) displaced by x=5mm and y=1mm and 6 bunches in the storage ring (5 to 10) assumed to be damped down to zero amplitude. The study was limited to 5 mm due to the use of a Taylor series map, proven accurate up to this amplitude using a symplectic integrator. Amplitudes up to 13 mm can be expected in reality. Tracking was done for 4096 turns and the final amplitudes are shown for bunch number 1 and 5.

A- Linear map

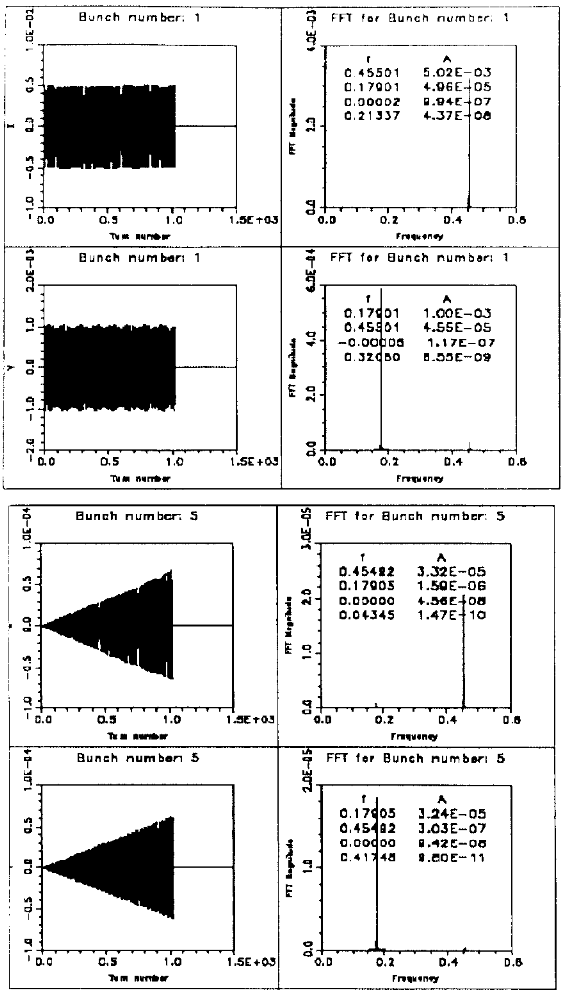

Figure 1: x and y amplitudes for bunch 1 and 5 .

In table 2a we present the effect on the final amplitudes of one cavity, of the resistive wall , and of both, respectively. For the resistive wall (R.W.) the wake field was stored for one turn in Table 2a and 500 turns in Table 2b.

	R.W. and CAV.		R.W. (1 wake)		CAVITY	
bunch #	x [mm]	y [mm]	x [mm]	y [mm]	x [mm]	y [mm]
1	5.0387	1.0156	5.026	1.015	5.012	1.0002
5	0.0237	0.0243	0.0342	0.0242	0.0107	0.0001

Table 2a.

bunch #	R.W. (500 wakes)	
	x [mm]	y [mm]
1	5.0387	1.0156
5	0.0237	0.0243

Table 2b.

