Computer Assisted Accelerator Tuning*

John K. Boyd
Lawrence Livermore National Laboratory
P.O. Box 808, Livermore, CA 94550

Abstract

The challenge of tuning an induction accelerator in real time has been addressed with the new TUNE GUIDE code. The code initializes a beam at a particular position using a tracer particle representation of the phase space. The particles are transported, using a matrix formulation, element by element along the beamline assuming that the field of a solenoid, or steering element is constant over its length. The other allowed elements are gaps and drift sections. A great deal of effort has been spent programming TUNE GUIDE to operate under the IBMPC Windows 3.1 system. This system features an intuitive, menu driven interface, which provides an ability to rapidly change beamline component parameter values. Consequently various accelerator setups can be explored and new values determined in real time while the accelerator is operating. In addition the code has the capability of varying a component value over a range and then plotting the resulting beam properties, such as radius or centroid position, at a down stream position. Element parameter editing is also included along with an on-line hyper text oriented help package.

I. INTRODUCTION

An induction accelerator can be tuned in real time with a responsive code having an intuitive graphical interface. The TUNE GUIDE code uses a matrix formulation to reduce the model complexity, and thus the required run time. TUNE GUIDE has been implemented in the Windows 3.1 system to obtain ease of use. The accelerator is modeled as a number of regions of solenoid focus, drift, dipole steering, or accelerating gap. Results are generated by repetitively applying beam line matrix elements to the initial condition. A real accelerator beamline element has a field with a peak value at its center and then diminishes to zero at large distances. The TUNE GUIDE code models beamline elements as constant fields over a specified length and then zero outside this region. The main part of this paper elaborates on the description of the beam line elements in the TUNE GUIDE code.

The matrix models used by TUNE GUIDE are similar to those of other codes [1]. The main difference is the addition of an element consisting of a steering coil and solenoid in the same location. Also to keep run time down and improve realism a method has been used to select input parameters that involves off line element characterization. The best agreement between the code and a real element is obtained by using an effective length and field strength parameter that differs from the actual physical value and is chosen to account for the step function behavior. Complete agreement is not possible with only one or two free parameters. However, a method is described that gives agreement for the basic effects of a solenoid and steering coil. For a solenoid, a procedure is given for selecting the length and the gauss/amp parameter that produces the correct phase advance and focusing strength. For a steering coil an equation is determined that provides a transverse gauss/amp parameter which results in the same steering kick as the real coil.

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II. SOLENOID BEAMLINE ELEMENT MODEL

A solenoid causes an electron beam to focus and also rotate. These effects begin occurring before the beam enters the starting position of the solenoid coil. In the TUNE GUIDE code model there is no focus or rotation until the beam enters a region of constant field. Also upon exiting the region of constant field the model ceases any focus or rotation. In order to preserve the focus and rotation that really results from the solenoid that is being modeled, it is necessary to calculate what values of solenoid length and gauss/amp should be used in TUNE GUIDE. The values that are derived guarantee equivalence along the axis.

A solenoid field $B(0,z) = GI_{ps}h(z)$ near the axis is modeled as a scaled profile function where G is the gauss/amp parameter, I_{ps} is the power supply current and $h(z)$ is a normalized axial variation function. The condition for equivalent focus is obtained by appealing to an axis expansion of the first order matrix for transport of a beam through a solenoid. The equivalence condition from this method requires that the integral of B^2 for the model must equal that of the real solenoid,

$$\alpha^2 G^2 I_{ps}^2 L_{eff} = G^2 I_{ps}^2 L \int_{-\infty}^{\infty} h^2(Z)\, dZ \qquad (1)$$

where α is a G modification factor resulting in focus strength equivalence (to be determined), L_{eff} is the effective model length of the solenoid and L is the actual solenoid length. Note that a normalized axial variable $Z = z/L$ is used in the Eq.(1) integral.

The condition for equivalent rotation is that the integral of B be the same,

$$\alpha G I_{ps} L_{eff} = G I_{ps} L \int_{-\infty}^{\infty} h(Z)\, dZ \qquad (2)$$

Equations (1) and (2) can be solved simultaneously to obtain,

$$L_{eff} = L\frac{\left[\int h dZ\right]^2}{\int h^2 dZ} \quad (3)$$

$$\alpha = \frac{\int h^2 dZ}{\int h dZ} \quad (4)$$

where the effective length and modification factor are specified in terms of the $h(Z)$ profile. The $h(Z)$ profile can be determined from analytical expressions for the field.

