Third-Order Bending Magnet Optics for Cartesian Coordinates

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A thorough analisys of the charged particles dynamics on the basis of the matrix formalism up to the 3-rd order incluisve for the sector dipole magnets is usually held in a curvelinear coordinate system. In those cases, whenever the dipole is not a sectoral one, transformation to the curvelinear coordinates, associated with the trajectory of the central particle of the beam, doesn't proove itself, because of the difficulty of the physical interpretation of the obtained results.That makes necessary to analyse the dynamics of the beam in the bending magnet in the cartesian (rectangular) coordinate system.

1. Motion Equation in the Rectangular (Cartesian) Coordinates. Linear Approximation

It is well known that the motion of the charged particle with the charge q and mass m in a magnetic field with the induction $\vec{B}$ is determined by the Lorentz force:

$$\frac{d}{dt}(\gamma m\vec{v}) = -q/c\cdot[\vec{v}\times\vec{B}], \qquad (1.1)$$

where $\vec{v}$ - speed of a particle, $\gamma=(1-\beta^2)^{-1/2}$, $\beta=v/c$, c - light speed. We examine the rectangular (Cartesian) coordinate system XYZ with the axis OZ, directing stright alone the motion of a particle. In the linear approximation the magnetic field $\vec{B}=(0, B_0, 0)$ we would obtain the non-linear differential equation of the plane trajectory $x(z)$

$$x'' = -\frac{1}{\rho_0}\cdot(1+x'^2)^{3/2}, \qquad (1.2)$$

where $1/\rho_0 \equiv h = qB_0/(\gamma pc)$ - the curvature of the trajectory, $p = mv$ - the particle momentum. The analytic solution of Eq. (1.2) with the initial conditions of $x(0)=x_0$, $x'(0)=x'_0$, is well known[1]

$$\begin{cases} x(z)=\sqrt{\rho_0^2-(z-c_1)^2}+c_2,\; x'(z)=\dfrac{z-c_1}{\sqrt{\rho_0^2-(z-c_1)^2}}, \\ c_1=x'_0\cdot\dfrac{\rho_0}{\sqrt{1+x'^2_0}},\; c_2=x_0-\dfrac{\rho_0}{\sqrt{1+x'^2_0}}. \end{cases} \qquad (1.3)$$

The determination of the higher order optics for the Eq. (1.1) in Cartesian coordinates is more complicated. It is easier to determine the higher than the first order optics in curvelinear coordinate system with furter transformation back to XYZ. Such transformation is adequally described by changing of the 2-d and 3-rd order abberation coeffs only.

Let $\vec{X^0}=(x_0, x'_0, y_0, y'_0, l, \delta)$ be the initial conditions vector. $\vec{X}(z)$ we would perform in the form of the decomposition on the initial parameters X^0_i $(i=1,2)$

$$X_i(z)=R_i(z,\vec{X})+\sum_{j=1}^{6}\sum_{k=j}^{6}T^c_{ijk}(z)X^0_jX^0_k+ +\sum_{j=1}^{6}\sum_{k=j}^{6}\sum_{l=k}^{6}U^c_{ijkl}(z)X^0_jX^0_kX^0_l \qquad (1.4)$$

The index "c" shows that the coeffs were obtained in rectangular (Cartesian) coordinate system. With $i=3,..,6$ functions $R_i(z,\vec{X})$ are linear on $\vec{X}$ and correspond with the general type (*TRANSPORT*-type). We define the decomposition coeffs (1.4) following the formalism [2].

2. The 3-rd Order Beam Optics of a Dipole Magnet

The solution of the Eq. (1.1) in the curvelinear coordinates (especially in its linear approximation) is well known. That's why we would review general parts briefly.

The right-oriented curvelinear coordinate system XYS is chosen so, that the ort $\vec{s}$ should be aimed on a tangent to some, chosen before, respondent to the predifined specific initial conditions, of the *arbitrary (relative)* trajectory.