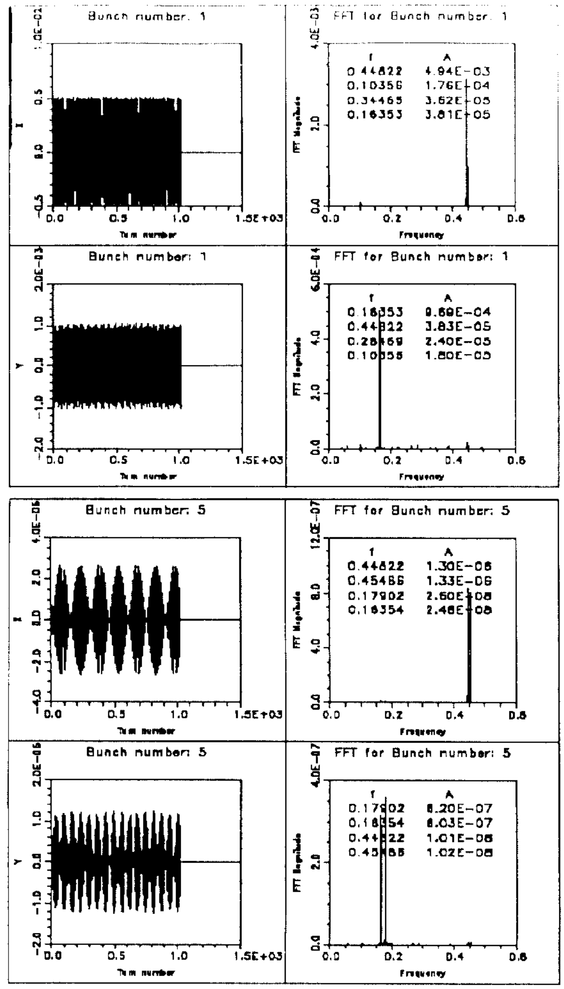

Figure 2: x and y amplitude for bunch 1 and 5 .

The amplitude of bunch 1 slowly increases to 5.038 mm in x-plane and 1.015 mm in y-plane. For bunch 5, the amplitude reaches 0.0237 mm in x and 0.0243 mm in y (Table 2a). Figure 1 shows the linear growth for the stored bunches. The use of one wake instead of five hundred wakes for the resistive wall is a good approximation and will be used for the following tracking studies (Table 2b).

B- Growth rates

H-Growth rate (s)		V-Growth rate (s)	
tracking	calculated	tracking	calculated
0.128	0.122	0.0902	0.0881
0.103	0.0979	0.0727	0.0706
0.0892	0.0848	0.0631	0.0611
0.0800	0.0761	0.0566	0.0548
0.0732	0.0696	0.0517	0.0502
0.0678	0.0646	0.0479	0.0466

Table 3.

Growth rates from tracking and estimates using the theory of section 2 were compared for the two cases of zero or non zero initial amplitude. In table 3 the results are shown for the stored bunches (5 to 10) with zero initial amplitude. We find very good agreement between tracking and calculation. Table 4 shows the results for the displaced bunches (1 to 4). The agreement is good.

H-Growth rate (s)		V-Growth rate (s)	
calculated	tracking	calculated	tracking
0.0208	0.0164	0.00569	0.00495
0.0208	0.0247	0.00569	0.00582
0.0208	0.0279	0.00569	0.00633
0.0208	0.0281	0.00569	0.00656

Table 4.

C- Nonlinear map

Table 5 shows the difference on the final amplitudes for bunch 1 and 5 when using a non-linear map

map	x1	y1	x5	y5
Linear	5.1048	1.0561	0.0602	0.0621
Non-linear	5.1006	1.0487	0.0027	0.0012

Table 5.

The non-linear map introduces a modulation of the amplitude for the stored bunches which leads to limited final amplitude (Figure 2). For the selected initial amplitudes, the change of the final amplitudes, after 4096 turns, are very small.

The FFT on the bunches 5 to 10 shows a double peak around the tunes. An explanation was found by numerical simulation of a driven anharmonic oscillator leading to the same double peak when Fourier analyzed (Figure 3).

