A COAXIAL-TYPE ACCELERATING SYSTEM WITH AMORPHOUS MATERIAL

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## Absrtact

In order to accelerate low-energy protons and heave-ions, a wide accelerating frequency range is necessary. Such applications of accelerating structures with amorphous cores loaded cavity have been studied by several laboratories. In this article a method of computation the impedance of a coaxial cavity loaded with amorphous toroidal cores is described. In this computation is shown, what a dominant role is played by the skin-effect in a core with a foliated structure. As a result of an analysis of electric and magnetic fields distribution, the cavity impedance is computed. The impedance depends on capacity and magnetic permeability, having real and imaginary parts. These parts depend on frequency, the sheet thickness, conductivity, and on permeability of the material at zero frequency. It is important that this impedance may be stabilized in some frequency range. Applications of accelerating systems of this type are discussed in different cases..

## I. INTRODUCTION

A very high magnetic permeability of amorphous metals permits to use it in various accelerating systems [1],[2]. In these systems a coaxial cavity louded with amorphous toroidal cores having a foliated structure is used. In case of homogenous material with zero conductivity havino constant magnetic and electric permeability in a coaxial-type cavity, electric and magnetic fields in most part of cavity have only  $\not \in h$  and  $\not H \varphi$  components on T mode. But in case of amorphous toroidal cores with a foliated structure and non-zero conductivity the longitudinal components electric field  $E_Z$  and current  $J_Z$ are presented too. As a result electromagnetic fields are non-uniform in each layer of metal. This skin-effect is small usualy in low frequency transformes, but it can play a dominant role at radio-frequency range. As a result of the analysis of electric and magnetic fields in layers of the metal and in isolation between layers the input cavity impedance will be calculated over wide frequency range.

# II. ANALYSIS OF THE FIELDS AND GENERAL RESULTS

The coaxial cavity with the foliated structure is shown in Figure 1. The time harmonic azimuthal symmetric mode must satisfy the Maxwell equations which can be solved in local layers of metal or isolation.

$$E_{z} = (\alpha_{s}/s_{s}) Z_{o} e^{j(\omega t - \kappa z)} (I)$$
$$E_{r} = (j\kappa/s_{s}) Z_{I} e^{j(\omega t - \kappa z)} (2)$$

$$H_{\varphi} = Z_{i} e^{j(\omega t - \kappa z)} \qquad (3)$$

$$H_{P|r=a} = I/2\pi a \qquad (4)$$

where

$$Z_{o} = A_{s} J_{o} (a_{s} r) + B_{s} N_{o} (a_{s} r)$$

$$Z_{i} = A_{s} J_{i} (a_{s} r) + B_{s} N_{i} (a_{s} r)$$

$$a_{s}^{2} = \omega^{2} \mathcal{E}_{s} \mathcal{H}_{s} - j \mathcal{H}_{s} \mathcal{E}_{s} \omega - \kappa^{2}$$

$$S_{s} = \mathcal{E}_{s} + j \mathcal{E}_{s} \omega; \quad S=1, 2, ...$$

 $\mathcal{M}_{S}$ ,  $\mathcal{E}_{S}$ ,  $\mathcal{B}_{S}$  - are magnetic permeability at zero frequency, dielectric permeability and conductivity in layer;  $\mathcal{J}_{O,I}$ ,  $\mathcal{N}_{O,I}$  - are the Bessel functions;  $\mathcal{A}_{S}$ ,  $\mathcal{B}_{S}$  - are constants for the layer; I - is the current in the inner tube.



Figure 1. Coaxial-type cavity with foliated toroidal cores.

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In Figure 1 we can see three layers (dielectric-metal-dielectric) at one radial period of core. We assume that in case of foliated structure the longitudinal component electric field is equale to zero just on the local surfaces.

$$E_{z(r_{i})} = E_{z(F)} = E_{z(r_{i})} = 0$$
 (5)

