Electron Beam Depolarization in a Damping Ring*

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Abstract

Depolarization of a polarized electron beam injected into a damping ring is analyzed by extending calculations conventionally applied to proton synchrotrons. Synchrotron radiation in an electron ring gives rise to both polarizing and depolarizing effects. In a damping ring, the beam is stored for a time much less than the time for self polarization. Spin flip radiation may therefore be neglected. Synchrotron radiation without spin flips, however, must be considered as the resonance strength depends on the vertical betatron oscillation amplitude which changes as the electron beam is radiation damped. An expression for the beam polarization at extraction is derived which takes into account radiation damping. The results are applied to the electron ring at the Stanford Linear Collider and are compared with numerical matrix formalisms.

1. Introduction

In an ideal synchrotron, the vertical polarization component of a polarized injected beam is conserved. Due to coupling of the spin to the orbital motion, however, the spin motion is perturbed. Depolarizing resonances occur whenever the electron spin tune, ν_s, equals a resonance tune, K, by satisfying $\nu_s = K \equiv n+mP+q\nu_x+r\nu_z+s\nu_{syn}$, where P is the superperiodicity, ν_x and ν_z are the horizontal and vertical betatron tunes, ν_{syn} is the synchrotron tune, while $m, n, q, r,$and s are integers. In the absence of any longitudinal and radial error fields, the spin tune, ν_s, is equal to $a\gamma$, where $a = 0.011596$ is the anomalous part of the electron magnetic moment and $\gamma = \frac{E}{m_e c^2}$, where E is the electron energy, m_e is the electron mass, and c is the speed of light. In this paper we study the effects of depolarizing resonances on the spin motion of polarized electrons injected into a damping ring.

2. Spin Precession and Depolarizing Resonances

The spin of an orbiting particle in a synchrotron obeys the Thomas-BMT equation[1], which describes the spin motion in the presence of electromagnetic fields in the laboratory frame. With no significant electric fields in the accelerator, the Thomas-BMT equation reduces to

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S} \times [(1+a\gamma)\vec{B}_\perp + (1+a)\vec{B}_\parallel], \qquad (1)$$

where $\vec{S} = (S_x, S_y, S_z)$ is the spin vector, e is the electric charge, and γ is the relativistic factor, while $\vec{B}_\perp$ and $\vec{B}_\parallel$ are the magnetic field components transverse to and parallel to the instantaneous velocity of the particle.

The magnetic fields in the Thomas-BMT equation may be expressed[2] in terms of the particle coordinates. The corresponding spinor representation is

$$\frac{d\Psi}{d\theta} = \frac{i}{2}\begin{pmatrix} -\kappa & -t-ir \\ -t+ir & \kappa \end{pmatrix}\Psi, \qquad (2)$$

where $\kappa, t,$ and r depend on the particle coordinates. The polarization components are obtained by taking the expectation value of the Pauli matrix vector, $\vec{\sigma}$; i.e. $S_i = \Psi^\dagger \sigma_i \Psi$. The off diagonal matrix elements characterize the effect of spin depolarization due to the coupling between the up and down components of the spinor wave function. Given the periodic nature of a synchrotron, the coupling term may be expanded in terms of the Fourier components; i.e.

$$t + ir = \sum \epsilon_j e^{-iK_j\theta} \qquad (3)$$

in which θ is the particle orbital angle, K_j is the value of the resonant tune for the j^{th} resonance, and ϵ_j is the resonance strength and is given by the Fourier amplitude

$$\epsilon_j = \frac{1}{2\pi}\int (t+ir)e^{iK_j\theta}d\theta \approx \frac{1+a\gamma}{2\pi}\sum \frac{\partial B_z/\partial x}{B\rho} z e^{iK_j\theta}. \qquad (4)$$

This corresponds to summing over the precession angles due to each radial error field.

In the single resonance approximation[3], the spin equation in the particle rest frame is given by

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} a\gamma & -\zeta \\ -\zeta^* & -a\gamma \end{pmatrix}\Psi \quad \text{with} \quad \zeta = \epsilon \cdot e^{-iK\theta}. \qquad (5)$$

Transforming[3] the spin equation to the resonance precession frame using $\Psi_K = e^{i\frac{K\theta}{2}\sigma_z}\Psi$, we obtain

$$\frac{d\Psi_K}{d\theta} = \frac{i}{2}(\delta\sigma_z + \epsilon_R\sigma_x - \epsilon_I\sigma_y)\Psi_K, \qquad (6)$$

where σ_i are the Pauli matrices, $\epsilon = \epsilon_R + i\epsilon_I$, and $\delta = K - a\gamma$ measures the nearness to the resonance.

