# Longitudinal Rf Matching during AGS-RHIC Beam Transfer\*

X. Pei

Brookhaven National Laboratory, Upton, NY 11973, USA

#### Abstract

The Relativistic Heavy Ion Collider is a 2.5 mile long superconducting colliding synchrotron currently under construction in the Brookhaven National Laboratory. It is capable of accelerating heavy ions such as gold to energies as high as 100 GeV/u. The collision of such high energy heavy ion beams is expected to demonstrate important statistical quark and gluon plasma phenomena. The RHIC will be injected with the existing Alternate Gradient Synchrotron (AGS) and the rf systems of both the Collider and the AGS must be matched to accomplish bucket to bucket beam transfer.

# I. INTRODUCTION

The designed RHIC circumference is 19/4 that of the AGS<sup>[1]</sup>. At this ratio, if we have 12 bunches in the AGS synchrotron and 57 bunches in the RHIC collider, we have, assuming isochronous bunch distribution:

#### 57/19 = 12/4

which means that both rings have the same bunch spacing.

After the initial commissioning, it is hoped that the RHIC have twice as many bunches. So its bunch number will be increased to 114<sup>111</sup>. In this case the RHIC bunch spacing is half that of the AGS. In both the 57 and 114 RHIC bunch cases, the RHIC rf system will operate at the 342nd harmonic of the beam revolution frequency and the AGS rf system will operate at the 12th harmonic of the beam revolution frequency.

## **II. KICKER CONTROL**

It is always possible to change the rf phase of the injecting ring to inject the *m*th bucket from the injecting ring into the *n*th bucket of the receiving ring. A phase shift equivalent to an effective delay is introduced after the phase locking of the first rf bunch of both rings:

$$\tau_{d} = ms_{inj} - ns_{rec} - lT_{inj}$$
(1)  
m,n,l = 1,2,3...

where  $s_{inj}$  and  $s_{rec}$  are the bunch spacings of the injecting and the receiving rings and  $T_{inj}$  is the period of the injecting ring. l is an integer to keep the delay within one revolution period of the injecting ring.

In reality, it is advantageous to minimize rf phase shift operations as the adiabatic phase change requirement slows

down the injection operation and complicates operation as well as hardware and software designs.

We therefore take advantage of the simpler bunch spacing relation in the AGS and the RHIC and divide the longitudinal matching into the kicker control part (called "cogging") and the rf phase match part (called "synchronization")<sup>[2]</sup>. In this scheme, rf phase locking needs only to be done once every AGS acceleration cycle.

Fig.1 is a block diagram showing the principle of such a scheme and the description is as follows:



Fig.1 Block diagram of cogging (enclosed by the dotted box) and synchronization between the RHIC and the AGS during beam transfer.

The rf cavity signals are obtained by vector-summing all the cavities in each ring. Since the RHIC has to be filled by the AGS in many acceleration cycles, the RHIC rf buckets are numbered by a counter. There is no need to number the AGS buckets as the simple bunch spacing relation will automatically align each AGS bucket to the RHIC bucket as if two gears are cogged. The counter has the modulus of the RHIC harmonic number and a reset starts counting. The cogging circuit controls the AGS extraction and the RHIC injection kicker triggers according to the RHIC rf pulses. The rf phase is assumed to be matched by the synchronization circuit. In the case of equal bunch spacing in both rings, the RHIC buckets are injected sequentially. At present, the plan is for the kickers to transfer one bunch of beam at a time. Because not every RHIC rf bunch is to be filled with beam, a digital state machine circuit will delay, in the case of 57 bunches, 6 rf pulses after each kick, so that the kicker trigger sequence will be at the 1st, 7th, 13th... etc. of the RHIC rf pulses. In the case that the RHIC bunch number is 114, its bunch spacing is half that of the AGS. Only every other RHIC rf bucket will be cogged to the AGS bucket. To minimize phase shift operations, the odd numbered RHIC beam buckets will be filled first. The injection kickers in this scheme can only kick one bucket into the RHIC --- otherwise the already injected beam in the RHIC can be kicked out. The kicker trigger sequence controlled by

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the state machine will therefore be the 1st, 7th,...13th, etc. of the RHIC rf pulses --- corresponding to the 1st, 3rd, .... and 113th RHIC beam bunches. The AGS rf phase is then changed to match the even numbered RHIC beam bunches and the kickers will be triggered at the sequence of the 2nd, 4th,... and 114th RHIC bunches.