We assume the continuity the tangential components electric and magnetic fields on bounderies between metal and isolation as well. In such case the longitudinal current  $\mathcal{J}_{\mathcal{Z}}$  passing in opposite directions in two zones of metal, and force lines of electric field have "snake-type" shape. This current passes from zone  $\mathcal{M}_{\mathcal{L}} \mathcal{F}$  to  $\mathcal{F}_{\mathcal{L}} \mathcal{F}$ along end face of the layer, and the gaps between cores do not disturb picture of the field. Using equations (1), (3), (5) and continuity the fields, we can write the linear equations system which contains six constants  $\mathcal{A}_{\mathcal{S}}$ ,  $\mathcal{B}_{\mathcal{S}}$ . Using this system we can write the dispersion equation in the following general form:

$$D = |Z_{m,n}| = 0$$
(6)  
m, n = 1, 2, 3... 6

where

D - is a determinant of the linear system, Zmn - are the Bessel functions with coefficient, or zero. During further investigations we assume fulfilment of the following condutions:

$$w \mathcal{E}_2 \ll \mathcal{E}_2$$
;  $1 \ll d_2 \mathcal{M}$  —in the metal;

 $A, A, \mathcal{L}_{1}, \mathcal{H}_{1} = \mathcal{H}_{0}; \mathcal{H}_{1} = \mathcal{O} - \text{in the dielectric;}$  $A_{1,2}/\mathcal{L}_{1}, A_{1,2}/\mathcal{L}_{1} = \text{in cores as a whole.}$ 

Using asymptotic form of the Bessel functions in zone of metal and expansion in series in zone of dielectric [3] we can present equation (6) in the following form:

$$G_4 d_1^4 + G_2 d_1^2 + G_0 = 0$$
 (7)

where

$$G_{4} = j(\Delta_{1}^{2}/4)(\mathcal{E}_{2}/\mathcal{E}_{1}\omega) \operatorname{sin}_{2}P/2P$$

$$G_{2} = (\Delta_{1}/\Delta_{2}) \operatorname{cos}_{2}P$$

$$G_{0} = \omega^{2}\mathcal{E}_{1}\mathcal{M}_{2} \operatorname{sin}_{2}P/P$$

$$P = (\Delta_{2}/2)(-j\mathcal{M}_{2}\mathcal{E}_{2}\omega)^{\frac{1}{2}} \qquad (8)$$

Dispersion equation (7) can be solved relative to  $\alpha_i$  and to

wave vector K

$$K^{2} = \omega^{2} \mathcal{E}_{i} \left[ \mathcal{M}_{o} + (\Delta_{z} / \Delta_{i}) \mathcal{M}_{z} t_{g} P / P \right] \quad (9)$$

Of equations (1), (3), (5), (7) we recieve the following consequence as well.

$$r_1 H_{\varphi}(r_1) = r_4 H_{\varphi}(r_4) \qquad (10)$$

As a result of relation (10) in dielectric dependence analogous to T mode in case with the homogenous material is obtained. But in metal drop of the magnetic field is observed. This drop depends on  $\mathcal{P}$  (B) because of the skin-effect. Using these result and addition condition (4) we can computate intensity of vortex U between inner and outer tubes. Summation of two waves having wave vector  $\pm K$ gives the cavity input impedance  $\mathbb{Z}$  and vector admittance

$$I U^{-1} = I \left[ \int_{a}^{b} E_{r} dr \right]^{-1} = Z^{-1} = Y \quad (11)$$

where

$$Y = -j \frac{2 \mathcal{T}}{l_n(6/q)} \sqrt{\frac{\tilde{\mathcal{E}}}{\tilde{\mathcal{X}}}} C t g(KL) \quad (12)$$

$$\widetilde{\mathcal{E}}=\mathcal{E}_1\left(\Delta_1+\Delta_2\right)/\Delta_1 \qquad (13)$$

$$\widetilde{\mathcal{M}} = \frac{\Delta_{i} \mathcal{M}_{0}}{\Delta_{i} + \Delta_{2}} + \frac{\Delta_{2} \mathcal{M}_{0}}{\Delta_{i} + \Delta_{2}} \frac{\pm g P}{P} \qquad (14)$$

$$\frac{lgP/P=P[1-llg\delta]}{p=(3h\overline{P}+3ln\overline{P})[\overline{P}(Ch\overline{P}+CO3\overline{P})]^{-1}}$$

$$\frac{lg\delta}{lg\delta}=(3h\overline{P}-3ln\overline{P})(3h\overline{P}+3ln\overline{P})^{-1}$$

