ANALYSIS AND APPLICATIONS OF QUADRATURE HYBRIDS AS RF CIRCULATORS*

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Abstract

The operation of a quadrature hybrid as a power combiner is analyzed. The analytical results are compared with data measured experimentally using a 211 MHz cavity. Graphical solutions of the measured cases are in good agreement with analytical predictions. The use of the 90°-hybrid as an RF circulator is also analyzed. The active operation of the harmonic cavity in the NSLS VUV-ring is used to demonstrate this application. This fourth-harmonic cavity is used to change the shape of the bucket potential to lengthen a stored bunch. Thus, a longer stored-beam lifetime can be achieved without compromising the high brightness of the VUV photon beam. If operated actively, the harmonic cavity would present a mismatched load to an RF generator. Thus, a need exists for a circulator. Similarities in operation between the 90°-hybrid and a circulator are discussed.

I. ANALYTICAL MODEL FOR THE HYBRID AS A POWER COMBINER

The quadrature hybrid [1], is a four-port network with 90° phase shift between two of the ports and no phase shift between the other two ports. Refering to Fig. 1, one can write its S-matrix as:

$$\begin{pmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -j \\ -j & 0 & 0 & 1 \\ 0 & -j & 1 & 0 \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{pmatrix} \qquad (1)$$

In this section we summerize the results of our analysis for a 90°-hybrid [2]. As shown in Fig.1, the hybrid analysed is used to combine the power from two transmitters feeding ports 1 and 4. A cavity is being connected to port 2, and a waster load is connected at port 3.

Assumptions:

a. The cavity represents an impedance mismatch of Γ_c.
b. The transmitters T_A and T_B have the same impedance mismatch represented as Γ_g
c. The two transmitters are providing two identical voltages except for a 90° phase shift,

$$V_{T_A} = V_g \angle 0° \; ; \; V_{T_B} = V_g \angle 90°$$

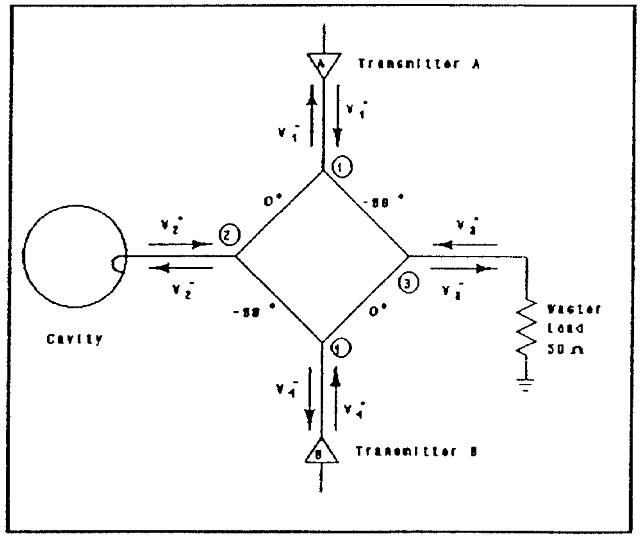

Figure 1. Configuration for the case analyzed.

d. A load ($Z_L = Z_o$ of the hybrid) is connected at port 4.

Approach:

Here we use the linearity of the system and apply the superposition principle. T_A is activated while T_B is OFF(case 1) . Then, T_B is activated while T_A is OFF (case 2), as follows:

Case 1 : Since the active port in this case is port 1, one needs to solve first for the ratio V_1^-/V_1^+ as seen by the transmitter T_A, when T_B is OFF. This is done by using Eq. 1 to solve for V_1^+,as follows:

$$\begin{pmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -j \\ -j & 0 & 0 & 1 \\ 0 & -j & 1 & 0 \end{pmatrix} \begin{pmatrix} V_1^+ \\ \Gamma_c V_2^- \\ 0 \\ \Gamma_g V_4^- \end{pmatrix} \qquad (2)$$

Solving for V_1^+,one can get ,

$$V_1^+ = \left(\frac{2+\Gamma_g\Gamma_c}{2}\right)\left(\frac{1-\Gamma_g}{2}\right) V_g\angle 0 \qquad (3)$$

Now, we can solve for the reflected voltages. This is done by solving Eq. (4).

