Emittance and Damping of Electrons in the Neighborhood of Resonance Fixed Points*

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Abstract

The stable fixed points generated by nonlinear field harmonics in a cyclic lattice define a multiturn stable orbit. The position of the orbit for each turn in each magnet of the lattice determines the betatron tunes and lattice dispersion functions describing the linear motion of charged particles with respect to the stable orbit. Since the position of the fixed points is dependent in part on the central orbit tune, it turns out that the multiturn orbit dispersion function depends to a large extent on the central orbit chromaticity. In particular, the horizontal partition number can be made to vary from values less than zero (horizontal antidamping for electrons) to values greater than three (longitudinal antidamping). The central orbit chromaticity therefore plays a major role in determining the characteristic emittance of an electron beam with respect to the multiturn orbit.

I. INTRODUCTION

Nonlinear resonance studies at the University of Wisconsin electron storage ring, ALADDIN, have been reported previously [1,2]. During those studies it was observed that electrons kicked to the neighborhood of the third integer resonance fixed points had betatron oscillations that damped to the fixed points. Subsequent experiments at ALADDIN have been devoted to measuring the island lifetimes and rates of diffusion of the beam from the islands to the central region [3]. Damping and quantum growth rates of the beam in the islands are determined by the linear characteristics of the betatron oscillations about the stable fixed points which define a three-turn closed orbit.

Figure 1 shows a computer simulation of the damping of two phase points with initial starting points almost (but not quite) equal. One particle damps to the separatrix and ends up damping to the central orbit. The other particle also damps to the separatrix but then proceeds to damp to the third integer resonance stable fixed points. In this simulation the damping rates have been artificially increased. The tracking is stopped before the damping is complete.

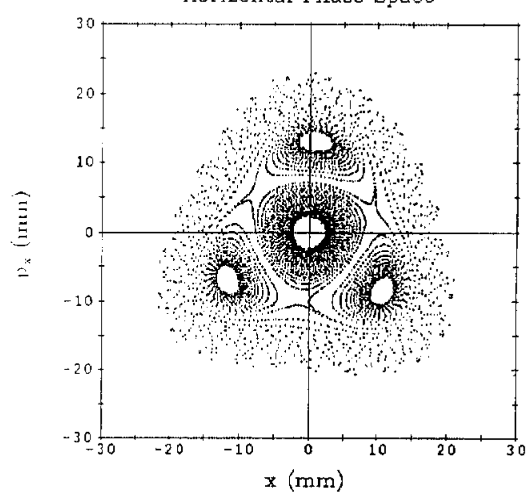

Figure 1. Damping of two almost equal phase points.

II. SIMULATIONS OF THE THREE-TURN CLOSED ORBIT

The ALADDIN storage ring has four long straight sections and normally operates with betatron tunes of $\nu_x = 7.14$, $\nu_y = 7.23$. Each of the four sectors contains sextupoles used to control the horizontal and vertical chromaticity. The third integral resonance is produced by changing the horizontal tune to about resonance is produced by changing the horizontal tune to about 7.33 and turning on an extra sextupole to produce the 22nd harmonic.

To determine the linear characteristics of oscillations about the fixed points one first uses a tracking program to find the position of the three-turn closed orbit at all of the magnets. These positions are used to define new elements which can be used to build a lattice for the three-turn machine. Each quadrupole or sextupole becomes a gradient bending magnet for the three-turn machine. For example, the bend in a quadrupole is determined by the change in angle of the fixed point orbit in passing through the quadrupole, i.e.

$$\theta = -\Delta x' = \frac{B'\ell}{B\rho} < x > = \frac{\ell}{< \rho >}$$

where $B'/B\rho$ is the quadrupole strength. The edge angles are also determined by slopes of the orbit on entering and leaving the magnet.

III. LINEAR CHARACTERISTICS OF THE THREE-TURN ORBIT

The lattice elements described in the preceding section describe a machine with well-defined betatron and dispersion functions. The horizontal tune is about 22.033. (One can verify the tune by tracking the oscillations about the stable fixed points using the central nonlinear machine.)

The interesting feature of the new three-turn machine is the dependence of the dispersion function on the chromaticity of

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the oscillations with respect to the original central orbit. This dependence on chromaticity is explained as follows. The position of the fixed point is determined by the central orbit tune and an average amplitude-dependent tune shift. Near resonance the amplitude-dependent tune shift is not very sensitive to change in the chromaticity correcting sextupoles. Therefore, the fixed points for a positive momentum particle are at either a greater or smaller amplitude depending on whether the chromaticity is respectively negative or positive. Figures 2 and 3 show the dispersion functions for normalized chromaticities of ± 0.3. In general, the two dispersion functions have a phase difference of about π. (Note that the dispersion function generated by the three-turn lattice can be verified by off-momentum tracking using the original nonlinear lattice.)

