Analysis of Beam Loading in Electrostatic Columns*

George J. Caporaso and John J. Barnard
Lawrence Livermore National Laboratory
Box 5508, Livermore, CA 94550 USA

Abstract

A lumped element circuit model is derived which accounts for both electric and magnetic coupling of the beam to the electrodes and drive circuitry of an electrostatic column. A modified one dimensional Poisson equation which incorporates two dimensional effects is discussed. An effective capacitance between the beam and the column electrodes which affects the electric coupling is estimated. Simple analytic cases which treat electric and magnetic coupling separately are solved and compared against a numerical simulation. Scaling laws are given for the magnitude of the beam loading.

I. INTRODUCTION

This paper briefly discusses a beam loading model for an injector column system and how 2-D effects can be incorporated in a one dimensional code. We will show that the loading arises from two different effects which can be tied to electrostatic and magnetic fields.

II. BEAM LOADING MODEL

We wish to consider beam loading in a cylindrical column such as shown figure 1. A large insulator supports several electrodes or plates. There are external resistors which help to grade the applied voltage to the electrodes. There is a substantial capacitance between each electrode. The column assembly is enclosed in a large tank so that there is an additional capacitance from each electrode to the tank (which is assumed to be at ground potential).

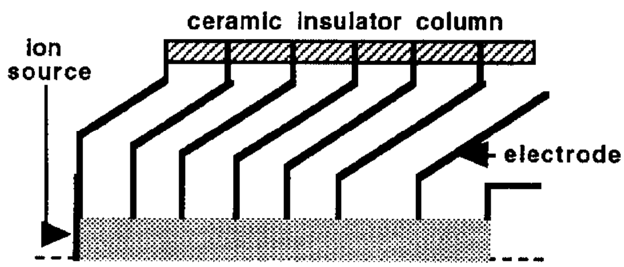

Figure 1. Typical ion injector column.

Figure 2 shows the effects of the beam's azimuthal magnetic field. Since the electrodes are good conductors the transient magnetic field generated by the beam cannot penetrate them on the time scale of the pulse. Ampere's law then requires that surface currents are generated in order to set up a magnetic field that cancels the beam's field inside the electrodes. The total surface current is equal and opposite to the beam current and can be approximately modeled as a current source in parallel with the inter-plate capacitance and resistance as shown in the bottom half of figure 2 (also shown in the circuit is a stray capacitance C_s between each plate and the outer tank (ground)).

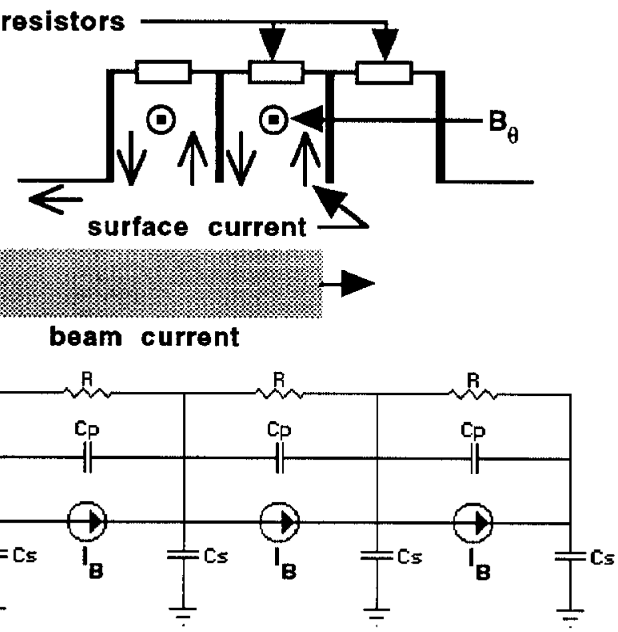

Figure 2. The beam's azimuthal magnetic field drives a return current on the electrode surfaces. The circuit equivalent is also shown.

Next consider the effect of the beam's radial electric field on the electrodes. If we imagine an isolated conducting aperture in a conducting cylinder as a uniform coasting beam passes through we would see electrons rushing in radially on the aperture as the head of the beam passes by.

