# NON LINEAR EFFECTS IN FERRITE TUNED CAVITIES

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#### Abstract

The phenomenon of dependence of the resonance shape and frequency on the RF power level in perpendicular biased ferrite-tuned cavities has been observed by G. Hulsey and C. Friedrichs in the the SSC test cavity experiment. This paper presents a theoretical as well as numerical analysis of this phenomenon and compares the results with experimental data. The effect of this nonlinearity on the SSC low energy booster prototype cavity is discussed.

### I. INTRODUCTION

The tuning of the rf cavities in the SSC Low Energy Booster (LEB) and in the TRIUMF Kaon factory is done using Perpendicular-Biased yttrium-garnet Ferrites (PBF) [1],[2]. In this configuration the rf magnetic field inside the ferrite is perpendicular to the bias magnetic field. The main advantage of the PBF configuration over the more conventional parallel-biased Ni-Zn ferrites is the higher achievable magnetic quality value  $(Q_m)$ , which in turn allows for higher energy densities and gap voltages. The main limitations to even higher gap voltages are the magnetic losses and nonlinear phenomena. The phenomenon of dependence of the resonance shape and frequency on the rf power level in PBF cavities has been observed by G. Hulsey and C. Friedrichs in the the SSC test cavity experiment [3]. This paper analyzes this nonlinearity and correlates it with the dependence of the permeability on the ratio of rf field to bias magnetic field amplitudes.

The paper is divided into five sections. The first section analyzes the permeability dependence on the rf field amplitude. The analysis is carried out in the quasi-static approximation due to the small ratio, in our experiment, of the rf drive frequency to the biased field gyro-frequency. This nonlinear term in the permeability is used in the second sec-

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tion to analyze the dependence of the frequency and resonance shape on the rf power level. Section three compares the analytical and experimental results. The analytical expression fits well with the observed data. Using this comparison we will try to estimate the nonlinear effects expected in the LEB and we conclude with a short summary.

# II. PERMEABILITY DEPENDENCE ON RF FIELD AMPLITUDE

Neglecting dissipation effects the equation of motion for the ferrite magnetization is given by

$$d\mathbf{M}/dt = -\gamma \,\mu_0 \,\mathbf{M} \,\mathbf{x} \,\mathbf{H}_{\rm eff} \tag{1}$$

where  $\gamma$  is the gyromagnetic ratio, which is about 2.8 MHz/Gauss for most materials. In Eq. (1)  $\mu_0$  is the permeability of free space and  $\mathbf{H}_{eff}$  is the effective magnetic field at the magnetization location at time t. To make the analysis tractable we solve equation (1) in the Cartesian coordinate system. We define the external bias field  $H_0$  to be in the z direction and the perpendicular rf field,  $H_{rf}$ cos( $\omega$ t), in the y direction. Neglecting also the difference between the external and the effective magnetic fields, the three components of equation (1) are

$$dM_{x}/dt = -M_{y} \omega_{0} + M_{z} \Omega(t)$$
 (2a)

$$dM_v/dt = M_x \,\omega_0 \tag{2b}$$

$$dM_z/dt = -M_x \Omega(t)$$
 (2c)

where  $\omega_0$  and  $\Omega$  are defined by

$$\omega_0 = \gamma \,\mu_0 \,H_0 \tag{3}$$

 $\Omega(t) = \Omega_{\rm rf} \cos(\omega t) = \gamma \,\mu_0 \,H_{\rm rf} \cos(\omega t)$ 

Eqs. (2a)-(2c) are nonlinear equations for the ferrite magnetization vector which preserve its magnitude  $M_0$ 

$$M_0 = \sqrt{M_x^2 + M_y^2 + M_z^2}$$
(4)

The general solution to eqs.(2a)-(2c) is a Fourier sum on all the harmonics of the drive frequency  $\omega$ . We shall limit the present analysis to the fundamental frequency only. Assume an exponential temporal behavior of the magnetization vector components  $M_x M_y$  in Eqs.(2a) (2b) and making use of Eq. (4) we obtain the following expression for the permeability

$$\mu = \mu_0 + \frac{M_0}{H_0} \cdot \frac{1}{1 - \frac{\omega^2}{\omega_0^2}} - \frac{3}{8} \frac{M_0 H_{r_f}^2}{H_0^3}$$
(5)

where we neglect terms of the order  $\omega^2 \Omega^2 / \omega_0^4$ . The permeability in Eq. (5) decreases with increasing rf field amplitude, causing a frequency upshift in ferrite tuned cavities.