Because of the well known symmetry relation of the scalar magnet potential, in relation to the mid-plane, a particle, that starts in that plane won't leave it.

$$\varphi(x, y, s) = -\varphi(x, -y, s) \qquad (2.1)$$

We would decompose Eq. (1.1). To obtain the abberation coeffs of the decomposition (1.4), it is necessary to carry out the following procedures:

1. Decompose $B(x, y, s)$ close to the arbitrary trajectory, taking into account the symmetry relation (2.1).
2. Decompose Eq. (1.1) up to the necessary order.
3. Carry out the substitution of (1.3) into the obtained equation.
4. Generate the differencial equations, by equating the coeffs of the identical members.
5. Find out the linear independent solutions of the obtained equations, Green function, after what integrate it order after order, with the right parts of the mentioned above equations.

2.1 Decomposition of the B(x,y,s) Field.

To decompose the $B(x, y, s)$ we would use the Maxwell equation and the symmetry relation.

We rewrite the scalar potential in the form, correspondent with Eq. (2.1). To obtain the recurrent equations between the coeffs in φ- decomposition we would use the Laplas equation. Further by identifying the decomposition coeffs with a well known decomposition of the central field $B_y(x, 0, s)$, we wuold perform those coeffs in terms of the valueless coeffs of the so called "multiplicative force" $k_1(s)$, $k_2(s)$ и $k_3(s)$

$$B_y(x,0,s)=B_y(0,0,s)\cdot[1-k_1\,hx+k_2\,h^2x^2+k_3\,h^3\,x^3+...],$$

$$\begin{cases} k_1 = -\left[1/hB_y\,\partial B_y/\partial x\right]_{x=y=0} & (quadrupole), \\ k_2 = \left[1/2h^2B_y\,\partial^2 B_y/\partial x^2\right]_{x=y=0} & (sextupole), \\ k_3 = \left[1/6h^3B_y\,\partial^3 B_y/\partial x^3\right]_{x=y=0} & (octupole). \end{cases} \qquad (2.1.1)$$

Then the decomposition of the field components up to the 3-rd order are the following:

$$B_x(x,y,s)=p_0c/q\,[-h^2k_1\,y+2\,h^3\,k_2\,x\,y+3\,h^3\,x^2\,y+ +(-h^4k_3-h^4k_2/3+2hh'k'_1/3+h^2k''_1/6+h'^2k_1/3+ +h\,h''\,k_1/3-h^4\,k_1/6+h'^2/6+h\,h''/3)\,y^3+...],$$

$$B_y(x,y,s)=p_0c/q\,[\,h-h^2\,k_1\,x+h^3\,k_2\,x^2+h^4\,k_3\,x^3+ +(-h^3\,k_2+h^3\,k_1/2-h''/2)\,y^2+(-3h^4\,k_3-h^4\,k_2+ +2hh'k'_1+h^2k''_1/2+h'^2k_1+hh''k_1-h^4k_1/2+h'^2/2+ +hh'')xy^2+..],$$

$$B_s(x,y,s)=p_0c/q\,[h'\,y+(-h^2\,k'_1-2\,h\,h'\,k_1-hh')\,xy+ +(-h^3\,k'_2/3-h^2\,h'\,k_2+h^3\,k'_1/6+h^2\,h'\,k_1/2)\,y^3+ +(h^3\,k'_2+3h^2h'k_2+h^3k'_1+2h^2h'k_1+h^2h')x^2y+...]. \qquad (2.1.2)$$

2.2 Decomposition of the Motion Equation.

The result form of the motion equation in the X- and Y-plane is the following

$$x''-(1-k_1)h^2x=h\delta-(1-2k_1+k_2)h^3x^2+h'xx'+$$
$$+(2-k_1)h^2x\delta+hx'^2/2+(h''-h^3(k_1-k_2))y^2+$$
$$+h'yy'-hy'^2/2-h\delta^2+(k_1-2k_2-k_3)h^4x^3-$$
$$-hh'x^2x'+(1-2k_1+k_2)h^3x^2\delta-(2-2k_1/3)h^2xx'^2-$$
$$-(h'^2/2+k_1(h^4/2+hh''+h'^2)+2hh'k'_1+h^2k''_1-$$
$$-3h^4(k_2+k_3))xy^2-(hk'_1+2h'k_1)hxyy'+$$
$$+h^2k_1xy'^2/2-(2-k_1)h^2x\delta^2+3hx'^2\delta-$$
$$-h^2k_1x'yy'-(h''-h^3k_1+2h^3k_2)y^2\delta/2-h'yy'\delta+$$
$$+hy'^2\delta/2+h\delta^3), \qquad (2.2.1)$$