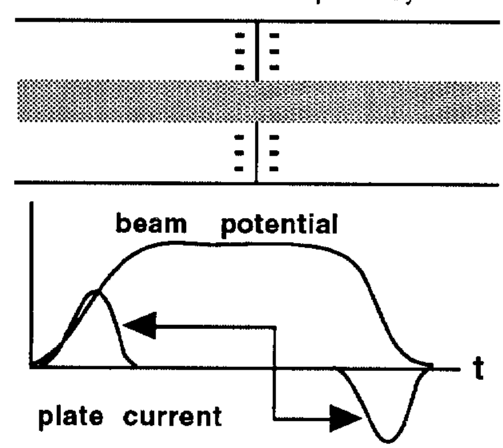

Figure 3. The beam's radial electric field draws in charges onto the plate. The current that flows in response to the passage of the beam appears to be a capacitive charging current.

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This current would look like the derivative of the potential difference between the axis of the system and the plate. If the beam is long and of constant density, no additional radial current will flow. As the tail of the beam passes underneath the aperture the electrons which initially rushed in will be expelled and the radial current will reverse sign as the potential difference between the axis and the plate returns to zero. The radial current looks like a capacitive charging current that is proportional to the time derivative of a the potential difference between the axis and the plate. This is represented as a capacitance C_g in series with a voltage source between the plate and ground. The value of the voltage source is the value of the potential on the axis.

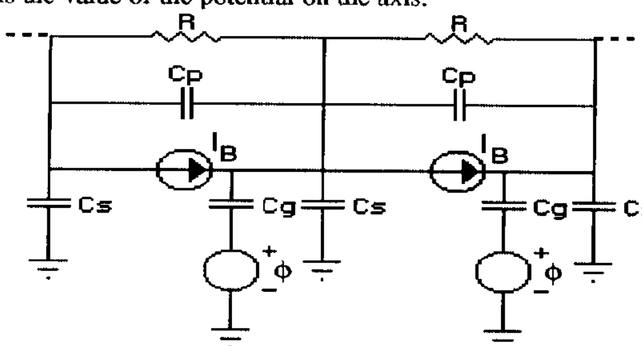

Figure 4. The circuit representation of the column showing both the return current and capacitive loadings.

In order to evaluate the loading we need to estimate both the value of the "beam capacitance" and the value of the potential along the axis.

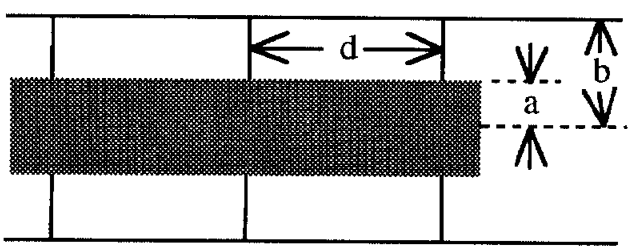

Figure 5. Geometry for the "beam capacitance" calculation.

We first look at estimating the value of the capacitance. We consider a uniform coasting beam just touching a periodic array of conducting apertures in an infinitely long cylinder. We will solve for the axial component of electric field and find the total charge induced on each aperture from the boundary condition $E_z=\sigma/\varepsilon_o$. We will then calculate the potential difference across the beam and define the "beam capacitance" as the total charge on the aperture divided by the potential difference across the beam. The capacitance is a function of the cylinder radius, aperture radius and aperture separation as is shown in figure 6. The presence of other apertures acts to reduce the local radial electric field of the beam thus reducing the amount of charge induced on a given aperture. Hence, the capacitance of a given plate will decrease as the inter-plate spacing decreases. For d/2b greater than approximately 2.0 the other plates have no effect and this solution is almost indistinguishable from that of a single plate in an infinite cylinder. The capacitance can be found analytically as $C_g=16\pi\varepsilon_0 b\, f(a/b,d/b)$ where $f(a/b,d/b)$ is given as

$$f\left(\frac{a}{b},\frac{d}{b}\right)=\sum_{n=1}^{\infty}\frac{J_o\left(\frac{x_{on}a}{b}\right)}{x_{on}^2 J_1(x_{on})}\left[1-\frac{a}{b}\frac{J_1\left(\frac{x_{on}a}{b}\right)}{J_1(x_{on})}\right]\tanh\left(\frac{x_{on}d}{2b}\right). \quad (1)$$