Fig.2 Rf bucket relation between the RHIC and the AGS. Each pulse is an rf bucket. In the RHIC, beam is injected into every 6th bucket (solid fill) during the 57 bunch operation and into both the solid filled and the stripe filled buckets in the 114 bunch operation. To inject the stripe-filled RHIC bucket, the AGS is phase shifted three RHIC rf pulses.

Note that triggering the kickers according to the RHIC rf pulses is only a necessary condition. A sufficient condition must also include the AGS rf phase being matched to the RHIC rf phase and the AGS beam being ready for extraction. Thus the signal from the state machine, the phase lock ok signal and the AGS beam ready signal (labeled as the "arm" input in Fig.1) should be coincidenced to trigger the kickers. Once a trigger to the kickers is sent, the state machine advances to the state corresponding to the next RHIC bunch. Finally, some delay adjustment is needed to take the beam TOF and signal transmission time into account.

### III. RF PHASE MATCH

The RHIC rf frequency is 6 times that of the AGS frequency so a divide-by-6 circuit is used to bring the RHIC rf frequency down to be phase compared to the AGS rf frequency. The error signal is integrated and drives a phase shifter to change the AGS rf phase.

In the case of 114 RHIC bunches, the difference in bunch spacings in two rings makes it necessary to switch from even-numbered RHIC bunch injection to odd-numbered bunch injection. A pulse removing circuit removes three RHIC rf pulses to insure the phase lock as shown in Fig.2 to flip between the states.

A question of concern is how fast the phase locking process can take place. Accompanying rf phase shifting, there always is a frequency shift which causes a bucket energy error. The following is the basic longitudinal equations describing particle motions:

$$\frac{dp}{dt} = \frac{QeV}{C} \sin\phi$$

$$\frac{d\phi}{dt} = 2\pi(hf - f_r)$$

$$\frac{df}{f} = (\frac{1}{\gamma^2} - \frac{1}{\gamma_1^2})\frac{dp}{p} = \eta \frac{dp}{p}$$
(2)

where C is the ring circumference, f the beam revolution frequency, h the harmonic number of the ring,  $f_r$  the rf frequency,  $\phi$  the vectored rf phase the beam sees and V the vectored rf peak voltages of the ring. Q and p are the charge and momentum of the beam at the velocity of  $\gamma$  and  $\gamma_r$  is the transition  $\gamma$  of the ring.

From the above we can map the traditional longitudinal phase space into a space where the energy axis is replaced by the fundamental revolution frequency f in a ring and the horizontal axis consists of the rf phase the beam sees. In this coordinate system, we have, for a stationary bucket<sup>[3]</sup>:

$$h\Delta f_{max} = 2f_s$$
 (3)

 $(\mathbf{1})$ 

where  $\Delta f_{max}$  is the half height of an rf bucket in terms of beam revolution frequency and  $f_s$  the beam synchrotron oscillation frequency at the harmonic *h*.

The above result shows that, in terms of beam frequency, the half height of a stationary bucket is just twice the synchrotron oscillation frequency!

