System Modeling for the Longitudinal Beam Dynamics Control Problem in Heavy Ion Induction Accelerators*

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Abstract

We address the problem of developing system models that are suitable for studying the control of the longitudinal beam dynamics in induction accelerators for heavy ions. In particular, we present the *preliminary* results of our efforts to devise a general framework for building detailed, integrated models of accelerator systems consisting of pulsed power modulator circuits, induction cells, beam dynamics, and control system elements. Such a framework will permit us to analyze and design the pulsed power modulators and the control systems required to effect precise control over the longitudinal beam dynamics.

I. INTRODUCTION

An important problem in the design of heavy ion accelerators for the inertial confinement fusion application is the control of the longitudinal beam dynamics. Space-charge-dominated beams are accelerated, compressed, and transported over large distances, and the interaction of the beam with the impedance of the induction modules gives rise to the longitudinal instability [1]-[3].

Conceptually, the acceleration waveforms consist of three components (cf. Fig. 1). First, there are the main acceleration pulses. Then bipolar "ear pulses" serve to compensate for the deleterious effects of space-charge forces. Finally, "fast" pulses correct for errors in the main acceleration waveforms and compensate for the interaction between acceleration modules and the beam or for other disturbances.

The induction cells and pulsed power modulators must be designed to provide these three components. In each case, the required pulses must meet stringent requirements on shape and timing, and these requirements vary as a function of location along the accelerator. Furthermore, accurate pulse-waveform tailoring and timing requires some form of closed-loop feedback control. The performance requirements become even more stringent for recirculators and multi-pulse accelerators. Then issues such as modulator pulse repetition rate, efficiency, cell reset, and pulse-to-pulse stability become critical. Clearly this is a complex analysis and design problem. To achieve a satisfactory design, many design parameters must be adjusted and system performance evaluated in terms of multiple, and possibly, conflicting design objectives.

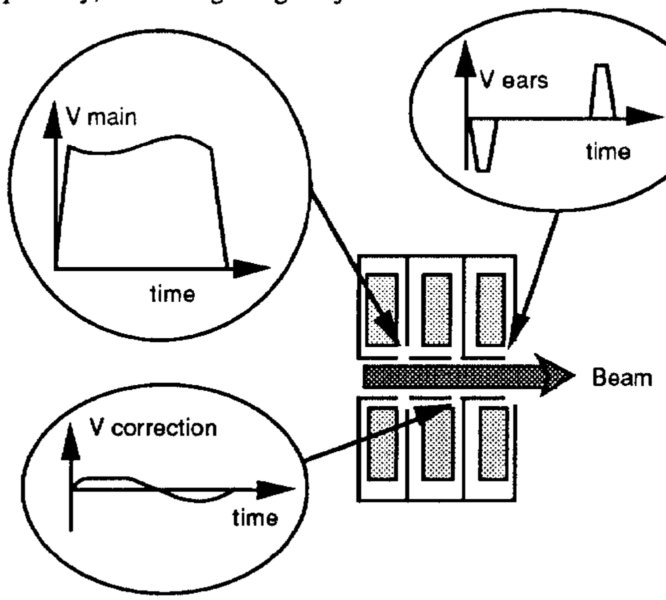

Fig. 1. Waveforms required for control of longitudinal beam dynamics.

To evaluate possible designs for main acceleration modules, ear pulsers and fast waveform correction modulators, the need arises for a computational modeling and design methodology possessing two features. First, it must permit the formulation of high fidelity, integrated system models comprised of multiple subsystems—pulsed power modulator circuits, induction cells, control system circuits, and beam dynamics (cf. Fig. 2). Second, it must provide an efficient means of exercising these models to explore the design parameter space, evaluate design-performance objectives, and arrive at optimal designs.

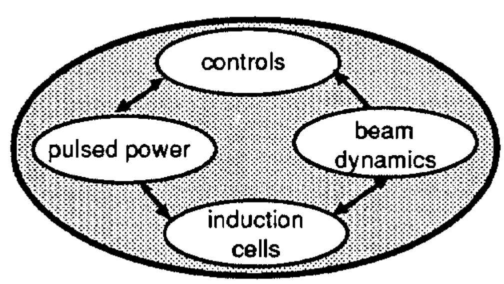

Fig. 2. Integrated system model comprised of four classes of subsystems.

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We are developing a general system modeling and design methodology that is well-suited for studying the engineering issues that arise in the control of longitudinal beam dynamics. This methodology will provide a general framework for building detailed, integrated models of accelerator systems. It will also permit us to design and evaluate possible modulator designs and control strategies and allow us to explore important design issues, such as requirements on sensor accuracy, control system bandwidth, and the number and location of sensors and correction modulators required to achieve satisfactory longitudinal beam control. It will ultimately enable us to apply modern control techniques to the longitudinal beam instability problem. In this paper, we give a brief overview of the methodology and its planned application in the study of the longitudinal beam dynamics control problem.

II. THE METHODOLOGY

The anatomy of the methodology that we are developing is depicted in Fig. 3. It consists of three stages. In the input stage, the system is defined in terms of its topology and its components or elements. Parameter "knobs" by which design parameters can be varied and performance objectives and constraints are also defined at this stage. Then the particular tool is selected for a desired analysis or design task. Next comes the computational stage, in which the system model and the selected analysis or design task is formulated mathematically. These mathematical models are then operated upon by specific algorithms for performing the particular task selected. Finally, the output stage provides the results of the computational phase.