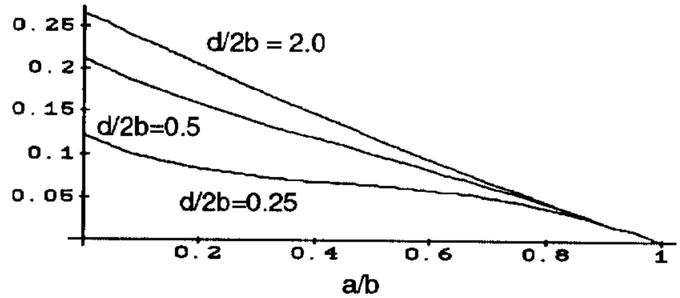

Figure 6. f(a/b,d/b) as a function of inter-plate spacing.

We next turn to the problem of calculating the potential along the axis. The correct way to do this of course is to solve the 2-D Poisson equation. However, we would like to use this model in a 1-D code so we need some way of accounting for 2-D effects. A "field equation" that fits this requirement was suggested by Langdon [1]. It basically approximates $\nabla_\perp^2\phi$ by the quantity $(V-\phi)/a^{*2}$ where V is the potential along the outer wall of the problem and ϕ is the potential on axis. The quantity a^* is defined as

$$a^* \equiv \frac{a}{2}\sqrt{1+2\ln\left(\frac{b}{a}\right)} \quad (2)$$

and is a characteristic length over which potential changes axially. Thus, the 2-D Poisson equation is to be replaced by the approximate equation

$$\frac{\partial^2\phi}{\partial z^2}+\frac{(V-\phi)}{a^{*2}}=-\frac{\rho}{\varepsilon_o} \quad (3)$$

where ϕ now represents the on-axis potential.

We can see how well this equation works on a test problem. Imagine a conducting cylinder with conducting end caps. Inside this cylinder we put a charged column of uniform radial density of radius a and arrange for the axial variation of density to correspond to that of steady state Child-Langmuir flow (the density varies as $z^{-2/3}$).

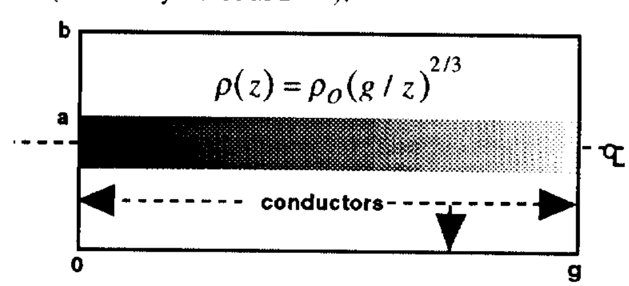

Figure 7. Geometry for Child-Langmuir flow density problem in a conducting cylinder with end plates.

than one then essentially the entire beam potential appears on the plate and raises the column voltage. When this ratio is less than one, however, the column is raised to something on the order of the maximum beam potential multiplied by this ratio.

We can solve this problem using the exact 2-D Poisson's equation and by using the "field equation". The solutions are overlaid on figure 8. We see that the field equation is remarkably accurate and correctly incorporates the 2-D effects of the end caps shorting out the radial electric fields.

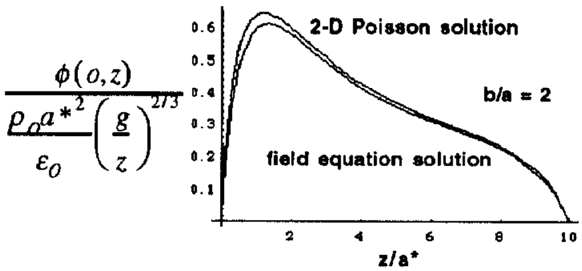

Figure 8. Comparison of field equation result with 2-D Poisson solution for a Child-Langmuir problem.