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$$\begin{pmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -j \\ -j & 0 & 0 & 1 \\ 0 & -j & 1 & 0 \end{pmatrix} \begin{pmatrix} \left(\frac{2+\Gamma_g\Gamma_c}{2}\right)\left(\frac{1-\Gamma_g}{2}\right) V_g\angle 0 \\ \Gamma_c V_2^- \\ 0 \\ \Gamma_g V_4^- \end{pmatrix} \qquad (4)$$

giving,

Reflected voltages *Incident voltage*

$$V_1^- = \frac{\Gamma_c}{2}\,\frac{(1-\Gamma_g)}{2}\,V_g\angle 0 \qquad V_1^+ = \left(\frac{2+\Gamma_g\Gamma_c}{2}\right)\left(\frac{1-\Gamma_g}{2}\right) V_g\angle 0$$

$$V_2^- = \frac{1}{\sqrt{2}}\,\frac{1-\Gamma_g}{2}\,V_g\angle 0 \qquad V_2^+ = \frac{\Gamma_c}{\sqrt{2}}\,\frac{(1-\Gamma_g)}{2}\,V_g\angle 0 \quad (5)$$

$$V_3^- = \frac{j}{\sqrt{2}}\,(1+\Gamma_g\Gamma_c)\frac{(1-\Gamma_g)}{2}\,V_g\angle 0 \qquad V_3^+ = 0$$

$$V_4^- = -j\,\frac{\Gamma_c}{2}\left(\frac{1-\Gamma_g}{2}\right) V_g\angle 0 \qquad V_4^+ = -j\,\frac{\Gamma_g\Gamma_c}{2}\left(\frac{1-\Gamma_g}{2}\right) V_g\angle 0$$

Case 2: Following the same steps as in Case 1, we solve for V_4^+, giving

$$V_4^+ = \left(\frac{2-\Gamma_g\Gamma_c}{2}\right)\left(\frac{1-\Gamma_g}{2}\right) V_g\angle 90$$

Similar to Case 1, we can solve for the incident and reflected voltages at the four ports for this case.

Total Incident and Reflected Voltage:

By applying the superposition principle, we can get the total voltages at the four ports when both transmitters are ON [2]. The resulting voltages are:

$$\begin{array}{llll} \textit{Transmitter A:} & V_1^+ = (1+\Gamma_g\Gamma_c)V_i & ; & V_1^- = \Gamma_c\,V_i \\ \textit{Cavity:} & V_2^+ = \sqrt{2}\,\Gamma_c V_i & ; & V_2^- = \sqrt{2}V_i \quad (6) \\ \textit{Load:} & V_3^+ = 0 & ; & V_3^- = \sqrt{2}\,\Gamma_c\Gamma_g V_i \\ \textit{Transmitter B:} & V_4^+ = j(1-\Gamma_g\Gamma_c)V_i & ; & V_4^- = j\Gamma_c\,V_i \end{array}$$

where

$$V_i = \frac{(1-\Gamma_g)}{2}\,V_g\,\angle 0^\circ$$

II. ANALYSIS OF MEASURED DATA

To verify the above analytical results, measurements were carried out using the configuration shown in Fig. 1. The forward and reflected powers were measured using directional couplers connected to a power meter. The measurement was done using a 211 MHz cavity. The power was measured at three different cavity phase settings (0°, -45°, and 45°). The results of the measurements are shown in Table 1 where the voltages are in volts.

Table I

Cav. Phase	V_1^+	V_1^-	V_2^-	V_2^+	V_3^-	V_4^+	V_4^-
0°	524 (0°)	63 (60°)	765	100	18.7	548 (95°)	55 (-55°)
-45°	990 (-20°)	530 (-150°)	1142	830	774	900 (100°)	485 (115°)
45°	1166 (20°)	485 (-55°)	1116	791	656	354 (40°)	524 (-150°)

To fully determine the reflection coefficients of the cavity (Γ_c) and the transmitters (Γ_g); we use a combination of analytical computation and graphical solutions where we use phasor diagrams and loci for different phasors. To illustrate that we will consider the phasor diagram in Fig. 2 which represents a sample case (cavity at 45°).

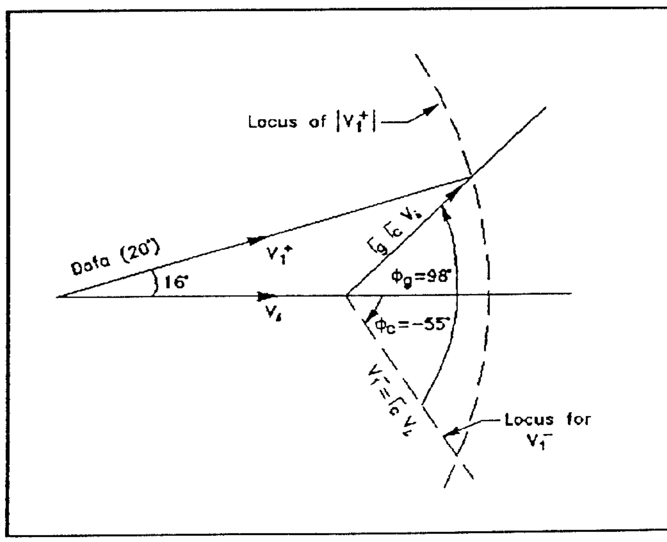

Figure 2. Phasor diagram for graphical solution

If we consider the phasor V_i as the reference, we can find ϕ_g (phase angle for Γ_g) and ϕ_c (phase angle for Γ_c) as shown in Fig 2. Using the data in Table 1, the graphical solution gives

$$\Gamma_c = 0.7\ \angle -55^\circ \quad \textit{and} \quad \Gamma_g = 0.83\ \angle 98^\circ$$

