

the state machine will therefore be the 1st, 7th,...13th, etc. of the RHIC rf pulses --- corresponding to the 1st, 3rd, and 113th RHIC beam bunches. The AGS rf phase is then changed to match the even numbered RHIC beam bunches and the kickers will be triggered at the sequence of the 2nd, 4th,... and 114th RHIC bunches.

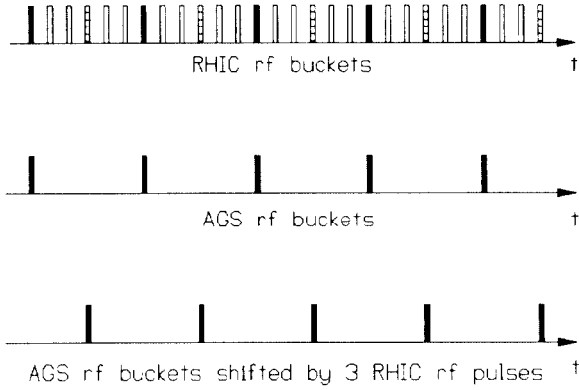


Fig.2 Rf bucket relation between the RHIC and the AGS. Each pulse is an rf bucket. In the RHIC, beam is injected into every 6th bucket (solid fill) during the 57 bunch operation and into both the solid filled and the stripe filled buckets in the 114 bunch operation. To inject the stripe-filled RHIC bucket, the AGS is phase shifted three RHIC rf pulses.

Note that triggering the kickers according to the RHIC rf pulses is only a necessary condition. A sufficient condition must also include the AGS rf phase being matched to the RHIC rf phase and the AGS beam being ready for extraction. Thus the signal from the state machine, the phase lock ok signal and the AGS beam ready signal (labeled as the "arm" input in Fig.1) should be coincided to trigger the kickers. Once a trigger to the kickers is sent, the state machine advances to the state corresponding to the next RHIC bunch. Finally, some delay adjustment is needed to take the beam TOF and signal transmission time into account.

III. RF PHASE MATCH

The RHIC rf frequency is 6 times that of the AGS frequency so a divide-by-6 circuit is used to bring the RHIC rf frequency down to be phase compared to the AGS rf frequency. The error signal is integrated and drives a phase shifter to change the AGS rf phase.

In the case of 114 RHIC bunches, the difference in bunch spacings in two rings makes it necessary to switch from even-numbered RHIC bunch injection to odd-numbered bunch injection. A pulse removing circuit removes three RHIC rf pulses to insure the phase lock as shown in Fig.2 to flip between the states.

A question of concern is how fast the phase locking process can take place. Accompanying rf phase shifting, there always is a frequency shift which causes a bucket energy error.

The following is the basic longitudinal equations describing particle motions:

$$\begin{aligned} \frac{dp}{dt} &= \frac{QeV}{C} \sin \phi \\ \frac{d\phi}{dt} &= 2\pi(hf - f_r) \\ \frac{df}{f} &= \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right) \frac{dp}{p} = \eta \frac{dp}{p} \end{aligned} \quad (2)$$

where C is the ring circumference, f the beam revolution frequency, h the harmonic number of the ring, f_r the rf frequency, ϕ the vectored rf phase the beam sees and V the vectored rf peak voltages of the ring. Q and p are the charge and momentum of the beam at the velocity of γ and γ_t is the transition γ of the ring.

From the above we can map the traditional longitudinal phase space into a space where the energy axis is replaced by the fundamental revolution frequency f in a ring and the horizontal axis consists of the rf phase the beam sees. In this coordinate system, we have, for a stationary bucket^[3]:

$$h\Delta f_{\max} = 2f_s \quad (3)$$

where $\Delta f_{\max}$ is the half height of an rf bucket in terms of beam revolution frequency and f_s the beam synchrotron oscillation frequency at the harmonic h .

The above result shows that, in terms of beam frequency, the half height of a stationary bucket is just twice the synchrotron oscillation frequency!

