Impedance Calculations for a Coaxial Liner

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Abstract

Cold beam pipes like in SSC or LHC require shielding tubes with pumping holes called liner. As a first approximation instead of holes a tube with an arbitrary number of rotationally symmetric interruptions is analyzed using the mode-matching-technique. Results are presented for the longitudinal coupling impedance.

I Introduction

For cold beam pipes like in SSC or LHC coaxial waveguides are required where the inner conductor shields the outer surface from synchrotron radiation. The pumping holes in the inner conductor represent a coupling impedance of the beam to these discontinuities. For small holes this impedance can be obtained according to the Bethe theory of diffraction [1]. To avoid limitations w.r.t. the size and number of holes as well as the thickness of the inner conductor we use the mode matching technique to get an accurate solution.
To fit the surfaces of the structure to coordinate surfaces $\varrho, \varphi, z = const.$ we have to restrict ourself to holes with rectangular cross section (Fig.1a). But as a first approximation we will treat the simplified arrangement as shown in Fig.1b. Later the analysis described below will be extended to the really three dimensional problem of holes with rectangular cross section.

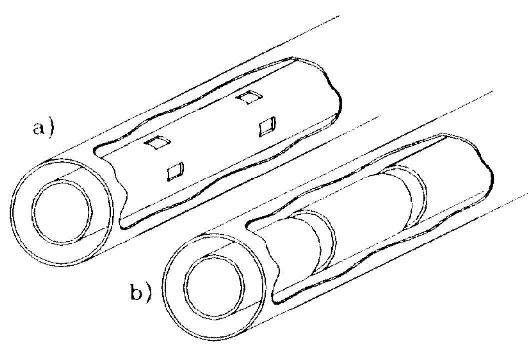

Figure 1: scetch of a liner

II Scattering matrix and excitation vector for one single cell

Let us assume a charged particle Q moving with the velocity of light parallel to the axis with an offset r. The m-th azimutal spectral components of the electromagnetic field in the frequency domain are well known if the surrounding vacuum chamber is smooth. To take the discontinuities into account we first separate the whole structure into cells (Fig.2). Due to the symmetry of one segment we split the exciting field E_s in two standing waves with a phase shift of 90^o w.r.t. time and position

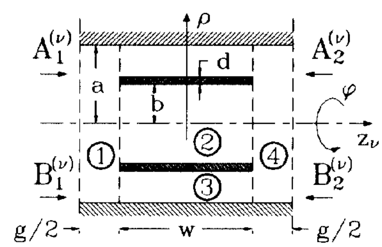

Figure 2: one segment of the rotational symmetric liner

$$\begin{matrix} E_s' \\ E_s'' \end{matrix} \sim \frac{1}{2} e^{-j(k_0 l/2+\Phi_\nu)} \{ e^{-jk_0 z_\nu} \pm e^{jk_0 z_\nu} \} \qquad (1)$$

where $k_0 = \omega / c_0$ and $\Phi_{\nu+1} = \Phi_\nu + k_0 l$.
In- and outgoing TE and TM waves exist at the considered two ports of one segment. The amplitudes of them we call $\mathbf{A}_{1,2}$ and $\mathbf{B}_{1,2}$ respectively. Now we introduce the wave amplitudes $\mathbf{A}'$, $\mathbf{A}''$, $\mathbf{B}'$, $\mathbf{B}''$ referring to the excitation (1). Then we have

$$\begin{matrix} \mathbf{A}' \\ \mathbf{A}'' \end{matrix} = \frac{\mathbf{A}_1 \pm \mathbf{A}_2}{2} \, ; \quad \begin{matrix} \mathbf{B}' \\ \mathbf{B}'' \end{matrix} = \frac{\mathbf{B}_1 \pm \mathbf{B}_2}{2} \, . \qquad (2)$$

On the other hand we can write the relation between $\mathbf{A}$ and $\mathbf{B}$ in matrix form

$$\begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{12} & \mathbf{S}_{11} \end{pmatrix} \begin{pmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{pmatrix} + \begin{pmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{pmatrix}$$
$$\begin{pmatrix} \mathbf{B}' \\ \mathbf{B}'' \end{pmatrix} = \begin{pmatrix} \mathbf{S}' & 0 \\ 0 & \mathbf{S}'' \end{pmatrix} \begin{pmatrix} \mathbf{A}' \\ \mathbf{A}'' \end{pmatrix} + \begin{pmatrix} \mathbf{W}' \\ \mathbf{W}'' \end{pmatrix} \qquad (3)$$