III. ANALYTIC SOLUTION AND SCALING LAWS

We now wish to put all these elements together to see the effects of beam loading. We will try a simplified problem so that we can solve it analytically to use as a check against the HINJ implementation [2]. We take the continuum limit of a discrete network in order to use differential equations in both z and t. We first look at the effects of the "beam capacitance" alone without the current source. In addition, to simplify the problem we assume that the column source impedance is zero and we eliminate the stray capacitance (that is in figure 4 we remove the current source and set C_S to zero. We take a coasting beam of constant density that is traveling at speed v. We take the length of the column to be 10 a* and assume that both ends of the column are grounded.

We plot the solution as a dimensionless voltage as a function of length along the column for successive times. We see that without a source impedance the column voltage rises reaching a maximum at about the time that the head of the beam reaches the end of the column (x=10) and then decays asymptotically to zero (for an infinitely long beam) as the resistance discharges the capacitors to ground. The quantity v_o is given by

$$v_o = \left(\frac{C_g a^{*2}}{C_p}\right)\left(\frac{\rho_o a^{*2}}{\varepsilon_o}\right) \qquad (3)$$

while x is defined as $x \equiv \beta c t / a^*$.

From this and other simple test problems (not discussed) we can draw some conclusions about beam loading. The beam capacitance and plate capacitance form a divider along which the beam potential appears. From our test problem for the field equation we see that the electric field from a sharply rising beam front actually spreads out over a distance roughly equal to a^*. The column forms a divider with series capacitance equal to C_p/a^* and with shunt capacitance equal to $C_g a^*$. When $C_g a^{*2}/C_p$ is greater than one then essentially the entire beam potential appears on the plate and raises the column voltage. When this ratio is less than one, however, the column is raised to something on the order of the maximum beam potential multiplied by this ratio.

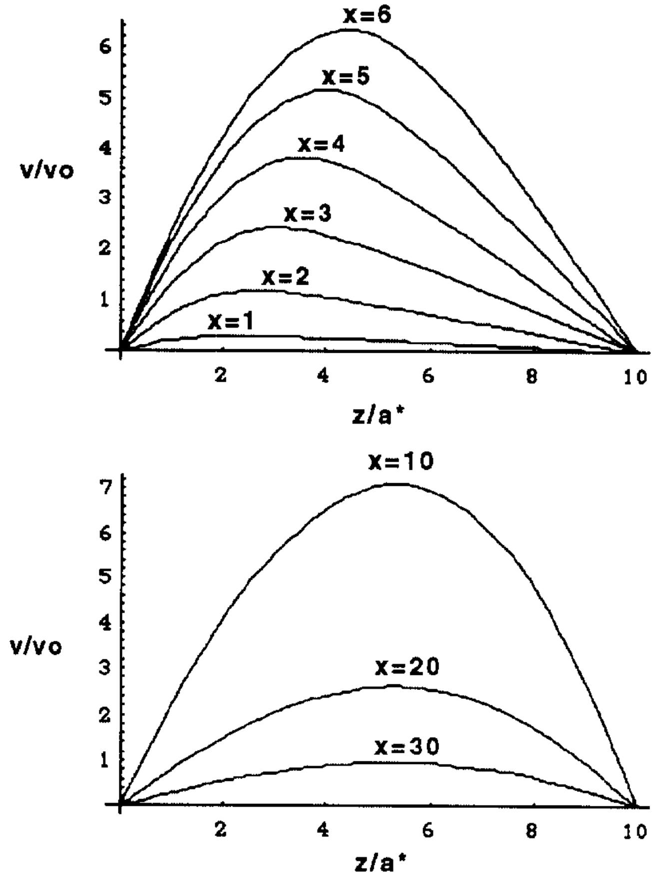

Figure 9. Solution to the simplified network discussed in the text.

The current sources will also raise the column voltage if there is no source impedance. The effect is qualitatively similar to the case of just voltage sources alone. In this case the effective length for the circuit is roughly given by the velocity of the beam and the RC time constant per unit length. Multiplying this length by the resistance per unit length gives an effective resistance through which the return current is flowing raising the voltage.

When the source impedance is included in the picture the column voltages now go negative (we are speaking of the *perturbation to the column voltages caused by the loading*). This occurs because the return current is flowing through this impedance and represents the mechanism by which the source of the column voltage is coupling energy into the beam.

IV. REFERENCES

[1] B. Langdon, Private communication.

[2] J. Barnard, G. Caporaso, S. Yu and S. Eylon, these proceedings.