The general solution of Eq.(6) can be expressed as a linear combination of two eigenmodes: $\Psi_K = C_1\Psi_{K\uparrow} + C_2\Psi_{K\downarrow}$ with $C_1^2 + C_2^2 = 1$. Let S_i be the magnitude of the injected polarization, S_f the magnitude of the extracted polarization, and S_z the vertical component of the stored polarization in the resonance precession frame. For a vertically polarized injected beam, the polarization can be obtained by taking the expectation value of σ_z giving

$$S_z = \frac{\delta}{\lambda}(|C_1|^2 - |C_2|^2) + \frac{|\epsilon|}{\sqrt{+}\sqrt{-}}|C_1C_2|\cos(\lambda\theta + \phi). \quad (7)$$

where the phase angle ϕ is given by $\phi = \arg(C_1^* C_2)$, $\sqrt{\pm} = \sqrt{(\lambda \pm \delta)^2 + |\epsilon|^2}$, and $\lambda = \sqrt{\delta^2 + |\epsilon|^2}$.

When $\epsilon = 0$ in Eq. (7) we have $\delta = \lambda$ and therefore $S_z = S_i = |C_1|^2 - |C_2|^2$, where S_i is the polarization far from resonance; i.e. the magnitude of the injected polarization. Averaging over many revolutions around the ring

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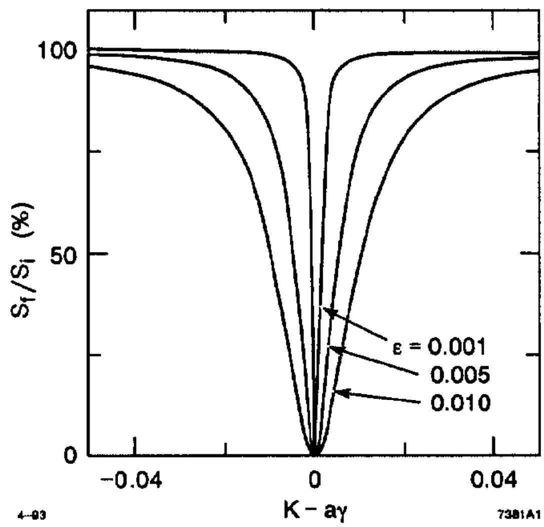

Figure 1. The ratio of the final to injected beam polarization as a function of the nearness to the resonance for three different resonance strengths.

we find, with $|C_1| = 1$ and $|C_2| = 0$ corresponding to a pure initial state, that

$$\frac{\langle S_z \rangle}{S_i} = \frac{K - a\gamma}{\sqrt{(K-a\gamma)^2 + |\epsilon|^2}}. \quad (8)$$

On resonance, where $K = a\gamma$, the polarization is zero. Just off resonance we may write $\frac{\langle S_z \rangle}{S_i} = \cos\alpha_i$ so the ratio $\frac{\langle S_z \rangle}{S_i}$ may be interpreted as the projection of the injected polarization onto S. For a constant beam emittance, $\alpha_i = \alpha_f$ and the final polarization, S_f, is given by projecting $\langle S_z \rangle$ onto the vertical direction:

$$S_f = \frac{\langle S_z \rangle}{S_i} S_i \cos\alpha_f = S_i \cos^2\alpha = \frac{(K-a\gamma)^2}{(K-a\gamma)^2 + |\epsilon|^2} S_i. \quad (9)$$

The dependence of the polarization on the resonance strength is shown in Fig. 1. The ratio of the final to injected polarization, $\frac{S_f}{S_i}$, is plotted as a function of $\delta = K - a\gamma$ for three different resonance strengths.