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After calculation of the reflection matrices $\mathbf{S}'$, $\mathbf{S}''$ and the excitation vectors $\mathbf{W}'$, $\mathbf{W}''$ as described below the matrices $\mathbf{S}_{11}$, $\mathbf{S}_{12}$ and vectors $\mathbf{W}_1$, $\mathbf{W}_2$ become

$$\begin{matrix}\mathbf{S}_{11}\\ \mathbf{S}_{12}\end{matrix} = \frac{\mathbf{S}' \pm \mathbf{S}''}{2} \;;\; \begin{matrix}\mathbf{W}_1\\ \mathbf{W}_2\end{matrix} = \mathbf{W}' \pm \mathbf{W}'' \,. \qquad (4)$$

Now we make the following ansatz in subspace 1 of Fig.2 for the beam induced transverse electromagnetic field with even symmetry using a compact matrix notation

$$\vec{E}_{t,1}^{(\nu)'} = \tfrac{Z_0 Q}{2\pi a}\vec{\mathbf{F}}^T(\tfrac{\varrho}{a},\varphi)\left\{\mathbf{Z}^+(z_\nu)\mathbf{A}_1^{(\nu)'} + \mathbf{Z}^-(z_\nu)\mathbf{B}_1^{(\nu)'}\right\}$$

$$\vec{e}_z \times \vec{H}_{t,1}^{(\nu)'} = \tfrac{Q}{2\pi a}\vec{\mathbf{F}}^T(\tfrac{\varrho}{a},\varphi)\mathbf{Y}_1\left\{\mathbf{Z}^-(z_\nu)\mathbf{B}_1^{(\nu)'} - \mathbf{Z}^+(z_\nu)\mathbf{A}_1^{(\nu)'}\right\}$$

$$\mathbf{Z}^{\pm} = \exp(\mp \mathrm{j}\mathbf{K}_1[z_\nu + l/2]) \qquad (5)$$

where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$. In (5) we have defined

- the vectorial column matrix

$$\vec{\mathbf{F}}(\xi,\varphi) = \vec{e}_\varrho \mathbf{F}_\varrho(\xi,\varphi) + \vec{e}_\varphi \mathbf{F}_\varphi(\xi,\varphi) \qquad (6)$$

  with elements

$$\begin{aligned} F_{\varrho,i}^{TE} &= \cos(m\varphi)\frac{m}{\varrho}J_m(j'_{mi}\xi)\\ F_{\varrho,i}^{TM} &= \cos(m\varphi)j_{mi}J'_m(j_{mi}\xi)\\ -F_{\varphi,i}^{TE} &= \sin(m\varphi)j'_{mi}J'_m(j'_{mi}\xi)\\ -F_{\varphi,i}^{TM} &= \sin(m\varphi)\frac{m}{\varrho}J_m(j_{mi}\xi)\end{aligned}$$

- the diagonal matrices $\mathbf{K}_1$ and $\mathbf{Y}_1$ with the modal wave numbers and wave admittances respectively

$$K_{1,i}^{TE} = \sqrt{k_0^2 - j'^2_{mi}/a^2} \quad ; \quad Y_{1,i}^{TE} = K_{1,i}^{TE}/k_0$$
$$K_{1,i}^{TM} = \sqrt{k_0^2 - j^2_{mi}/a^2} \quad ; \quad Y_{1,i}^{TM} = k_0/K_{1,i}^{TM}$$

J_m are the Besselfunctions, J'_m their derivatives and j_{mi}, j'_{mi} their zeros. The superscript (^{T}) describes the transpose of a matrix. In a similar way we can make an ansatz for subspace 2 and 3. Note that in subspace 3 a TEM-field must be taken into account. After matching the field in the planes $z_\nu = \pm w/2$ we get the matrices $\mathbf{S}'$, $\mathbf{S}''$, and vectors $\mathbf{W}'$, $\mathbf{W}''$. All appearing coupling integrals can be solved analytically. For example the coupling matrix for the transition from subspace 2 to subspace 1 reads

$$\mathbf{M}_{21} = \frac{1}{ab}\int_0^b \int_0^{2\pi} \vec{\mathbf{F}}(\tfrac{\varrho}{b},\varphi)\cdot\vec{\mathbf{F}}^T(\tfrac{\varrho}{a},\varphi)\varrho\,\mathrm{d}\varphi\,\mathrm{d}\varrho \qquad (7)$$

