

Investigation of Spin Resonance Crossing in Proton Circular Accelerators

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Abstract

The general approach to the problem of the symplectic integration of Hamilton's equations, which is presented in paper [1] and which uses only the common properties of Lie groups, is developed to obtain the numerical tracking methods for both orbital and spin motions. The integration step of these methods is an explicit canonical map. (Here we use the term "canonical" instead "symplectic" for the systems with the spin Poisson brackets.) Some methods of such type are realized in computer code VasiLIE [2]. With help of this code it is possible to study the dynamics of the polarization during acceleration. The numerical simulation of crossing the depolarizing resonances depending on the different parameters was performed for a lattice under study for the TRIUMF KAON Factory Booster [3]. The effect of nonlinear elements is also discussed.

I. INTRODUCTION

The spin resonance crossing is the main process which does not allow efficiently to accelerate polarized proton beam to high energy. The influence of spin resonances can be so great that in the end we can obtain a completely depolarized beam. This depends on concrete conditions of a passage through resonances. The behaviour of a semiclassical spin of a particle far from resonances is described well by a vector $\vec{n}$ and a spin tune ν_s introduced by Ya.S.Derbenev and A.M.Kondratenko [4]. For every given orbital trajectory a projection of a spin vector on the direction $\vec{n}$ is preserved and the projection on the transverse plane is rotated with a tune ν_s (the correct mathematical definition of $\vec{n}$ and an algorithm of its calculation with help of one turn Taylor map see in [5]). In general case in nonresonance situation it is possible to conserve the beam polarization only along the direction $\vec{n}$ because $\vec{n}$ and a spin tune ν_s depend on the betatron and synchrotron oscillations.

Let consider the situation of one isolated resonance crossing. In this case we have two directions $\vec{n} = \vec{n}_1$ and $\vec{n} = \vec{n}_2$ which describe the spin motion before and after resonance. If we know the angle between an image of $\vec{n}_1$

after resonance and $\vec{n}_2$, we can predict the value of the depolarization of the particle beam. For the first time the depolarization due to passage through a resonance was estimated by M. Froissart and S. Stora [6]:

$$P_2 = P_1 \cdot (2 \exp(-\pi |w_k|^2 / 2\alpha) - 1) \quad (1)$$

This formula gives a small value of depolarization in cases of very slow or very fast passage of spin resonances. What will happen when we have the intermediate situation or cross one after another several near resonances? The numerical integration methods are a useful addition to analytical investigations of this problem. In this paper we present the numerical integration method, the integration step of which is an explicit canonical map.

II. NUMERICAL INTEGRATION METHOD

The classical spin-orbit equations of the motion in circular accelerators have the form of a Hamiltonian system if we use the Poisson bracket:

$$\{F(\vec{z}), Q(\vec{z})\} = F_{\vec{q}} \cdot Q_{\vec{p}} - F_{\vec{p}} \cdot Q_{\vec{q}} + [F_{\vec{s}} \times Q_{\vec{s}}] \cdot \vec{S} \quad (2)$$

and the Hamiltonian:

$$H = H_{orb}(\vec{x}, \tau) + \vec{W}(\vec{x}, \tau) \cdot \vec{S}$$

where $\vec{z} = (\vec{x}, \vec{S})$ and $\vec{x} = (\vec{q}, \vec{p})$ are canonical orbit variables, $\vec{S} = (S_1, S_2, S_3)$ is a classical spin vector, τ is the generalized machine azimuth or the time (see details in [5]).

So we will use a general approach to the numerical integration of Hamiltonian systems which is presented in paper [1] and which uses only the common properties of Lie groups and Poisson brackets. For simplicity one consider the case when the Hamiltonian does not depend on τ . Usually the effect of the spin on an orbital motion is not taken into account. In this case we can obtain the solution of the system with the Hamiltonian $\vec{W} \cdot \vec{S}$ in the evident form. It means that it is possible to reduce the initial problem to the problem of symplectic integration of an