$$y''+h^2k_1y=2(k_2-k_1)h^3xy+h'xy'-h'x'y+hx'y'+$$
$$+h^2k_1y\delta-(k_1-4k_2-3k_3)h^4x^2y-hh'x^2y'+(2h'k_1+$$
$$+hk'_1)hxx'y-(2-k_1)h^2xx'y+2(k_1-k_2)h^3xy\delta-$$
$$-h^2k_1x'^2y/2+h'x'y\delta+hx'y'\delta+(hh''/3+h'^2-$$
$$-(h^4/6-hh''/3-h'/3)k_1++2hh'k'_1+h^2k''_1/6-$$
$$-h^4k_3)y^3-3h^2k_1yy'^2/2-h^2k_1y\delta^2+\ldots \qquad (2.2.2)$$

2.3 The Differencial Equations of the Matrix Elements.

The further algorithm of soluting the Eqs. (2.2.1)-(2.2.2) is evident. To determine the linear independent solutions we examine the linear parts of those equations:

$$\begin{cases} x''-(1-k_1)h^2x=h\delta, \\ y''+k_1h^2y=0. \end{cases} \qquad (2.3.1)$$

Each of the mentioned above equations has at least two linear independent solutions. The general solution of the equations, as it is well known, is the linear combination of the linear independent solutions with the arbitrary coeffs. Besides all the equation for the determination of the $x(s)$ with the right part has also a private solution, that is one of the components of the general one. Let's define the most general form of the mentioned solutions in the bending plane as:

1. *Sine-like function* $s_x(s): s_x(0)=0, s'_x(0)=1, \delta=0$.
2. *Cosine-like function* $c_x(s): c_x(0)=1, c'_x(0)=1, \delta=0$.
3. *Dispersion function* $d_x(s): d_x(0)=0, d'_x(0)=0, \delta=1$.
4. *Sine-like function* $s_y(s): s_y(0)=0, s'_y(0)=1, \delta=0$.
5. *Cosine-like function* $c_y(s): c_y(0)=1, c'_y(0)=1, \delta=0$.

Those functions define the so-called *characteristic rays* of the arbitary magnetic system and all of it's abberation coeffs. The common solutions of the Eq. (2.3.1) with the initial conditions (ch. 1) is the following:

$$\begin{cases} x(s)=c_x(s)\cdot x_0+s_x(s)\cdot x'_0+d_x(s)\cdot\delta, \\ y(s)=c_y(s)\cdot y_0+s_y(s)\cdot y'_0. \end{cases} \qquad (2.3.2)$$

That common form of the generated solutions makes evident that the Green function $-G(s,\xi)=s(s)\,c(\xi)-c(s)\,s(\xi)$, and the private solution of the nongomogenius equation $q''+k^2q=f$: should be generated by means of the integral

$$q=\int_0^s G(s,\xi)f(\xi)\,d\xi. \qquad (2.3.3)$$

2.4 Differencial Equations of the Abberation Coeffs.

The abberation coeffs of the matrices $R_{ij}(s)$, $T_{ijk}(s)$, $U_{ijkl}(s)$ are the solutions of the nonhomogenius garmonic oscillation differential equations of the Eq. (2.3.3) form with the null initial conditions. We would state that the right parts of the equations for obtaining the elements T_{ijk} - are the square forms of the 1-st order coeffs. The driving forces of the 3-rd order coeffs $U_{ijkl}(\xi)$ have a more complicated form. We notice that the "angle" elements of the matrix *(i=2,4)* are calculated by differentiating on s of the "coordinate" elements *(i=1,3)*.