III Field of the entire structure

Let us now consider an infinitely long shielding tube with N interruptions. This can be simulated by combining $N-2$ cells, as shown in Fig.2, and two cells at the end of our tube

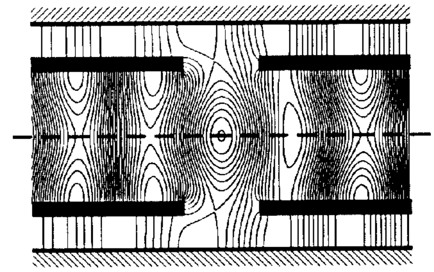

Figure 3: lines of force for two segments, $k_0 a = 1.76$

with $w \to \infty$. Then, the relation between incoming and outgoing waves at port 1 of segment $\nu+1$ is given by

$$\mathbf{B}_1^{(\nu+1)} = \mathbf{R}^{(\nu+1)}\mathbf{A}_1^{(\nu+1)} + \mathbf{V}^{(\nu+1)} \qquad (8)$$

where $\mathbf{R}$ and $\mathbf{V}$ include the influence of the entire structure to the right of segment $\nu+1$. Taking into the account the ν'th segment results in the recurrence formula

$$\mathbf{R}^{(\nu)} = \mathbf{S}_{11}^{(\nu)} + \mathbf{S}_{12}^{(\nu)}\mathbf{Q}^{-1}\mathbf{R}^{(\nu+1)}\mathbf{S}_{12}^{(\nu)}$$
$$\mathbf{V}^{(\nu)} = \mathbf{W}_1^{(\nu)} + \mathbf{S}_{12}^{(\nu)}\mathbf{Q}^{-1}(\mathbf{R}^{(\nu+1)}\mathbf{W}_2^{(\nu)} + \mathbf{V}^{(\nu+1)})$$
$$\mathbf{Q} = 1 - \mathbf{R}^{(\nu+1)}\mathbf{S}_{11}^{(\nu)} \quad . \qquad (9)$$

Iterating through the whole structure we get finally at the interface between segment 1 and 2 the equations

$$\{1 - \mathbf{R}^{(1)}\mathbf{R}^{(2)}\}\mathbf{B}_2^{(2)} = \mathbf{R}^{(1)}\mathbf{V}^{(2)} + \mathbf{V}^{(1)}$$
$$\mathbf{A}_2^{(1)} = \mathbf{R}^{(2)}\mathbf{B}_2^{(1)} + \mathbf{V}^{(2)} \quad . \qquad (10)$$

The wave amplitudes at all other interfaces can now be determined by a forward recurrence using (3).
As an example Fig.3 shows the lines of force for N=2.

IV The coupling impedance

Here, we will restrict ourself to the evaluation of the longitudinal coupling impedance for $m = 0$. Using the abbreviation

$$\mathcal{F}_{1,2}^{(\nu)}(z) = -\mathrm{j}k_0(a,b)^2\sum_i \frac{1}{j_{0i}^2}\left[E_{z1,2}^{(\nu)}\right]_i + \frac{Z_0 Q}{2\pi}\ln\frac{a}{b} - U_{TEM}^{(\nu)} \qquad (11)$$

we get

$$Z = -\tfrac{1}{Q}\sum_{\nu=1}^{N-1}\left\{\mathcal{F}_2^{(\nu)}([\nu-1]l - \tfrac{g}{2}) - \mathcal{F}_1^{(\nu+1)}([\nu-1]l - \tfrac{g}{2})\right.$$
$$\left. + \mathcal{F}_1^{(\nu+1)}([\nu-1]l + \tfrac{g}{2}) - \mathcal{F}_2^{(\nu+1)}([\nu-1]l + \tfrac{g}{2})\right\}. \qquad (12)$$

It can easily be shewn that U_{TEM} in (10) is essentially given by the voltage between the inner and outer conductor, i.e.

$$U_{TEM}^{\nu} = e^{\mathrm{j}k_0 z}\int_{b+d}^{a} E_{\varrho 3}^{(\nu)}\,\mathrm{d}\varrho \qquad (13)$$

where only the TEM field contributes. $[E^{(\nu)}_{z1,2}]_i$ describes the i'th mode of the longitudinal electric field in subspace 1 and 2 respectively.
Fig.4 - Fig.7 show impedances for some configurations. Note that the value in Fig.4 for $\omega \to 0$ is given by

$$\Re[Z(\omega \to 0)] = \frac{Z_0}{\pi} \ln \frac{a^2}{b^2 + bd} \quad .$$

The frequency independent behaviour of the real part of the impedance in Fig.4a for $d = 0$ agrees with the results in [2].