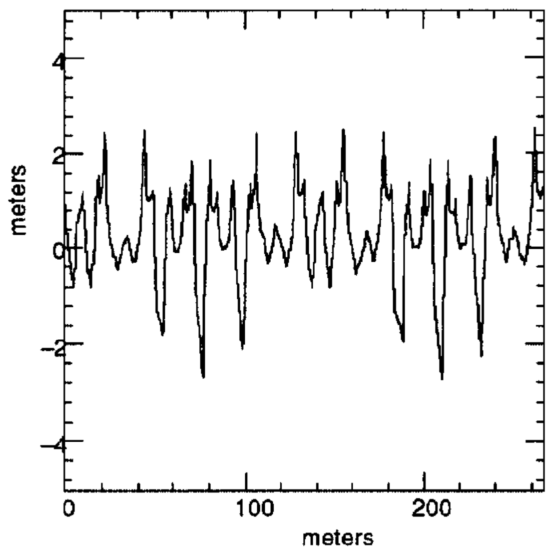

Figure 2. Dispersion function with normalized chromaticity = .3.

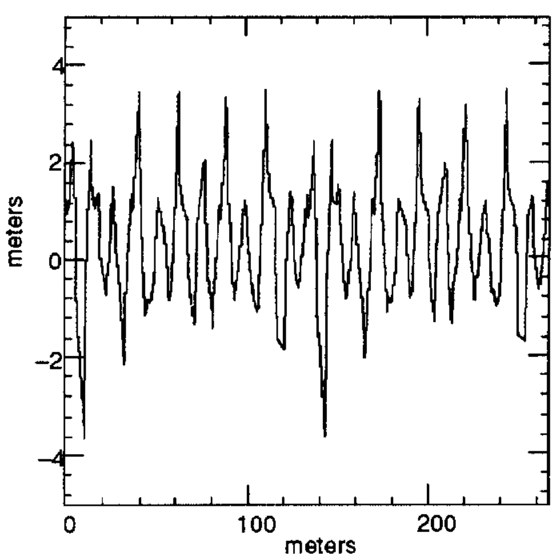

Figure 3. Dispersion function with normalized chromaticity = –.3.

This dependence of the dispersion function on the central orbit chromaticity has a large effect on the damping partition numbers J_x and J_E. The damping time constants are given by [4]

$$\tau_i = \frac{2E_o}{J_i P_\gamma} \qquad i = x,\ y,\ E$$

$$J_x = 1 - D \quad J_y = 1 \quad J_E = 2 + D$$

$$D = \frac{-\int \eta G\ (G^2 + 2K)\ ds}{\int G^2 ds}$$

where η is the dispersion function, G is the reciprocal bend radius, and K is the gradient focusing force in the magnet.

For the central orbit (and almost all machines), D is a small quantity. However the gradient terms in the new dipoles defining the three-turn machine are not small. In addition, the sign of the contributions to D in these magnets depends on the sign of the dispersion which in turn depends on the central orbit chromaticity.

Figures 4 and 5 show the contributions to the integrand for D for opposite signs of the central chromaticity. It turns out that the dependence of D (and hence J_x) on the chromaticity is almost linear as shown in Figure 6.

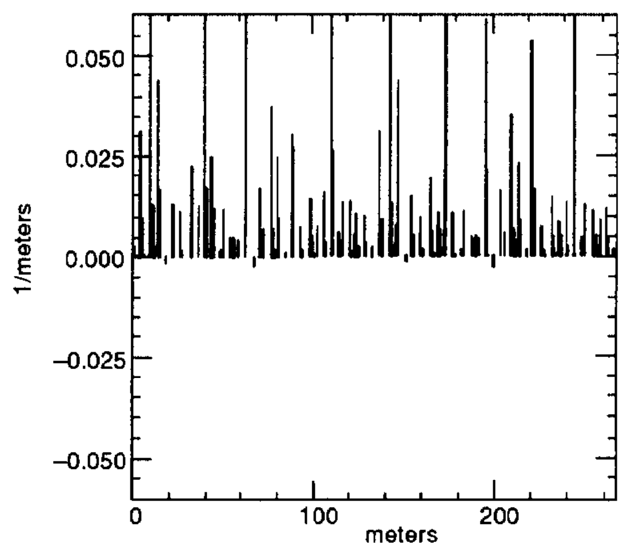

Figure 4. Contribution to the integrand of D with chromaticity = .3.

This dependence of the dispersion function on the central orbit chromaticity has a large effect on the damping partition numbers J_x and J_E.

On the other hand, the quantum fluctuation emittance growth rate depends on the square of the dispersion function. Hence, the plot of this function versus the chromaticity shows a minimum near small positive values (Figure 7). The resultant natural emittance (Figure 8) determined by the growth rate and the damping time constant shows a very shallow minimum at a moderate positive value of the chromaticity. The minimum value is about 10% smaller than the corresponding emittance for the central orbit.