$$\overline{P}=\Delta_2 (\mathcal{M}_2 \mathcal{S}_2 \mathcal{W}/2)^{1/2}$$

$$\frac{lg\delta}{lg\delta}=(2\mu_2 \mathcal{S}_2 \mathcal{W}/2)^{1/2}$$

In these formulas  $\mathcal{K} = \mathcal{O}(\mathcal{EH})^{n/2}$  is the complex wave vector (9);  $\mathcal{E}$  - is the resulting dielectric permeability;  $\mathcal{H}$  - is the resulting magnetic permeability in the foliated core. Formula (14) generalizes result [4] in case of foliated core.

#### III. THE SPECIAL CASES

On of the typical conditions:

$$\Delta_2 \simeq 10^{-5} \text{m}; \quad \omega = 2\pi \cdot 10^6 \text{s}^{-1}; \\ \mathcal{B}_2 \simeq 10^6 \text{ohm}^{-1} \text{m}^{-1}; \quad \mathcal{M}_2 = \mathcal{M}_0 \cdot 10^4$$

parametr  $\bar{P} \simeq 2$  and as a result  $\ell g S \simeq 1$ . In this case in zone  $\phi < 1$  ("short cavity") the impedance and the general formulas (12), (14) will be as follows:

$$\widetilde{\mathcal{M}} = |\widetilde{\mathcal{M}}| e^{-\widetilde{\mathcal{J}}\widetilde{\mathcal{T}}/4}$$
(15)

$$Z' = Y = \frac{2 \pi}{\ell n (6/0)} \sqrt{\frac{\tilde{\varepsilon}}{1 \tilde{\mathcal{U}}_{1}}} \left[ \Psi_{1} + j \Psi_{2} \right]$$
(16)

where

$$|\tilde{\mathcal{A}}| = (\Delta_1 + \Delta_2)^{-1} \sqrt{\frac{2 M_2}{97 G_2 f}}$$
 (17)

$$Y_{1} = A \cos(\pi 7/8) + B \sin(\pi 7/8)$$

$$Y_{2} = A \sin(\pi 7/8) - B \cos(\pi 7/8)$$

$$A = Sh \Theta_{s} / [Ch \Theta_{s} - CO3 \Theta_{c}]$$

$$B = \sin \Theta_{c} / [Ch \Theta_{s} - CO3 \Theta_{c}]$$

$$\Theta_{s} = 2 \Theta_{s} in(\pi 7/8); \Theta_{c} = 2 \Theta_{c} \cos(\pi 7/8)$$

$$\Phi = 1 K | L = 2\pi f (E | Z_{1})^{1/2} L$$

frequency The result (17) gives attenuatio characteristic  $|\mathcal{M}|$  at high frequency. Bath parts cooputation result of the vector admittance is shown in Figure 2.



Figure 2. Characteristics of the vector admittance and shunt impedance  $R_{S} \sim \Psi_{i}^{-1}$ 

vector admittance will be as follows:

$$Z^{-1} = Y = \frac{\Delta_1 + \Delta_2}{2L \ln(\ell/q)} \sqrt{\frac{\pi \delta_2}{M_2 f}} (1-j)$$

In zone  $\mathcal{3} \prec \mathcal{O}$  ("long cavity") the impedance and vector admittance are low-dependent on  $igsildsymbol{eta}$  or  $igsildsymbol{\mathcal{F}}$  as a result of an attenuation over the length  $\angle I$  . In this zone impedance matching with generator over wide frequency range is more simple, but the losses may be high.

Resonance takes place only in one point  $arphi \! pprox \! \mathscr{T} / \! \mathscr{L}$ . In this zone the losses are low, but impedance matching may be more complicated. The operating zone and cavity length may be selected with taking into account the acceleraiting gap capacity Cq

$$\overline{Z}^{-1} = \overline{Y} = Y + j_{2} \mathcal{T} f C_{g}$$

dissipative power  $P_d$ 

$$P_{d} = (U^{2}R_{s})/2 = \frac{U^{2}T}{\ell_{n}(\delta/a)} \sqrt{\frac{\mathcal{E}}{|\tilde{\mathcal{U}}|}} \Psi_{i},$$

and output generator impedance.

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