3. Spin Dynamics in an Electron Ring

Synchrotron radiation gives rise to both polarizing and depolarizing effects. Spin flip radiation tends to polarize the beam on a time scale which is long relative to the damping time and hence the store time. Spin flip radiation need therefore not be considered. However, radiation damping of the betatron oscillations is important because the resonance strength is proportional to the vertical betatron oscillation amplitude. The time dependence of the orientation of the spin vector is accounted for by noting that the turn by turn spin precession is adiabatic. The final polarization, S_f, is then a projection of the precessed spin vector onto the vertical. Thus $S_f = S_i \cos\alpha_i \cos\alpha_f$ or

$$S_f = \frac{K - a\gamma}{\sqrt{(K-a\gamma)^2 + |\epsilon_i|^2}} \frac{K - a\gamma}{\sqrt{(K-a\gamma)^2 + |\epsilon_e|^2}} S_i, \quad (10)$$

where $|\epsilon_i|$ is the resonance strength at injection and $|\epsilon_e|$ is the strength at extraction.

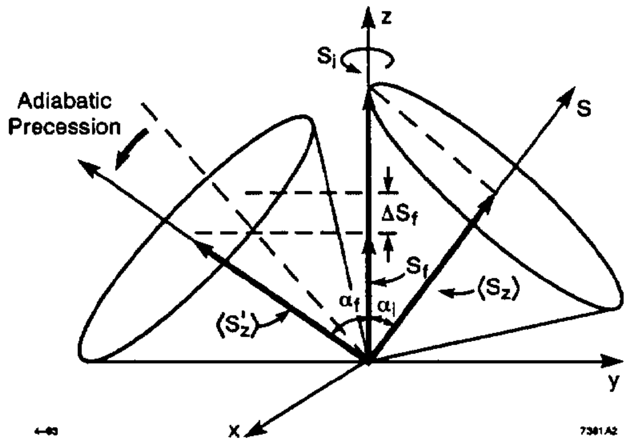

Figure 2. Graphical representation of Eq. (10).

A graphical representation of Eq. 10 is shown in Fig. 2. The injected polarization, S_i, is first projected onto the stable spin axis, the orientation of which is determined by the resonance. This projection, $\langle S_z \rangle$, precesses adiabatically to $\langle S_z' \rangle$ as the resonanance strength changes. Then $\langle S_z' \rangle$ is projected back to the vertical to obtain the polarization at extraction, S_f.

We now average over a Gaussian particle density distribution. In emittance space, the distribution function is given by $\rho(\eta) = \frac{1}{\eta_{rms}} e^{-\frac{\eta}{\eta_{rms}}}$, where η is the phase space occupied by a single particle and η_{rms} is the beam emittance. The final polarization is then

$$\frac{S_f}{S_i} = \int\int \frac{(K-a\gamma)\rho(\eta_i)d\eta_i}{\sqrt{(K-a\gamma)^2 + |\epsilon_i|^2}} \frac{(K-a\gamma)\rho(\eta_e)d\eta_e}{\sqrt{(K-a\gamma)^2 + |\epsilon_e|^2}} \quad (11)$$

where $\rho(\eta_i)$ and $\rho(\eta_e)$ are the distribution functions for the injected and the extracted beam, respectively.

4. Application to the SLC Damping Ring

During the 1992 physics run, polarized electron beams were created and transported to the interaction point for the first time[4]. Vertically polarized electron beams were injected into the damping ring at a nominal energy of 1.153 GeV. The store time was 8.33 ms while the damping time was about 3.7 ms. The resonance strengths at injection and extraction were estimated using DEPOL[2] which calculates the resonance strength based on Eq. 4. An injected to extracted emittance ratio of about 20 was assumed with the normalized beam emittance at extraction equal to 15 mm-mrad. The betatron tunes were 8.23 in x and 3.25 in y. Plotted in Fig. 3 is the extracted polarization as a function of $a\gamma$, where the solid line at $a\gamma = 2.6176$ corresponds to the operating energy of 1.153 GeV. The curve is calculated based on Eq. 11 which takes into account the damping of the betatron oscillations. If the vertical betatron tune is lowered, then both intrinsic resonances could cause slight depolarization. With these tunes however the amount of depolarization is minimal. We also considered the energy spread of the injected beam and the effect of a nonzero chromaticity. The energy spread, which was taken to equal the energy acceptance of the ring, was determined not to