Thus a linear phase shift in T , which corresponds to a shift of rf frequency by $\delta f = (1/2\pi)\Delta\phi/T$, will move a synchronous particle in the bucket by:

$$\frac{\delta f}{2f_s} = \frac{1}{2\pi} \frac{\Delta\phi}{T} \quad (4)$$

So for a phase shift of 2π , the percentage error caused by a linear phase shift is simply $(1/2)(T_s/T)$ of the total bucket, where $T_s = 1/f_s$. To control the energy error within 5 percent of the bucket height, for example, the linear phase shift needs to last 10 synchrotron oscillation periods.

Once the beam is shaken out of the center of the bucket, it starts undamped synchrotron oscillation. Should the oscillation be fairly linear, when the phase shift is stopped after an integer number of oscillations, the energy error will disappear and so will the oscillation. Particular care should be taken to avoid a phase shift that lasts half integer synchrotron periods, such as 0.5, 1.5, 2.5 times T_s , as this will double the oscillation amplitude and the oscillation will persist when the phase shift is over.

The longitudinal oscillation problem can be effectively treated with beam feedback. The feed back loop effectively

introduces a damping term for the oscillation and automatically shifts the rf phase adiabatically.

Fig.3 is a beam phase feedback loop that is nested in a larger radial feedback loop to be implemented for the RHIC. The reference signal actually comes from the beam radial sensor. For simplicity we won't go into details about its behavior in this paper.

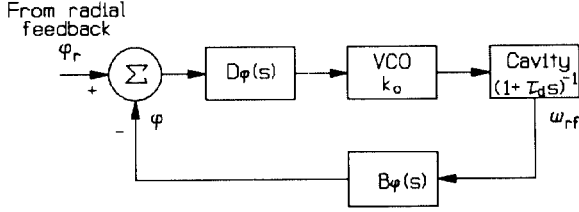


Fig.3 The beam feedback loop nested in the beam radial feedback system. In this loop, ϕ_r is obtained from the beam radial feedback and serves as a reference signal. ϕ is the beam phase. D_ϕ is the transfer function of an error amplifier that converts the phase error to the VCO control voltage. k_o is the gain of the VCO. The cavity's delay constant is τ_d . The beam's phase change in response to a frequency change of the cavity is B_ϕ .

We let the products of all the parts determined by electronics be K and the response of the system in s domain is just:

$$\phi = \frac{KB_\phi}{1 + KB_\phi} \phi_r \quad (5)$$

Since B_ϕ is simply the particles' undamped small phase motion in response to a sudden frequency or energy change of the bucket:

$$\ddot{\phi} + \omega_s^2 \phi = \dot{\omega}_{rf} \quad (6)$$

Its Laplace transform is just:

$$B_\phi(s) = \frac{s}{s^2 + \omega_s^2} \omega_{rf}(s) \quad (7)$$

Substituting the above into Eq.5, we get:

$$\phi(s) = \frac{Ks}{s^2 + Ks + \omega_s^2} \phi_r(s) \quad (8)$$

The first power s term in the denominator shows that there could be damping. By adjusting the constant K we can change the beam response as desired.

An important concern is the deviation of the RHIC

ring from the designed 19/4 circumference ratio with respect to the AGS due to construction tolerances. Let's estimate the effect of such errors on the injection process. Denote 4/19 of the RHIC circumference to be C , which is the supposed AGS circumference, and f the beam revolution frequency in the AGS. We have:

$$C = \frac{\beta}{f}, \quad \frac{\partial C}{\partial f} = -\frac{\beta}{f^2} \quad (9)$$

which leads to:

$$\frac{\Delta C}{C} = -\frac{\Delta f}{f} \quad (10)$$

The circumference error thus results in a reduction of the AGS momentum acceptance:

$$\frac{\Delta p}{p} = \frac{1}{\eta} \frac{\Delta f}{f} = -\frac{1}{\eta} \frac{\Delta C}{C} \quad (11)$$

We hope that the deviation of the designed circumference ratio is small so that the AGS will have as large a momentum aperture as possible. If the deviation results in unacceptable momentum aperture loss, the bending field of the AGS can be varied. This, however, will change the particle revolution frequency in the AGS. The rf frequency ratio will be more complicated between the two rings. Modern frequency synthesis technologies, fortunately, are able to lock two frequencies with any rational ratios steadily. The additional concern will be only to choose the appropriate time periods to make the beam transfer as the phase difference between the two rings is a function of time.

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V. REFERENCES

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