A. Azimuthal current sheet model

For the case of a solenoid consisting of a single thin layer of azimuthal current, there is an analytical solution for the magnetic field on axis,

$$B = GI_{ps}\sqrt{\frac{1}{4}+A^2}\left[\frac{Z_m}{\sqrt{Z_m^2+A^2}}+\frac{Z_p}{\sqrt{Z_p^2+A^2}}\right] \quad (5)$$

where $Z_p = 1/2+Z$, $Z_m = 1/2-Z$ and the normalized solenoid radius is $A = a/L$. From the form of Eq.(5) it is readily apparent that,

$$h(Z) = \sqrt{\frac{1}{4}+A^2}\left[\frac{Z_m}{\sqrt{Z_m^2+A^2}}+\frac{Z_p}{\sqrt{Z_p^2+A^2}}\right] \quad (6)$$

Using Eq.(6) in Eq.(3) and (4),

$$\frac{L_{eff}}{L} = \frac{2}{\int_0^\infty\left(\frac{Z_p}{\sqrt{Z_p^2+A^2}}+\frac{Z_m}{\sqrt{Z_m^2+A^2}}\right)^2 dZ} \quad (7)$$

$$\alpha = \sqrt{A^2+\frac{1}{4}}\int_0^\infty\left(\frac{Z_p}{\sqrt{Z_p^2+A^2}}+\frac{Z_m}{\sqrt{Z_m^2+A^2}}\right)^2 dZ \quad (8)$$

Equations (7) and (8) are used to produce values of L_{eff}/L and α as a function of A. These values can be used to determine input for the TUNE GUIDE code.

B. Rectangular cross section current model

At the next level of refinement the analytic model for the solenoid can be based on an azimuthal current that has constant current density in a rectangular cross section. The constant value is,

$$J_\theta = \frac{c}{2\pi L}GI_{ps} \quad (9)$$

where $G = 2\pi N/(10L(R_2-R_1))$, N is the number of turns, R_1 and R_2 are the normalized inner and outer radius respectively. The corresponding component of the vector potential is,

$$A_\theta(R, Z) = \frac{GI_{ps}L}{2\pi}\iiint\frac{\cos\theta' R' dR' d\theta' dZ'}{\sqrt{D(R, R', 0)}} \quad (10)$$

where $D(x, y, \theta) = x^2 - 2xy(\cos\theta\cos\theta' - \sin\theta\sin\theta') + y^2 + (Z-Z')^2$ and the coordinates are normalized to the solenoid length. Since the magnetic field on axis is desired, the above expression is expanded about $R = 0$ keeping only the first two terms. Then using the scaled

coordinate field definition $B_z = (LR)^{-1}\frac{\partial RA_\theta}{\partial R}$, it is found,

$$B_z = \frac{GI_{ps}}{\pi}\iiint dR' d\theta' dZ'\left[\frac{(R'\cos\theta')^2}{(R'^2+(Z-Z')^2)^{3/2}}\right] \quad (11)$$

Equation (11) can be integrated to obtain the profile function for this model.

$$h(Z) = \frac{Z_m\log\frac{R_2+\sqrt{R_2^2+Z_m^2}}{R_1+\sqrt{R_1^2+Z_m^2}}+Z_p\log\frac{R_2+\sqrt{R_2^2+Z_p^2}}{R_1+\sqrt{R_1^2+Z_p^2}}}{\log\frac{R_2+\sqrt{R_2^2+\frac{1}{4}}}{R_1+\sqrt{R_1^2+\frac{1}{4}}}} \quad (12)$$

Equation (12) is used in Eq.(3) and (4) to determine L_{eff}/L and α as a function of R_1 where some relationship between the inner and outer radius $R_2 \propto R_1$, must be assumed. For any particular solenoid both the inner and outer radius are known, so the effective length and α can be determined numerically.