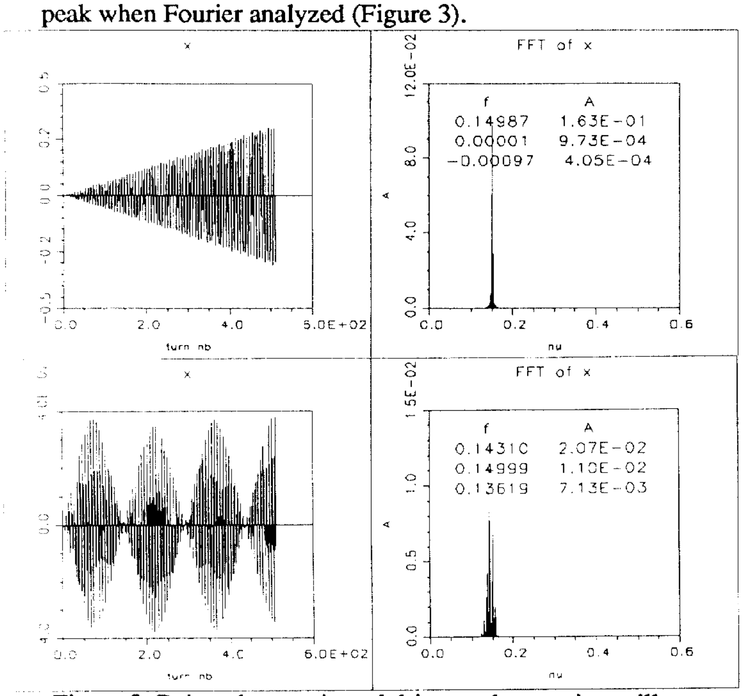

Figure 3. Driven harmonic and driven anharmonic oscillator.

V. INJECTION DYNAMICS

The bunches are injected from the booster to the storage ring with an amplitude of approximately 13 mm in the horizontal and 1 mm in the vertical plane. We have studied the behavior of the beam just after the injection, for 15,000 turns (the damping time is approx. 15 ms), neglecting wake fields but simulating the full 6-dimensional dynamics [7].

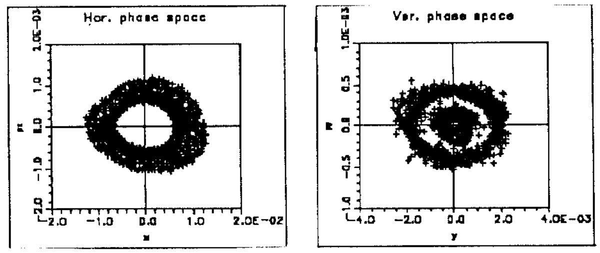

Figure 4. Phase space at injection.

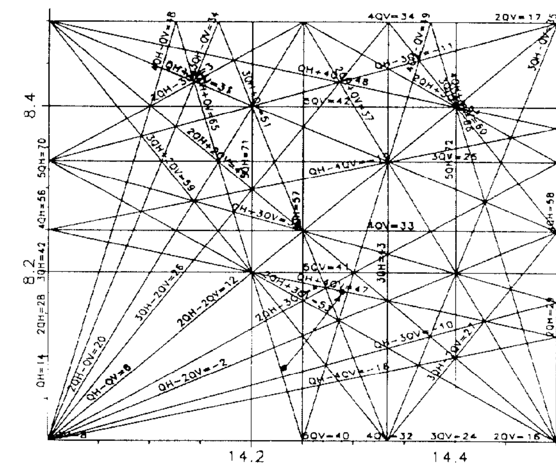

Figure 5. Tune diagram at injection.

Damping is observed in the horizontal plane together with beating in the vertical plane leading to large amplitudes. The large horizontal amplitude drives the vertical plane due to coupling. The change of amplitude due to radiation damping leads to a change of tune and crossing of resonances due to large amplitude dependent tune shifts.

VI. CONCLUSION

The tracking shows the importance of correct modeling of the lattice in the estimate of growth rates. Using a non-linear map for the lattice led to modulation of the amplitude, instead of linear growth, for an initially non-displaced bunch. However, the growth rates obtained in both planes are small. Study of the injection dynamics shows beating in the vertical, driven by the horizontal plane, leading to crossing of resonances.

VII. REFERENCES

[1] K. Bane.et al, AIP Conf. Proc. No 127, chap.2, 1983.
[2] K. A. Thompson and R. D. Ruth, SLAC-PUB 4962.
[3] K.A. Thompson, Asym. B-factory collider note ABC-57.
[4] M. Meddahi and J. Bengtsson, LBL-CBP Note/017.
[5] LBL-Pub-5172 Rev.
[6] J. Byrd and J. Corlett, ESG Tech Note-220.
[7] E. Forest, to appear in Part. Accel.