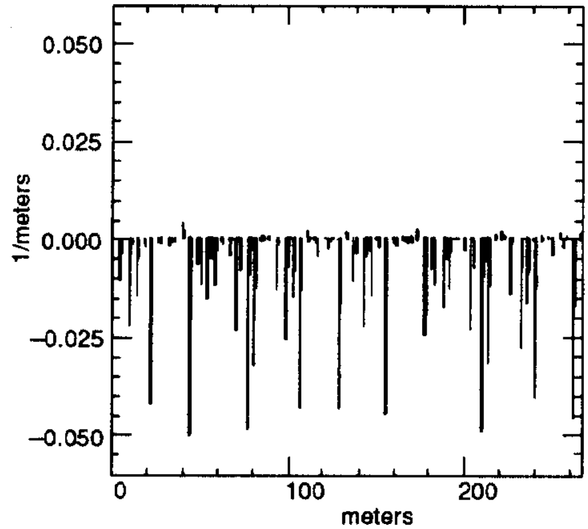

Figure 5. Contribution to the integrand of D with chromaticity = –.3.

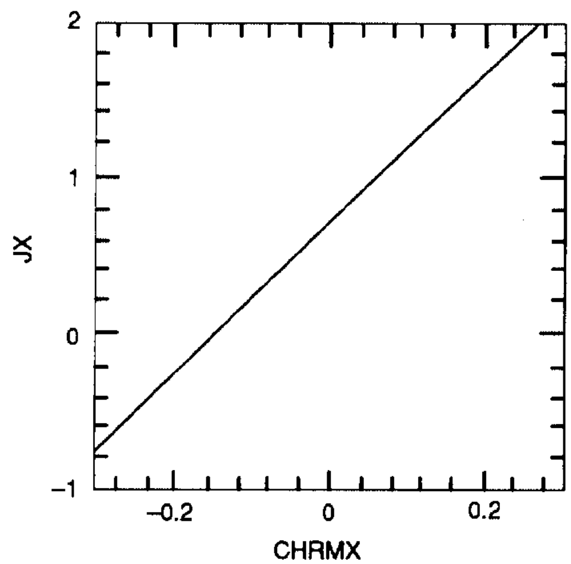

Figure 6. Dependence of J_x on chromaticity.

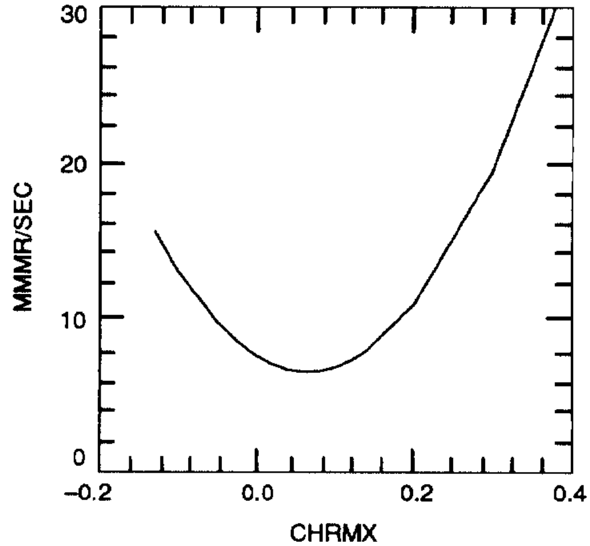

Figure 7. Quantum growth rate as a function of chromaticity.

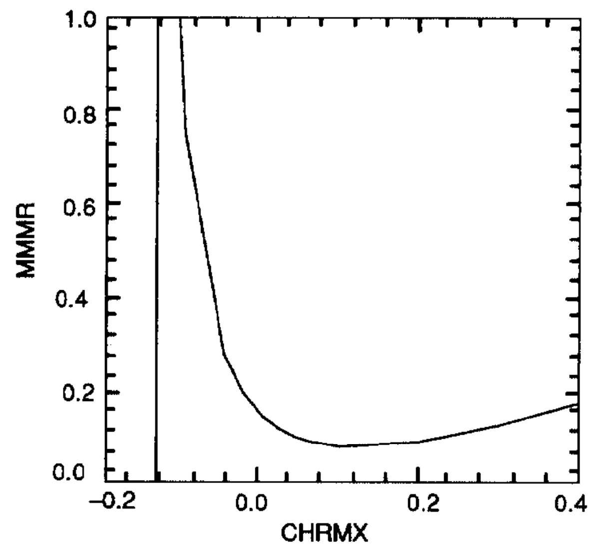

Figure 8. Island natural emittance as a function of chromaticity.

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