3. Linear Approximation

Magnetic field of the "pure" dipole $B=B(0,B_0,0)$, $h(s)=1/\rho_0=const$, $k_1=k_2=k_3=0$.

$$\begin{cases} s_x=\rho_0\sin s/\rho_0\,,\ c_x=\cos s/\rho_0\,, \\ s_y=y\,,\ c_y=1\,. \end{cases} \qquad (3.1)$$

Green functions on projections $(s\geq\xi)$

$$\begin{cases} G_x(s,\xi)=\rho_0\sin((s-\xi)/\rho_0)\,, \\ G_y(s,\xi)=s-\xi\,. \end{cases} \qquad (3.2)$$

Dispersional function $d_x(s)=\rho_0(1-c_x(s))$.

The non-zero matrix elements R_{ij}

$R_{11}=c_x$, $R_{12}=s_x$, $R_{16}=\rho_0(1-c_x)$, $R_{21}=-s_x/\rho_0^2$, $R_{22}=c_x$, $R_{26}=s_x/\rho_0$, $R_{33}=1$, $R_{34}=s$, $R_{44}=1$, $R_{51}=s_x/\rho_0$, $R_{52}=\rho_0(1-c_x)$, $R_{55}=1$, $R_{56}=s-s_x$, $R_{66}=1$.

4. Nonlinear Abberation Coeffs

The non-zero coeffs of the 2-d order:

$T_{111}=-(1-c_x^2)/2\rho_0$, $T_{112}=s_xc_x/\rho_0$, $T_{116}=s_x^2$, $T_{122}=\rho_0s_x(1-c_x)$, $T_{126}=s_x(1-c_x)$, $T_{144}=-\rho_0(1-c_x)$, $T_{166}=-s_x^2/2\rho_0$, $T_{314}=s\,s_x/\rho_0$, $T_{324}=\rho_0s(1-c_x)$, $T_{346}=s-s_x$.

The 3-rd order:

$U_{1111}=c_x^3h^4/8-c_xh^4/8+c_x^5h^2/8-c_x^3h^2/4+c_xh^2/8$,

$U_{1112}=c_x^2h^4s_x/8+c_x^4h^2s_x/8-c_x^2h^2s_x/8$,

$U_{1116}=h^3s_xs/2-hs_xs/2-11c_xh^5s_x^4/8-h^5s_x^4/2+$
$+c_x^5s_x^4/2+c_xh^3s_x^4+5c_xh^5s_x^2/8-c_x^3h^3s_x^2/2+$
$+h^3s_x^2+c_x^3hs_x^2/2+c_x^5/h/4+c_x^4h/2-c_x^2h/2-c_xh/4$,

$U_{1122}=--h^2s_xs/4-3c_x^2h^2s_x/8-3c_x^4s_x/8-c_x^3s_x/8-$
$-c_xs_x+3s_x/2-c_x^3h^2/8+c_xh^2/8-c_x^5/8+c_x^3/3+$
$+c_x^2-11c_x/8$,

$U_{1126}=-c_xhs/2+c_xs/2h+c_x^2h^3s_x/4+c_x^4hs_x/4-$
$-c_x^4s_x/4h-c_x^2s_x/4h$,

$U_{1144}=-h^2s_xs^2/4+h^2s_xs/2+c_xs/4-s_x/4+c_x^2/2-1/2$,

$U_{1166}=-h^2s_xs/2+s_xs/2+c_xh^4s_x^4+h^4s_x^4-c_xh^2s_x^4-$
$-c_xh^4s_x^2/2++c_x^3h^2s_x^2/2+c_xh^2s_x^2/2-2h^2s_x^2-$
$-c_x^3s_x^2/2-c_x^4+c_x^2$,

$U_{1222}=c_xs/4+c_x^2h^2s_x/4+c_x^4s_x/4-s_x/2-3c_x^5/8h^2+$
$+5c_x^3/4h^2-c_x^2/h^2+c_x/8h^2-3c_x^3/8+3c_x/8$,

