Correction of Longitudinal Errors in Accelerators for Heavy-Ion Fusion*

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Abstract

Longitudinal space-charge waves develop on a heavy-ion inertial-fusion pulse from initial mismatches or from inappropriately timed or shaped accelerating voltages. Without correction, waves moving backward along the beam can grow due to the interaction with their resistively retarded image fields, eventually degrading the longitudinal emittance. A simple correction algorithm is presented here that uses a time-dependent axial electric field to reverse the direction of backward-moving waves. The image fields then damp these forward-moving waves. The method is demonstrated by fluid simulations of an idealized inertial-fusion driver, and practical problems in implementing the algorithm are discussed.

I. Introduction

Axial confinement of the high-current beams needed for heavy-ion fusion (HIF) must be provided by the accelerating waveforms. The longitudinal electric field required for this confinement ideally is proportional to the axial derivative of the beam line-charge density in the beam frame, and if it could be applied continually, it would have no effect except to balance the axial space-charge force of the beam. However, these so-called "ear" fields can only be applied periodically in induction accelerators, at an amplitude that gives the correct average force. Moreover, the high cost of time-dependent pulsers favors the widest allowable spacing of these "ear cells."

Numerical modeling [1,2] indicates that the periodic application of ear fields initiates low-amplitude space-charge waves near the beam ends, even if the fields are applied every lattice period. The waves moving toward the beam head are shown theoretically to decay, but waves moving back from the head grow exponentially due to the "longitudinal instability," which is driven by the interaction of a line-charge perturbation with its resistively retarded image field. These growing waves can increase the longitudinal emittance of the beam and thereby frustrate the final focus of the beam onto a target. Additional sources of space-charge waves on ion pulses are the inevitable errors in measuring the line-charge density, the imperfect generation of ear fields, and the timing errors in applying them.

In this paper, a simple algorithm is proposed for correcting errors in either the line-charge profile or average longitudinal velocity of a HIF pulse. The method is briefly described in the next section, and it is demonstrated using a one-dimensional fluid code in Section III. Some warnings about the limited applicability of the method are mentioned in a final section.

II. Model

A. Basic Equations

Beam longitudinal dynamics is modeled here by treating slices of the beam as Lagrangian fluid elements. This approach is acceptable for studying longitudinal perturbations because of the long time scales involved and because there are no significant single-particle effects. In adopting a cold-fluid model, we implicitly assume that the beam has a negligible longitudinal temperature and that the slices remain approximately collinear. An approximate equation for the longitudinal velocity v is obtained by retaining only the electrostatic force in the single-particle motion equations and averaging the axial component over the beam cross-section. For a beam with a line-charge density λ, an ion mass M, and charge state q transported in a straight lattice, we obtain

$$\frac{dv}{dt} = \frac{qe}{M}\left(E_{ext} - g\frac{\partial\lambda}{\partial z} - \eta\lambda v\right). \tag{1}$$

Here, E_{ext} is the radially averaged axial component of the external electric field, and the following term accounts for the radially averaged longitudinal space-charge field of the beam, with the coupling factor g being given by

$$g \approx \frac{1}{4\pi\epsilon_0}\ln\left(\frac{R^2}{r_0}\right) \tag{2}$$

for a beam-pipe radius R and a matched beam radius of r_0. In deriving this space-charge field, the radial electrostatic field is assumed to vary over a much shorter scale length than λ, and we have used the fact that the charge density of an equilibrium beam is approximately constant except near the ends. The final term on the right side of Eq. (1) models the electric field that results when the image current in the accelerator wall is retarded due to an average resistance per unit length η [3]. In this simple description, the beam transverse dynamics only enter through the logarithmic coupling factor g. We treat this factor as a constant here to obtain a one-dimensional description. This choice is equivalent to assuming a matched beam with uniform axisymmetric focusing. An independent equation for λ is obtained by averaging the continuity equation over the beam cross section:

$$\frac{\partial\lambda}{\partial t} + \frac{\partial(\lambda v)}{\partial z} = 0. \tag{3}$$