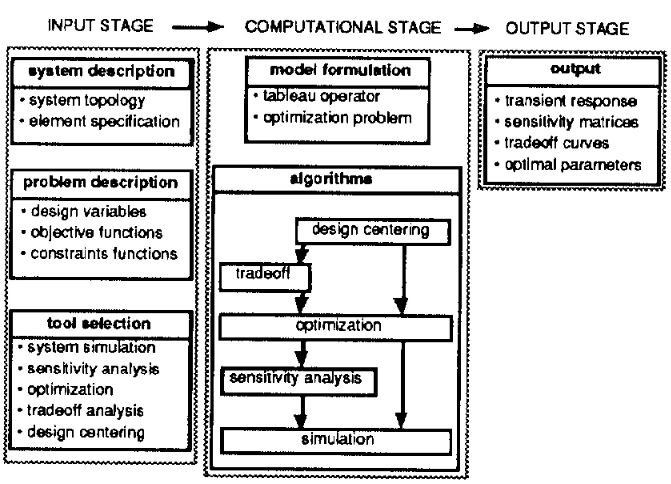

Fig. 3. Simplified anatomy of the methodology.

The analysis or design tasks are performed by one or more of five tools or algorithms, as shown in Fig. 3. The *simulation tool* computes the time-domain responses of all system variables or "states." It is the foundational tool in the sense that it serves as the "calculator" for the remaining four tools. The *sensitivity analysis tool* ascertains the sensitivity of a given point-design to variations in design parameters. The *optimization tool* permits us to tune design parameters in order to optimize a given performance measure. The *tradeoff tool* allows us to identify tradeoffs and arrive at designs that achieve an acceptable compromise among multiple, conflicting design objectives. Finally, the *design-centering tool* seeks design parameter values that insure that design objectives are met even in the presence of component tolerances and parameter variations.

To date, we have built a prototype code called PRISMA, which realizes the simulation tool and provides the capability of building models of systems comprised of diverse subsystem types. We have used this code successfully to analyze pulsed power problems at LLNL, particularly in the area of magnetically-switched modulators [4]-[5]. The code presently possesses a collection of basic circuit component models (e.g., capacitors, inductors, sources, transmission lines, nonlinear magnetic cores) and control system element models (e.g., transfer functions, integrators, saturation, dead zone, etc.). To this repertoire of components, we are now adding those elements that are required to model accelerator systems (e.g. beam dynamics, beam current monitors, acceleration gaps, drift sections, etc.).

The code utilizes a free-format input language for describing the topology and components of the system to be simulated. The user simply describes the system in a "net list," each line of which specifies a component type and its name, its connection points to the system (nodes) and its defining parameters. From this description of the system, the mathematical model of the complete system is constructed.

The simulator employs a sparse tableau formulation [6] of the system equations. In this formulation, the system model takes the form of a system of simultaneous differential-algebraic equations

$$\begin{aligned} \mathbf{f}(\mathbf{x}(t), \dot{\mathbf{x}}(t), t) &= 0 \qquad t \geq t_0 \\ \mathbf{x}(t_0) &= \mathbf{x}_0 \end{aligned} \tag{1}$$

The vector function $\mathbf{f}$ includes the topological constraints of the system (e.g., Kirchhoff's current and voltage laws for circuits) and the component constitutive equations. The vector $\mathbf{x}$ consists of node variables and component variables that are defined or constrained through each component's mathematical model. The advantage a general network and systems formulation has over a problem or application specific formulation is that changes in system topology, components, and parameters can be made and evaluated easily.

We obtain the solution of (1) by discretizing the differentiation operator by a backward differentiation formula. We then solve the resulting system of nonlinear difference equations by a modified Newton-Raphson method, which exploits the sparsity of the Jacobian of $\mathbf{f}$. A stiffly stable, adaptive step-size, adaptive order solver permits the simulation of highly nonlinear and stiff dynamical systems.

III. BEAM MODEL

At present, we model the longitudinal beam dynamics by a simple one-dimensional cold-fluid model [1]-[3], consisting of the continuity equation and the momentum transfer equation, which in the laboratory frame (z,t) take the form

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial z} = 0 \tag{2}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = \frac{F}{m} \tag{3}$$

with

$$I = v\lambda \tag{4}$$

where $\lambda(z,t)$ is the line charge density and $v(z,t)$ is the fluid velocity, and $I(z,t)$ is the total beam current. The force F is given by

$$F = qE - gq\frac{\partial \lambda}{\partial z} + F^{a} \tag{5}$$

where E is the longitudinal field induced by the interaction of the beam current with the induction gaps. In general, interaction of E and I can be modeled by a circuit model of the the induction module. The second term in (5) is the space-charge force, and F^{a} is the force applied by induction cells.

To incorporate the beam model into the tableau equation (1), we transform equations (2)-(5) into a Lagrangian coordinate system and discretized them in both time and space. Eventually, we plan to solve the equations by the method of lines to take advantage of the adaptive time-step algorithm of the stiffly stable solver in PRISMA.

IV. CONCLUSION

Once the beam model and models for other accelerator system components are fully integrated into PRISMA, we plan to study the longitudinal beam control problem. In particular, we plan to use the code to explore control strategies and to analyze modulator designs for ear-pulse and fast correction waveform generation. In the near future we also hope to add the sensitivity analysis and optimization tools, which should greatly enhance the effectiveness of the code as a engineering tool.

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