#### III. FREQUENCY AND AMPLITUDE DEPENDENCE ON RF FIELD

To estimate the resonance frequency shift and amplitude distortion as a result of the permeability dependence on the rf field we shall follow Slater's analysis [4]. We assume that the rf fields near resonance can be described by

$$E = V(t) E_{r}(x)$$

$$H = I(t) H_{r}(x)$$
(6)

where  $E_{\rm T}$  and  $H_{\rm r}$  are the resonance eigenfunctions for the eigenvalue  $\omega_{\rm r}$  Substituting eq. (6) in Maxwell's equations we end with the following equation for the current I(t) in the cavity

$$D = \omega_{\rm r} I (1+3\beta I^2/2) + \omega_{\rm r}/2Q \dot{I} + \ddot{I} - 3\beta I \dot{I}^2$$
(7)

where D is a drive function, Q is the cavity quality value and  $\beta$  is defined by

$$\beta = \mu_0 M_0 / H_0^3 \int_T d^3 x H_r^4 / W_m$$
(8)

The above integral is evaluated over the tuner volume and  $W_m$  is defined by

$$W_m = \int d^3x \,\mu_0 (1 + M_0/H_0) \,H_r^2$$

Assuming a periodic drive function of the form  $D = \overline{D}\cos(\omega t)$ 

in Eq. (7) and following Landau and Lifshitz analysis [5], we find that the resonance frequency

shift is

$$\Delta \omega_{\rm r} / \omega_{\rm r} = 3/16 \,\beta \bar{\rm I}^2(\omega_{\rm r}) \tag{9}$$

where  $\bar{I}$  is the magnetic field amplitude satisfying the following equation

$$\bar{I}^{2} = \frac{\bar{D}^{2}}{4\omega_{r}^{2}} \frac{1}{(\omega_{r} + \Delta\omega_{r} - \omega)^{2} + \omega_{r}^{2}/(16Q^{2})}$$
(10)

Equation (9) describes a second order dependence of the the frequency shift on the rf amplitude which is directly related to the gap voltage  $V_{gap}$ . It also shows an inverse dependence of the shift on the third power of the bias magnetic field. The nonlinear resonance shape is described in Eq. (10).



Figure 1. Frequency shift vs. maximum gap voltage  $I_b = 105 \text{ A}$ 

#### IV. COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

Figure (1) plots the frequency shift vs. cavity gap voltage for 105 A bias current. The stars are the experimental points and the continuous curve is a fit of the form

$$\Delta f / f = 7.0 \times 10^{-14} V_{gap}^{2}$$
 (11)

where  $V_{gap}$  is in volts. An excellent fit with experimental data is observed. To numerically estimate the coefficient of proportionality between the frequency shift and the gap voltage we used the 3D electromagnetic code HFSS. We end up with the value  $1.7*10^{-14}$  for this particular biasing (about four times lower than the experimental result). The discrepancy can be attributed to the sensitivity of this coefficient on the voltage transformation ratio between tuner to gap. Also a very good fit has been observed in the dependence of the frequency shift on bias magnetic field using the same correction factor.

Figure (2) presents the gap voltage dependence on frequency for various tetrode current drives using Eq. (10) with the transformation of I to  $V_{gap}$ . A resonance shift as well as resonance distortion with increase drive is observed.



Figure 2. Gap voltage vs. frequency for various drives

Figure (3) presents two oscilloscope traces of the gap voltage vs. frequency. The symmetric case relates to maximum gap voltage of 7 kV and the asymmetric one to 70 kV. The traces can be described very well with Eq. (10) using the frequency shift of Eq. (9).



Figure 3. Oscilloscope traces for gap voltages of 7 and 70 kV

Making use of the theory presented in sections II and III together with the experimental correction factor we estimated the frequency shift expected in the LEB. A maximum shift of 23.6 KHz is expected at 50 MHz for about 100 kV gap voltage.

## V. SUMMARY

A theoretical analysis of the dependence of the resonance frequency on rf power level in PBF cavities is presented. A good fit with experimental data is obtained. A correction factor of 4.12 is needed to fit the numerical results with experiment. The results of this analysis has been used to predict a maximum frequency shift of 23.6 KHz in the LEB cavity.

## **VI. REFERENCES**

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