III. STEERING BEAMLINE ELEMENT MODEL

A steering element causes an electron beam to deflect to the left, right or up, down, depending on its orientation. The deflection begins occurring before the beam actually arrives at the start of the physical coil. To determine the parameter that allows equivalent steering in TUNE GUIDE, it is sufficient to concentrate on a steering coil set that creates vertical field. The goal is then to select the ζ, gauss/amp parameter so the left or right deflection is equivalent to the on axis value which is caused by a real steering element. The condition to be satisfied is given by Eq(13),

$$\zeta I_{ps} L = L \int_{-\infty}^{\infty} B_y dZ \tag{13}$$

On the left side of Eq.(13) is the expression for the steering kick due to a TUNE GUIDE element which only steers over a finite distance. The right side of Eq.(13) is the expression for the total steering caused by a real element. In order to perform this integral the magnetic field of the steering coil is expressed in terms of components of the vector potential.

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \tag{14}$$

The steering coil currents that flow in the theta direction contribute to A_x and the currents in the axial or z direction contribute to A_z. Using Eq.(14) in Eq.(13),

$$\zeta I_{ps} = A_x(\infty) - A_x(-\infty) - \int_{-\infty}^{\infty} \frac{\partial A_z}{\partial x} dZ \tag{15}$$

The first two terms on the right side of Eq.(15) are steering field contributions at great distance from the steering coil. The formula to within a constant, for A_x is given below.

$$A_x = \int_0^{2\pi} d\theta' \int_{-\frac{L}{2}}^{L(\theta')} \frac{dZ' \sin\theta'}{\sqrt{D(r, A, \theta)}} \tag{16}$$

The integral on the right of Eq.(16) has Z in the denominator and thus as Z gets large the integral becomes small, so the A_x terms of Eq.(15) vanish. This means that ζ is completely specified by A_z. To determine this component of the vector potential the longitudinal current of the steering coil is modeled as a sheet,

$$J_z = -\frac{2NI_{ps}}{\pi a} \delta(a - r') \tag{17}$$

where a is the sheet radius and N is the number of turns. The sheet current has a variable length in angle about the axis and this feature appears below in the z limits of integration. The axial component of the vector potential is then,

$$A_z = -\left(\frac{2NI_{ps}}{\pi c}\right) \int d\theta' \int \frac{dZ'}{\sqrt{D(r, A, \theta)}} \tag{18}$$

and using this expression in Eq(14),

$$B_y = -\left(\frac{2NI_{ps}}{\pi c L}\right) \int_0^{2\pi} d\theta' \int_{Z_1}^{Z_2} \frac{(x - A\cos\theta')\, dZ'}{(D(r, A, \theta))^{3/2}} \tag{19}$$

The field at the axis is obtained by applying symmetry and performing the Z' integration,

$$B_y(0, 0, Z) = -\frac{8NI_{ps}}{\pi c a} \int_0^{\frac{\pi}{2}} \cos\theta' d\theta' \left(\Xi(2) - \Xi(1)\right) \tag{20}$$

where $\Xi(k) = (Z - Z_k(\theta')) / \sqrt{A^2 + (Z - Z_k(\theta'))^2}$ and the angle dependent axial integration limits are,

$$Z_1(\theta') = -\frac{1}{2} + \left(1 - \frac{\hat{L}}{L}\right)\frac{\theta'}{\pi}$$
$$Z_2(\theta') = \frac{1}{2} - \left(1 - \frac{\hat{L}}{L}\right)\frac{\theta'}{\pi} \tag{21}$$

An unrolled steering coil has current turns which run in the axial direction. The shape of these turns is that of a trapezoid. In Eq.(21) L corresponds to the length of the base of the trapezoid and $\hat{L}$ corresponds to the shorter length of the top of the trapezoid. Using Eq.(20) with (21) in Eq.(13), it is found (for current now expressed in amps),

$$\zeta = \frac{1.6N}{\pi a}\left[1 - \left(1 - \frac{2}{\pi}\right)\left(1 - \frac{\hat{L}}{L}\right)\right] \tag{22}$$

This expression is the gauss/amp factor for the steering coil. In the presence of ferrite it should be increased by approximately a factor of two. Using typical steering coil values of 13.5 and 36.2 cm for the lengths, 44 turns and a radius of 7.3 cm, gives ζ = 2.37 gauss/amp. This compares favorably with the value from reported measurements [2] where 420 gauss-cm at 5 amps is recorded. This is 420[gauss-cm]/(5[amp]x36.2[cm]) or 2.32 gauss/amp.

IV. REFERENCES

[1] A.C. Paul, TRANSPORT: an Ion Optic Program, Lawrence Berkeley Lab., LBL-2697, UC-32, TID-4500-R62 (Feb. 1975).

[2] J. Zentler, private communication