$U_{1226}=hs_xs/4+3c_x^2hs_x/8+3c_x^4s_x/8h+c_x^2s_x/8h-$
$-c_xs_x/h+s_x/2h+c_x^3h/8-c_xh/8+c_x^5/8h-c_x^3/2h-$
$-c_x^2/2h+7c_x/8h$,

$U_{1244}=s_xs^2/4-c_xs/4h^2-c_xs/2+s_x/4h^2+c_xs_x/2+$
$+c_x/h^2-1/h^2$,

$U_{1446}=c_x/2h-c_x^2/2h$, $U_{1666}=(1-c_x^2)/2h$,

$U_{3114}=-h^2s^2/4+c_xh^2s_xs/4+h^2s_xs+h^2s/4-$
$-c_xh^2s_x/4-h^2s_x$,

$U_{3124}=h^2s^3/12+s^2/4-c_xs_xs/4-2s_xs-c_x^2s/4+$

$+5\,s/4+s_x+c_x^2/2+2\,c_x-5/2,$

$U_{3146}=h\,s^2/4-c_x\,h\,s_x\,s/4-3\,h\,s/4+s/2h+$
$+3\,c_x\,h\,s_x/4-c_x\,s_x/2h\,,$

$U_{3224}=-s^3/12+c_x^3\,s/4h^2-s/4h^2-s/2+c_x\,s_x/2+$
$+c_x^2/2h^2-2\,c_x/h^2+3/2h^2\,,$

$U_{3246}=-h\,s^2/4-s^2/4h+c_x\,s_x\,s/4h+s/h-s_x/h-$
$-c_x^2/4h-c_x/h+5/4h,\ \ U_{3444}=-s_x/2-s/2.$

Nine elements T_{5jk} and nineteen elements U_{5jkl} are not listed.

5. The Cartesian Coordinates

To gain the physically trustworthy results it is necessary to transform the data to the rectangular (Cartesian) coordinate system XYZ. We would agree on the following designations:

$$\begin{cases} x_1=x\,,\ x_2=dx/dz=x'/(\,1+hx)\,,\ x_3=y\,, \\ x_4=dy/dz=y'/(\,1+hx)\,,\ x_5=l\,,\ x_6=\delta\,. \end{cases} \quad (5.1)$$

It is important to notice that results obtained in different coordinate systems differ only with the non-linear members. It won't be difficult to obtain the Cartesian elements T_{ijk} and U_{ijkl} of the matrix, having marked them with the "c". Here we discuss only those elements that differ from one system to another.

$T^c{}_{112}=T_{112}+h\,s_x\,,\ T^c{}_{211}=T_{211}-h\,c_x\,c'_x\,,$

$T^c{}_{212}=T_{212}+h\,s'_x-h\,(\,c_x\,s'_x-c'_x\,s_x)\,,$

$T^c{}_{216}=T_{216}-h\,(\,c_x\,d'_x+c'_x\,d_x)\,,\ T^c{}_{222}=T_{222}-hs_xs'_x,$

$T^c{}_{226}=T_{226}-h\,(\,s_x\,d'_x+s'_x\,d_x)\,,$

$T^c{}_{266}=T_{266}-h\,d_x\,d'_x\,,\ T^c{}_{314}=T_{314}+h\,s_y\,,$

$T^c{}_{413}=T_{413}-h\,c_x\,c'_y\,,\ T^c{}_{414}=T_{414}+h\,s'_y-h\,c_x\,s'_y\,,$

$T^c{}_{423}=T_{423}-h\,s_x\,c'_y\,,\ T^c{}_{424}=T_{424}-h\,s_x\,s'_y\,,$

$T^c{}_{436}=T_{436}-h\,c'_y\,d_x\,,\ T^c{}_{446}=T_{446}-h\,s'_y\,d_x,$

$T^c{}_{512}=T_{512}+h\,R_{52}\,.$

The 3-rd order:

$U^c{}_{1112}=U_{1112}+T_{112}\,h\,,\ U^c{}_{1122}=U_{1122}+2\,T_{122}\,h\,,$

$U^c{}_{1126}=U_{1126}+T_{126}\,h\,,\ U^c{}_{1134}=U_{1134}+T_{134}\,h\,,$

$U^c{}_{1144}=U_{1144}+2\,T_{144}\,h\,,$

$U^c{}_{2111}=U_{2111}+c_x^2\,c'_x\,h^2-(\,T_{111}\,c'_x+T_{211}\,c_x)\,h\,,$

$U^c{}_{2112}=U_{2112}+((c_x^2-c_x)h^2-T_{111}h)s'_x+((2c_x-1)\,c'_xh^2-$
$-T_{211}h)\,s_x-(T_{112}c'_x-T_{212}c_x-T_{212})\,h,$

$U^c{}_{2116}=U_{2116}+(\,c_x^2\,d'_x+2\,c_x\,c'_x\,d_x)\,h^2-(\,T_{111}\,d'_x+$
$+T_{211}\,d_x+T_{116}\,c'_x+T_{216}\,c_x)\,h\,,$

$U^c{}_{2122}=U_{2122}+((2c_x-2)\,h^2\,s_x\,T_{112}h)\,s'_x+c'_x\,h^2\,s_x^2-$
$-T_{212}h\,s_x-(\,T_{122}\,c'_x+T_{222}\,c_x-2T_{222})\,h,$

$U^c{}_{2126}=U_{2126}+((2c_x-1)\,h^2\,d_x-T_{116}\,h)\,s'_x+(((2c_x-$
$-1)\,d'_x+2\,c'_xd_x)\,h^2-T_{216}h)\,s_x-(\,T_{112}d'_x+T_{216}d_x+$
$+T_{126}\,c'_x+T_{216})\,h\,,$

$U^c{}_{2133}=U_{2133}-(\,T_{133}\,c'_x-T_{233}\,c_x)\,h\,,$

$U^c{}_{2134}=U_{2134}-(\,T_{134}\,c'_x+T_{234}\,c_x-T_{234})\,h\,,$

$U^c{}_{2144}=U_{2144}-(\,T_{144}\,c'_x+T_{244}\,c_x-2\,T_{244})\,h\,,$

$U^c{}_{2166}=U_{2166}+(\,c_x\,d_x\,d'_x+c_x\,d_x^2)\,h^2-(\,T_{116}\,d'_x+$
$+T_{216}\,d_x+T_{166}\,c'_x+T_{266}\,c_x)\,h\,,$

$U^c{}_{2226}=U_{2226}+(\,2\,d_x\,h^2\,s_x-T_{126}\,h)\,s'_x-d'_x\,h^2\,s_x^2-$
$-T_{226}\,h\,s_x-(\,T_{122}\,d'_x+T_{222}\,d_x)\,h\,,$

$U^c{}_{2233}=U_{2233}-(\,T_{133}\,s'_x+T_{233}\,c_x)\,h\,,$

$U^c{}_{2234}=U_{2234}-(\,T_{134}\,s'_x+T_{234}\,c_x)\,h\,,$

$U^c{}_{2244}=U_{2244}-(\,T_{144}\,s'_x+T_{244}\,c_x)\,h\,,$

$U^c{}_{2266}=U_{2266}+(\,d_x^2\,h^2-T_{166}\,h)\,s'_x-(\,2\,d'_x\,d'_x\,h^2-$
$-T_{266}\,h\,)\,s_x-(\,T_{126}\,d'_x+T_{226}\,d_x)\,h\,,$