Data measured were generally consistant with predictions from the analytical model represented in Eq. (6), except for a systematic phase difference of 10°.

III. THE 90°-HYBRID AS A CIRCULATOR FOR THE VUV RING HARMONIC CAVITY

A fourth-harmonic cavity is being used in the VUV-storage ring at the NSLS to change the shape of the bucket potential. The harmonic cavity would present a mismatched load to a generator if operated actively. The need exists for a circulator to steer any power reflected from the harmonic cavity to a waster load. In this section, we demonstrate the use of the 90°-hybrid as circulator. We use the parameters for the harmonic cavity given below as a numerical example [3],.

Main Cavity	*Harmonic Cavity*
V_{MC} = 80 KV	V_{HC} = 20 KV
Ψ_s = 79°	Ψ_H = -92.8°
(Rsh = 1 MΩ)	(Rsh = 300 kΩ)

To achieve this goal, we will analyze one of three approaches that we considered [4] for the active operation of the VUV harmonic cavity. In the tuner-compensation approach, the cavity is phased such that the beam current is in quadrature with the cavity gap-voltage. The tuner is operated to compensate for the beam-induced reactive power. The generator current is in-phase with the gap voltage. Lowest generator power requires matched coupling loop to the cavity. Since other practical considerations may require deviations from this ideal case; we have analyzed a general case where coupling can be made different from matched. Referring to Fig 3, we can define the coupling coefficient as

$$\beta \equiv \frac{R_{sh}}{n^2(50)} \tag{7}$$

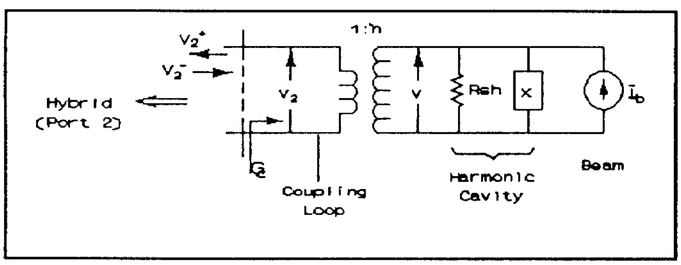

Figure 3. Cavity-port equivalent circuit.

For the conditions discussed above, and analyzed in detail in [4], we can obtain the powers at the four ports as,

Cavity Power: The power dissipated in the cavity is

$$P_w = \frac{1}{2}\frac{V^2}{50\, n^2\, \beta} = \frac{1}{2}\frac{V^2}{R_{sh}} \tag{8}$$

Load Power The power dissipated in the waster load is

$$P_l = P_w \left[\frac{(\beta - 1)^2}{4\beta}\right] \Gamma_g^2 \tag{9}$$

Transmitter A

$$P_A = P_w \frac{[(\beta + 1) + \Gamma_g (\beta - 1)]^2 - (\beta - 1)^2}{4\beta}$$

Transmitter B

$$P_B = P_w \frac{[(\beta + 1) - \Gamma_g (\beta - 1)]^2 - (\beta - 1)^2}{4\beta}$$

Combined Generator Power

The combined power from transmitter A and transmitter B is

$$P_g = P_A + P_B = P_w \frac{[(\beta + 1)^2 - (\beta - 1)^2 (1 - \Gamma_g^2)]}{4\,\beta} \tag{10}$$

From the above equations the power balance gives,

$$P_g - P_l = P_w \tag{11}$$

From this analysis it can be shown that, under certain conditions (for example the case where the generators can be considered as ideal current sources ,Γ_g = 1), the hybrid's operation is identical to that of a ferrite circulator [4].

IV. CONCLUSION

In this report we have analyzed in detail the performance of a 90°-hybrid. The analysis is based on 4-port scattering matrices. Analytical results were compared with experimental measurements. Graphical solutions based on the measured data gave good agreement with our analytical results. The use of the 90°-hybrid as a circulator was demonstrated using the harmonic cavity in the NSLS VUV-ring as an example.

Acknowledgement

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