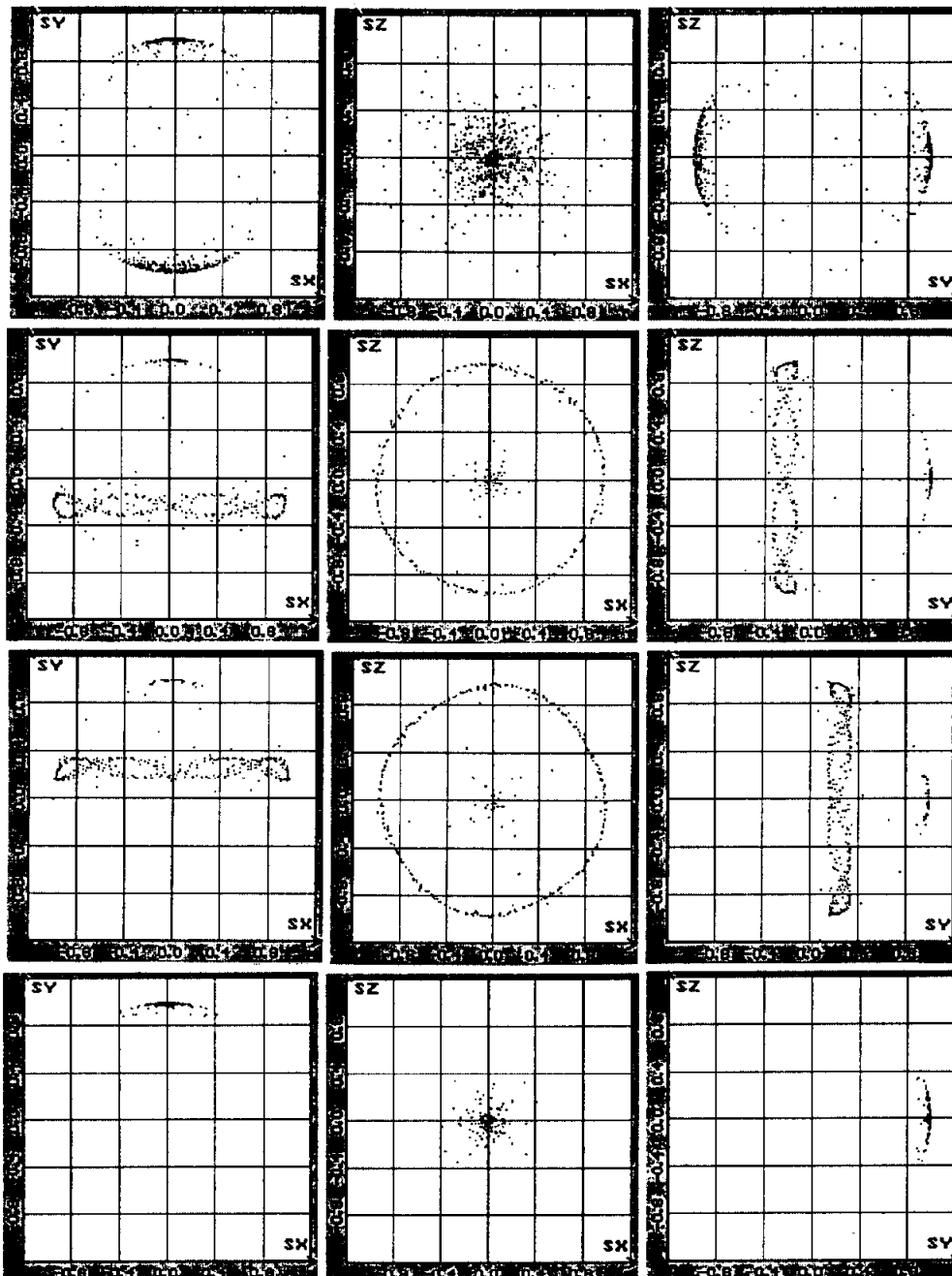


Figure 1:

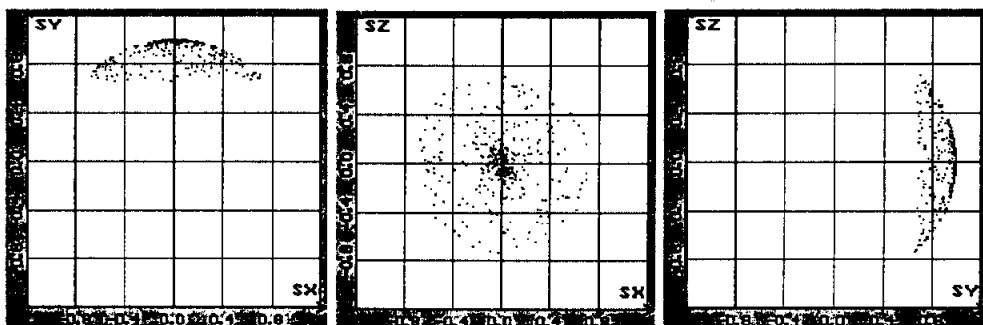


Figure 2:

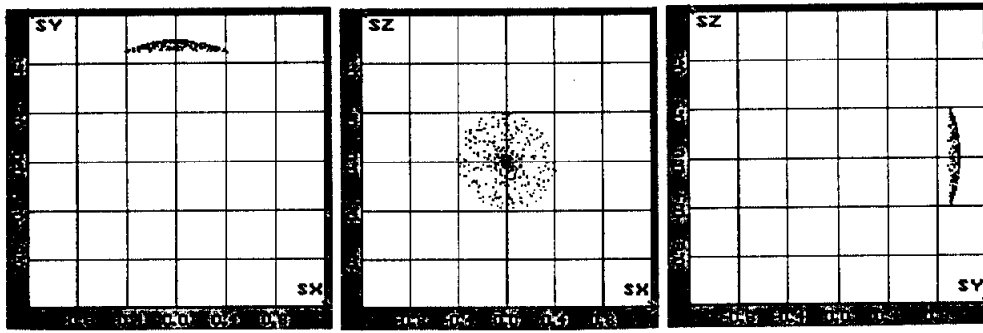


Figure 3:

orbital motion only. Using the Campbell-Baker-Hausdorff formula one can introduce a new vector

$$\vec{W}_1(\vec{x}, h) = \vec{W} + h \cdot \vec{U}_1 + h^2 \cdot \vec{U}_2 + \dots$$

so that for any given order k

$$\begin{aligned} \exp(-\frac{h}{2}H_{orb}) \cdot \exp(-h\vec{W}_1 \cdot \vec{S}) \cdot \exp(-\frac{h}{2}H_{orb}) = \\ = \exp(-h(H_{orb} + \vec{W} \cdot \vec{S})) + O(|h|^k) \end{aligned} \quad (3)$$

where h is the size of the integration step.

If we use the combination of some symplectic integration method of order k for orbital motion [1] and an evident formula for central Lie exponent in the left side of (3), we will obtain the numerical method of order k which preserves the Poisson bracket (2). Some methods of such type are realized in computer code VasiLIE [2].

III. NUMERICAL INVESTIGATIONS OF SPIN RESONANCE CROSSING

There are 6 intrinsic spin resonances in the acceleration region of a racetrack lattice under study for the TRIUMF KAON Factory Booster [3]. A part of them is in the intermediate region when a crossing speed is not very slow or very fast. In this part we present the results of numerical investigations of the resonance $\gamma G = 2 - \nu_y$ which is passed at the energy $E_k = 757 \text{ MeV}$.

Fig.1 shows turn-by-turn evolution of three spin projections during resonance crossing for a particle with $\delta p/p = 0$. These pictures were obtained with help of computer code VasiLIE [2] and correspond to different speeds α in formula (1).

The spin flip takes place with small depolarization for the crossing speed $\alpha \simeq 2 \times 10^{-4}$ (an upper part of Fig.1). It corresponds to a parameter $I = (\pi |w_k|^2)/(2\alpha) \simeq 0.6$ in formula (1). The bottom of Fig.1 shows the result of a fast resonance crossing with $I \simeq 0.01$. The rest of pictures show the intermediate cases for $I = 0.17$ and 0.09 .

At a sine change of a momentum during acceleration which is proposed for the Booster lattice, $I \simeq 0.3$.

The effect of nonlinearity influence is considered for a resonance $\gamma G = 2\nu_y$. Without sextupoles this resonance is due to the edge magnetic fields and is weak. So this resonance is nonlinear we have taken a large emittance to see its effect more clearly. Fig.2 shows the resonance crossing in the Booster lattice with sextupoles and Fig.3 without sextupoles.

ACKNOWLEDGMENTS

We would like to thank Prof. M. Craddock for support of this work. We would also like to thank Prof. Yu. Senichev, Prof. A.M. Kondratenko and Dr. A. Iliev for helpful discussions.

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