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To obtain tractable equations, we assume that the unperturbed beam has a constant and uniform line-charge density λ_0 and fluid velocity v_0. If we then change variables to the beam-frame distance $\zeta \equiv z - v_0 t$ and assume perturbations of the form $v = v_0 + \tilde{v}$ and $\lambda = \lambda_0 + \tilde{\lambda}$, then we obtain a pair of linearized equation for $\tilde{v}$ and $\tilde{\lambda}$:

$$\frac{\partial \tilde{\lambda}}{\partial t} + \lambda_0 \frac{\partial \tilde{v}}{\partial \zeta} = 0 \qquad (4a)$$

$$\frac{\partial \tilde{v}}{\partial t} = \frac{qe}{M}\left[\tilde{E}_{ext} - g\frac{\partial \tilde{\lambda}}{\partial \zeta} - \eta(\lambda_0 \tilde{v} + v_0 \tilde{\lambda})\right]. \qquad (4b)$$

Here, we have included an unspecified correction field $\tilde{E}_{ext}$ that depends on the correction algorithm.

B. Dynamics of Uncorrected Perturbations

For a perfectly conducting accelerator with no correction field, the perturbed equations of Eq. (4) reduce to a homogeneous wave equation for $\tilde{v}$ or $\tilde{\lambda}$. The general solution can be written in the form

$$\tilde{\lambda}(\zeta, t) = F_+(\zeta + v_p t) + F_-(\zeta - v_p t) \qquad (5a)$$

$$\tilde{v}(\zeta, t) = -\frac{v_p}{\lambda_0} F_+(\zeta + v_p t) + \frac{v_p}{\lambda_0} F_-(\zeta - v_p t), \qquad (5b)$$

where the wave phase velocity $v_p = (qeg\lambda_0/M)^{1/2}$ is typically small compared with v_0. This solution describes a slow wave moving backward at v_p in the beam frame and a fast wave moving forward at v_p. The two simplest examples are a pure velocity perturbation, which initially has $F_+(\zeta) = -F_-(\zeta)$, and a pure density perturbation with $F_+(\zeta) = F_-(\zeta)$. In the examples shown in this paper, a pure parabolic velocity perturbation is always used, but equivalent results are obtained with a density perturbation.

A non-zero resistance has been shown to cause bunching of backward waves and damping the forward waves [3]. If we assume that perturbations depend on ζ and t according to $\exp(ik\zeta - i\omega t)$, then the resulting dispersion relation shows for small v_p/v_0 that backward waves grow with a growth rate $\Gamma = \eta v_0 v_p / 2g$, while forward waves damp with a decay rate $-\Gamma$. This "longitudinal instability" can be seen in the numerical solution of the perturbed equations of Eq. (4) shown in Fig. 1. For this illustration, parameters resembling those of a HIF driver have been used, with an ion mass M of 200 amu, a charge state q of unity, an ion kinetic energy of 10 GeV, and a beam current $\lambda_0 v_0$ of 3 kA. The coupling factor g has been taken to be 1.4×10^10 m/F, and a resistance $\eta = 150\ \Omega$/m has been used. For these parameters, the initial perturbation is expected to grow by a factor of about 5.8 during the 8 μs duration of the simulation, and the calculated value is in almost exact agreement. The forward wave is seen in the figure to damp by a similar factor.

C. Correction Algorithm

The strategy adopted here for correcting longitudinal perturbations is to apply a suitable axial electric field $\tilde{E}_{ext}$ to reverse any backward waves, relying on the accelerator resistance to subsequently damp them. The required velocity change is calculated by noting the relation between line-charge density and velocity for a forward going wave:

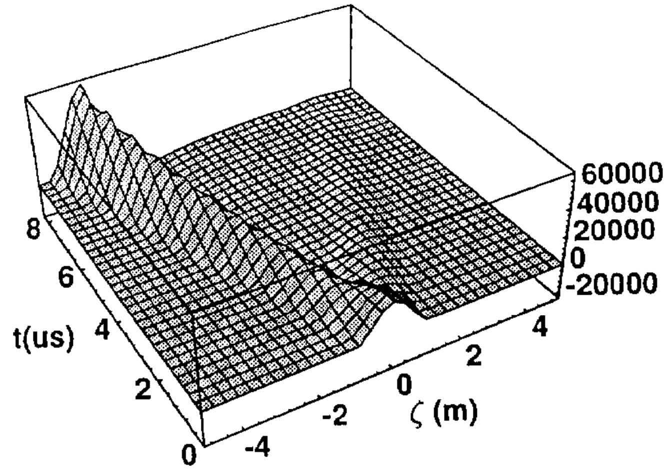

Fig. 1 Uncorrected evolution of a parabolic velocity perturbation in an accelerator with $\eta = 150\ \Omega$/m resistance.

$$\tilde{\lambda}(\zeta, t) = F(\zeta - v_p t) \qquad (6a)$$

$$\tilde{v}(\zeta, t) = \frac{v_p}{\lambda_0} F(\zeta - v_p t). \qquad (6b)$$

To correct perturbations at some time t_c, we then take $\tilde{\lambda}_{new}(\zeta, 0) = F(\zeta) = \tilde{\lambda}_{old}(\zeta, t_c)$ and change the velocity so that $\tilde{v}_{new}(\zeta, 0) = (v_p/\lambda_0)F(\zeta)$. The required velocity change is then

$$\Delta\tilde{v}(\zeta) = \frac{v_p}{\lambda_0}\tilde{\lambda}_{old}(\zeta, t_c) - \tilde{v}_{old}(\zeta, t_c). \qquad (7)$$

From Eq. (6), this velocity change is seen to vanish for purely forward-going waves, and it equals $2(v_p/\lambda_0)\tilde{\lambda}_{old}$ for purely backward perturbations.

As written, this velocity change requires an electric field to be applied simultaneously to the full length of the beam. Such application is difficult because the accelerating field in induction accelerators is confined to relatively short gaps. Instead, we use the fact that v_p/v_0 is normally small to replace the ζ-dependent field at t_c with a time-dependent field in a gap of length L_g located at the beam-head position at t_c. The required electric field is then

$$\tilde{E}_{ext}(t) = \frac{M v_0}{qe L_g}\ \Delta\tilde{v}[\zeta = v_0(t - t_c)]. \qquad (8)$$

III. Results

When the correction field from Eq. (8) is applied to an idealized perturbation in the absence of resistance, the method works as expected. Since the correction field is zero when the forward wave is traversing the gap, that portion of the wave is unaffected, but the backward wave is seen to change direction as the velocity perturbation changes sign The final state is a pair of undamped perturbations moving forward in the beam frame at v_p. If an accelerator resistance of 150 Ω/m is included, the correction

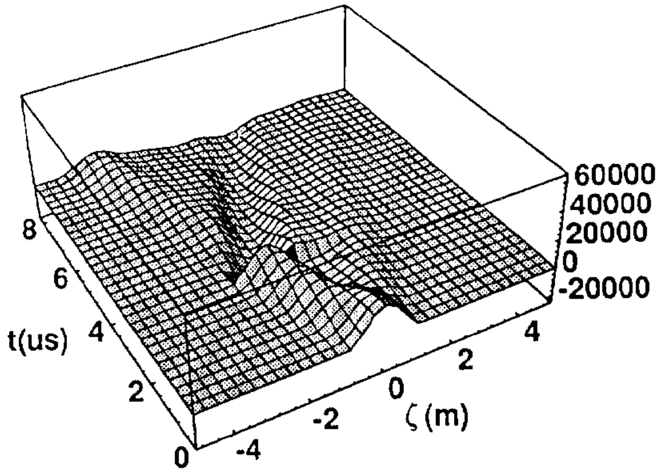

Fig. 2 Evolution of a parabolic velocity perturbation in an accelerator with $\eta = 150\ \Omega/\mathrm{m}$ making a single correction.