V Conclusion and outlook

In the present paper we have proposed an analytical method to calculate the impedance of annular interruptions in the inner conductor of a coaxial waveguide. The written computer code is an extension of existing codes. The advantage of the method used here is that we can change transverse dimensions from cell to cell. So in principle we are also able to analyse the so called detuned iris loaded waveguide while choosing $b + d = a$. In this case and if a, b are constant for all cells, we obtain exactly the same results as presented in [3].
The generalization to holes with rectangular cross section will be presented next.

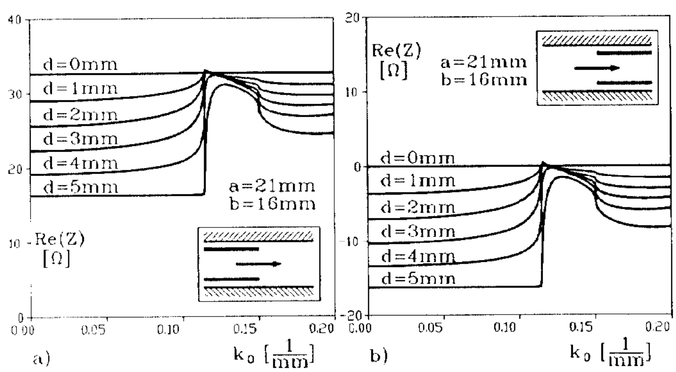

Figure 4: real part of the impedance of a single discontinuity a) for a charge leaving the inner conductor, b) entering the inner conductor

References

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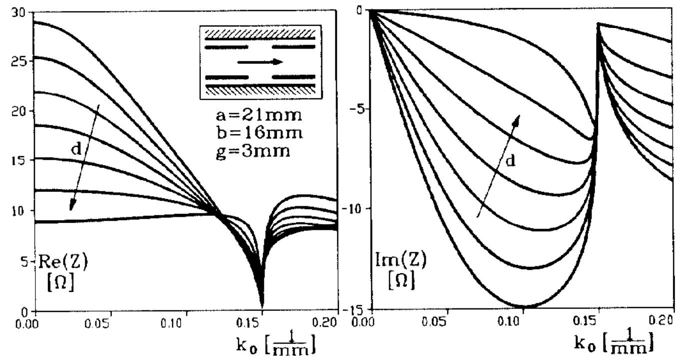

Figure 5: impedance of an annular interruption in the inner conductor with d=0.5 ,1.0 ,1.5 ,2.0 ,2.5 ,3.0 ,3.5 mm

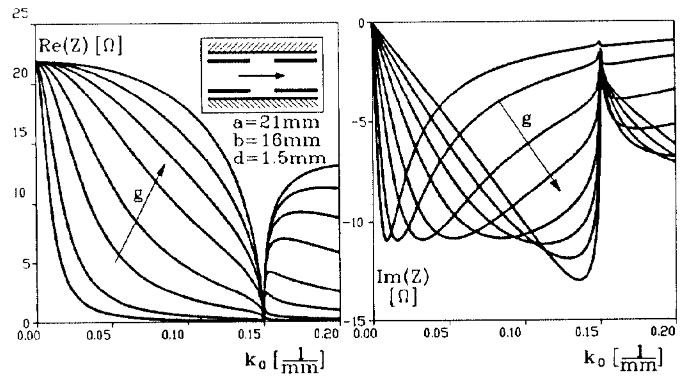

Figure 6: as Figure 5, but with varying gap in the inner conductor g=0.1, 0.5, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0 mm

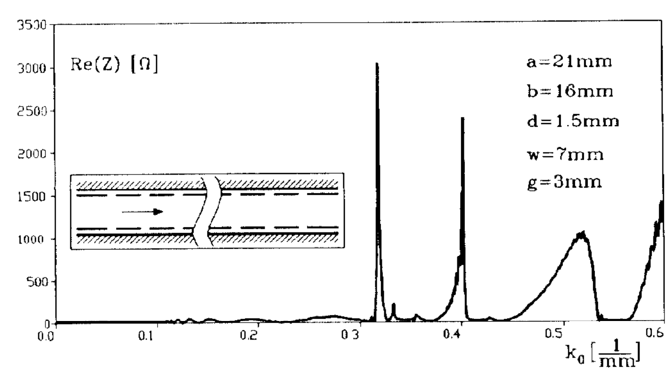

Figure 7: real part of the impedance for 20 annular interruptions