$U^c{}_{2336}=U_{2336}-(\,T_{133}\,d'_x+T_{244}\,d_x)\,h\,,$

$U^c{}_{2346}=U_{2346}-(\,T_{134}\,d'_x+T_{234}\,d_x)\,h\,,$

$U^c{}_{2446}=U_{2446}-(\,T_{144}\,d'_x+T_{244}\,d_x)\,h\,,$

$U^c{}_{2666}=U_{2666}+d_x^2\,d'_x\,h^2-(\,T_{166}\,d'_x+T_{266}\,d_x)\,h\,,$

$U^c{}_{3114}=U_{3114}+T_{314}\,h\,,\ U^c{}_{3123}=U_{3123}+T_{323}\,h\,,$

$U^c{}_{3124}=U_{3124}+2\,T_{324}\,h\,,\ U^c{}_{3146}=U_{3146}+T_{346}\,h\,,$

$U^c{}_{4113}=U_{4113}+c_x^2\,c'_y\,h^2-(\,T_{111}\,c_y^2+T_{413}\,c_x)\,h\,,$

$U^c{}_{4114}=U_{4114}+((c_x^2-c_x)h^2-T_{111}h)s'_y+(T_{414}-T_{414}c_x)h\,,$

$U^c{}_{4123}=U_{4123}+((2\,c_x-1)\,c'_yh^2-T_{413}\,h)\,s_x-(\,T_{112}c'_y+$
$+T_{423}\,c_x-T_{423})\,h\,,$

$U^c{}_{4124}=U_{4124}+((2c_x-2)s_xh^2-T_{112}h)s'_y-(\,T_{414}s_x+$
$+T_{424}\,c_x-2\,T_{424})\,h\,,$

$U^c{}_{4136}=U_{4136}+2c_xc'_yd_xh^2-(T_{413}d_x+T_{116}c'_y+T_{436}c_x)h\,,$

$U^c{}_{4146}=U_{4146}+((2c_x-1)\,d_xh^2-T_{116}\,h)\,s'_y-(\,T_{414}d_x+$
$+T_{446}\,c_x-T_{446})\,h\,,$

$U^c{}_{4223}=U_{4223}+c'_y\,s_x^2\,h^2-(\,T_{112}\,c'_y+T_{423}\,s_x)\,h\,,$

$U^c{}_{4224}=U_{4224}+(\,s_x\,h^2-T_{122}\,h)\,s'_y-T_{424}\,h\,s_x\,,$

$U^c{}_{4236}=U_{4236}+(2c'_yd_xh^2-T_{436}h)s_x-(T_{423}d_x+T_{126}c'_y)h\,,$

$U^c{}_{4246}=U_{4246}+(2s_xd_xh^2-T_{126}h)s'_y-(T_{446}s_x+T_{424}d_x)h\,,$

$U^c{}_{4333}=U_{4333}-T_{133}\,c'_y\,h\,,$

$U^c{}_{4334}=U_{4334}-(\,T_{133}\,s'_y-T_{134}\,c'_y)\,h\,,$

$U^c{}_{4366}=U_{4366}+c'_y\,d_x^2\,h^2-(\,T_{166}\,c'_y+T_{436}\,d_x)\,h\,,$

$U^c{}_{4444}=U_{4444}-T_{144}\,s'_y\,h\,,$

$U^c{}_{4466}=U_{4466}+(\,d_x^2\,h^2-T_{166}\,h\,)\,s'_y-T_{446}\,d_x\,h\,,$

$U^c{}_{5112}=U_{5112}+T_{512}\,h\,,\ U^c{}_{5122}=U_{5122}+2\,T_{522}\,h\,,$

$U^c{}_{5126}=U_{5126}+T_{526}\,h\,,\ U^c{}_{5134}=U_{5134}+T_{534}\,h\,,$

$U^c{}_{5144}=U_{5144}+2\,T_{544}\,h\,.$

We notice that such algorithm might be applied to the research of any multipoles of the higher order.

6. The Realisation of the Method

The model described above was used as a basis of it's program realisation on IBM PC/AT. The main problem, as we stated before, was that the decomposition of the motion equation and field components was held within the arbitrary trajectory (in the curvelinear coordinates XYS), though the results had to be represented in a Cartesian coordinates XYZ. The correction of the 2-d and 3-rd order abberation coeffs eliminates only part of the problem. The transformation to the Cartesian coordinate system involves the difinition of the equation of the arbitrary trajectory, thus the solution Eq. (1.2) was found, with the substitution of the linear part of the decomposition $\bar{X}(z)$ (1.4). Such substitution is prooved with the fact, that the generated relative trajectories of the beams with the momenta of more than *1 GeV/c* are of a low difference with the geometrical axis of a magnet.

Referencies

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