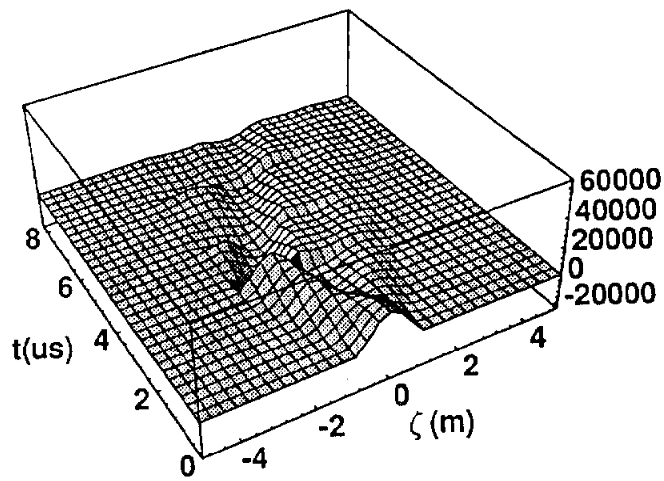

Fig. 3 Evolution of a parabolic velocity perturbation in an accelerator with $\eta = 150\ \Omega/\mathrm{m}$ making three corrections.

method works imperfectly, as seen in Fig. 2. Although the backward wave is substantially reversed, a small backward component remains and is the dominant perturbation by the end of the run. Reversal of the backward wave is incomplete in this case because the wave-equation solution Eq. (5), from which the correction field of Eq. (8) is obtained, is only exact in the absence of resistance.

Regrowth of backward waves can be controlled by periodically applying corrections with the form of Eq. (8). Fig. 3 shows the same initial velocity perturbation corrected at three locations about 170 m apart. At the end of the simulation there is no visible backward wave, although there has not been sufficient time after the last correction for significant regrowth. The main conclusion from this case is that periodic correction can control but probably not eliminate backward waves.

IV. Discussion

It should be stressed that the numerical results here are the best that might be obtained using the proposed correction algorithm. Perfect measurement of the perturbations was assumed, and the exact correction field was applied. In fact, measuring $\tilde{v}$ independently from $\tilde{\lambda}$ is difficult with currently available diagnostic techniques. Current loops can measure $I_b = \lambda v$ with an accuracy of about $\pm 1\%$ for the currents levels expected near the end of a HIF driver, and methods for obtaining the line-charge density λ, such as capacitive probes, are less accurate. Consequently, any scheme for combining these measurements to obtain the relative velocity error $\tilde{v}/v_0$ will have an error greater than 0.01, whereas final focus requirements limit $\tilde{v}/v_0$ to less that 0.005. Probably some as yet unproven method like laser "tagging" of ions is needed to measure $\tilde{v}$ directly. Generating the required correction field $\tilde{E}_{ext}$ is also challenging because of the magnitude and complicated time dependence of the correction signal. If we assume that the 1% uncertainty in I_b represents the smallest measurable perturbation, then the minimum correction signal from Eq. (8) for a gap length L_g of 3 cm is about 5×10^7 V/m. To avoid electrical breakdown, this voltage may have to be applied piecemeal in several successive cells.

The proposed correction algorithm effectively reduces the level of space-charge waves on a beam provided that the growth rate Γ is sufficiently high. When backward waves are repeatedly reversed with an interval δt between corrections, the peak amplitude of space-charge waves is reduced by about $\exp[\Gamma(\delta t - L_b/v_p)]$ compared with an uncorrected beam, where L_b is the beam length. For a growth length v_p/Γ equal to L_b, this reduction factor is at most about 0.37, making the utility of the correction scheme questionable. Furthermore, since incompletely damped perturbations reflect coherently at the beam head and begin to regrow, repeated corrections in effect trap the waves in a region approximately $v_p\delta t$ long near the beam head, most likely causing excessive emittance growth there as the waves phase mix. Because of these problems, the method is not useful at low energy, because $v_p/\Gamma \sim v_0^{-1}$. Also, work by Lee and Smith [4] shows that inclusion of a realistic amount of cell capacitance in the electric-field model substantially increases the growth length by reducing Γ, again reducing the effectiveness of the method.

References

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