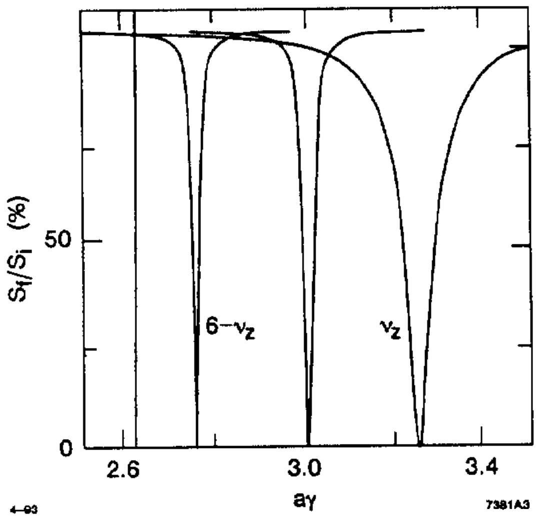

Figure 3. The ratio of the final to injected beam polarization as a function of $a\gamma$ calculated from Eq. 11.

cause significant depolarization. The nonzero chromaticity shifts the $6-\nu_z$ resonance downwards only slightly.

To compare the analysis with numerical simulation, we make the conservative approximation that the contributions from individual resources add coherently. Then summing over the contributions due to each resonance at a given $a\gamma$ gives, from Eq. (9)

$$S_f = [1 - \Sigma \frac{|\epsilon_j|^2}{(K - a\gamma)^2 + |\epsilon_j|^2}] S_i. \qquad (12)$$

Eq. (12) was used in Fig. 4a to calculated the ratio of the final to the initial polarization as a function of $a\gamma$.

These results include first order linear resonances only. To check the importance of higher order resonances and depolarization due to spin diffusion, we compare the results to those obtained using SLIM[5], which calculates the resulting equilibrium polarization; that is, the polarization one would observe after injecting unpolarized beam and allowing the beam to polarize due to spin flip radiation. The simulation using SLIM is shown in Fig. 4b. Shown in Fig. 4c is a simulation made with SMILE[6], which includes nonlinear resonances.

5. Conclusion

Using the spinor formulation of the Thomas-BMT equation, we emphasized that the measurable polarization depends on the projection of the polarization vector onto the stable spin direction at injection. This direction depends both on the resonance strength and the nearness of the operating energy to the resonance. We then considered the effects arising from synchrotron radiation. We obtained an expression for the final polarization taking into account the damping of the betatron oscillations. The solution was based on the realization that the polarization vector precessed adiabatically as the transverse distribution was damped. We then integrated over an assumed Gaussian transverse particle density distribution and predicted the final polarization at extraction. To test the effect of higher order resonances on the spin motion, we compared the predictions to numerical simulations under equilibrium conditions.

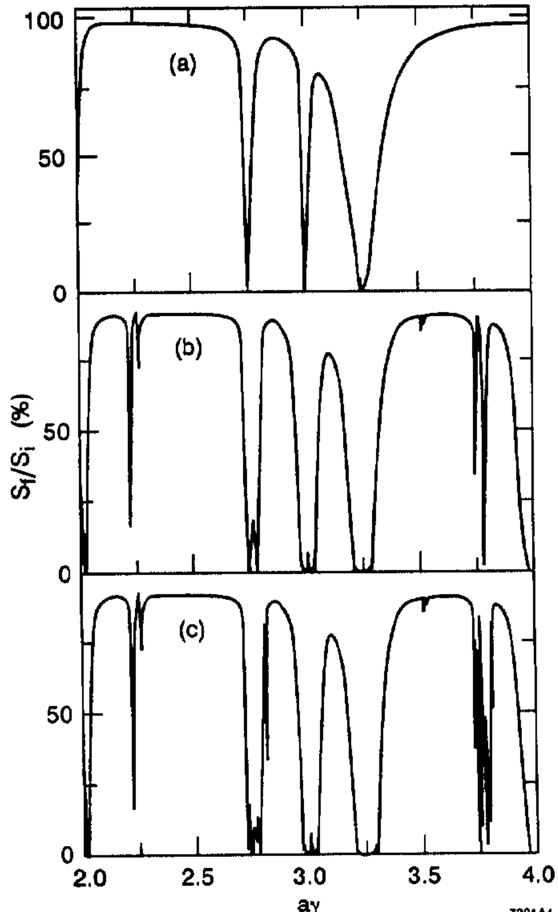

Figure 4. Comparison of ratio of final to injected polarization as a function of $a\gamma$ obtained from analytic calculations (4a), SLIM (4b), and SMILE (4c).

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