Thus a linear phase shift in *T*, which corresponds to a shift of rf frequency by  $\delta f = (1/2\pi)\Delta \phi/T$ , will move a synchronous particle in the bucket by:

$$\frac{\delta f}{2f_s} = \frac{1}{2\pi} \frac{\Delta \phi}{2f_s}$$
(4)

So for a phase shift of  $2\pi$ , the percentage error caused by a linear phase shift is simply  $(1/2)(T_s/T)$  of the total bucket, where  $T_s=1/f_s$ . To control the energy error within 5 percent of the bucket height, for example, the linear phase shift needs to last 10 synchrotron oscillation periods.

Once the beam is shaken out of the center of the bucket, it starts undamped synchrotron oscillation. Should the oscillation be fairly linear, when the phase shift is stopped after an integer number of oscillations, the energy error will disappear and so will the oscillation. Particular care should be taken to avoid a phase shift that lasts half integer synchrotron periods, such as 0.5, 1.5, 2.5 times  $T_s$ , as this will double the oscillation amplitude and the oscillation will persist when the phase shift is over.

The longitudinal oscillation problem can be effectively treated with beam feedback. The feed back loop effectively

introduces a damping term for the oscillation and automatically shifts the rf phase adiabatically.

Fig.3 is a beam phase feedback loop that is nested in a larger radial feedback loop to be implemented for the RHIC. The reference signal actually comes from the beam radial sensor. For simplicity we won't go into details about its behavior in this paper.



Fig.3 The beam feedback loop nested in the beam radial feedback system. In this loop,  $\phi_r$  is obtained from the beam radial feedback and serves as a reference signal.  $\phi$  is the beam phase.  $D_{\phi}$  is the transfer function of an error amplifier that converts the phase error to the VCO control voltage.  $k_o$  is the gain of the VCO. The cavity's delay constant is  $\tau_{ct}$ . The beam's phase change in response to a frequency change of the cavity is  $B_{\phi}$ .

We let the products of all the parts determined by electronics be K and the response of the system in s domain is just:

$$\phi = \frac{KB_{\phi}}{1 + KB_{\phi}}\phi_r \tag{5}$$

Since  $B_{\phi}$  is simply the particles' undamped small phase motion in response to a sudden frequency or energy change of the bucket:

$$\dot{\phi} + \omega^2 \phi = \dot{\omega}$$

in

Its Laplace transform is just:

$$\boldsymbol{B}_{\boldsymbol{\phi}}(s) = \frac{s}{s^2 + \omega_s^2} \omega_{\boldsymbol{\eta}}(s) \tag{7}$$

Substituting the above into Eq.5, we get:

$$\phi(s) = \frac{Ks}{s^2 + Ks + \omega_s^2} \phi_R(s) \tag{8}$$

The first power s term in the denominator shows that there could be damping. By adjusting the constant K we can change the beam response as desired.

An important concern is the deviation of the RHIC

ring from the designed 19/4 circumference ratio with respect to the AGS due to construction tolerances. Let's estimate the effect of such errors on the injection process. Denote 4/19 of the RHIC circumference to be *C*, which is the supposed AGS circumference, and *f* the beam revolution frequency in the AGS. We have:

$$C = \frac{\beta}{f}, \qquad \frac{\partial C}{\partial f} = -\frac{\beta}{f^2}$$
 (9)

which leads to:

$$\frac{\Delta C}{C} = -\frac{\Delta f}{f} \tag{10}$$

The circumference error thus results in a reduction of the AGS momentum acceptance:

$$\frac{\Delta p}{p} = \frac{1}{\eta} \frac{\Delta f}{f} = -\frac{1}{\eta} \frac{\Delta C}{C}$$
(11)

We hope that the deviation of the designed circumference ratio is small so that the AGS will have as large a momentum aperture as possible. If the deviation results in unacceptable momentum aperture loss, the bending field of the AGS can be varied. This, however, will change the particle revolution frequency in the AGS. The rf frequency ratio will be more complicated between the two rings. Modern frequency synthesis technologies, fortunately, are able to lock two frequencies with any rational ratios steadily. The additional concern will be only to choose the appropriate time periods to make the beam transfer as the phase difference between the two rings is a function of time.

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