Synchrobetatron Resonances Due to Crab Cavities

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Abstract

Perturbation of the particle motion by a crab-cavity can excite the synchrobetatron resonances. We estimate the the tolerances of the residual dispersion function in the crab-cavity as well as of the chromatic distortion of the phase advance between cavities due to these resonances.

I. INTRODUCTION

An important option of the B-factories with close by spacing bunches inside the beam is the use of a large crossing angle collision schemes [1,2]. In order to avoid the loss of the luminosity and synchrobetatron resonances, which are specific to the conventional crossing angle schemes [3], it was suggested [4] to use the so-called crab-crossing scheme, which initially was invented for linear colliders [5]. In this report we discuss the tolerances for the ring imperfections, related to synchrobetatron resonances due to crab-cavity. We assume the scheme, where bunches are tilted in the horizontal plane by RF-kickers, placed at points, with $\mp\pi/2$ phase advances of the horizontal betatron oscillations from the collision point. For numerical estimations we use the parameters reported in [2].

II. DISPERSION IN A CRAB-CAVITY

TM110-mode with a transverse deflecting voltage

$$V = \frac{cE\phi}{\Omega\sqrt{\beta_x^* \beta_{crab}}} \tag{1}$$

can give a necessary kick. Here, E is the particle energy, 2ϕ crossing angle, Ω RF-frequency of the cavity, β_x^* β-function at the IP and β_{crab} β-function at the crab-cavity. To produce the crossing angle of 50 mrad, the deflecting voltages must be 0.82 MV for LER and 1.9 MV for HER. For the sake of simplicity we neglect the effect of the edge fields of the crab-cavity and assume that a vector potential of the deflecting TM110-mode is

$$\mathbf{A}_\perp = 0, \quad A_\parallel = -V\frac{2x}{k_1 r}J_1(k_1 r)\Delta(s)\sin(\Phi). \tag{2}$$

Here, $\Phi = \Omega t + h\varphi_0$, φ_0 is the phase of the synchronous particle, h the RF-harmonic number, $J_1(x)$ the Bessel function of the 1st order, γ_1 its first root ($\gamma_1 \simeq 3.832$), $k_1 = \gamma_1/b$, b the radius of the cavity; $r^2 = x^2 + z^2$;

$$\Delta(s) = \delta(s + L/2) + \delta(s - L/2), \tag{3}$$

where L is the distance between cavities. We take that the oscillations of a particle near the closed orbit are described by the following equations ($\tau = \omega_0 t$ is taken as an independent variable, $\gamma\alpha \gg 1$)

$$\begin{gathered}
x = x_b + \eta\frac{\Delta p}{p}, \quad x_b = a_x\cos\psi_x, \quad z = a_z\cos\psi_z \\
\theta = \tau + \varphi, \quad \varphi = \varphi_s\cos\psi_s, \quad \varphi' = \nu_s\varphi_s\sin\psi_s, \\
I_{x,z} = \frac{p(\nu a^2)_{x,z}}{2R_0} = \frac{pJ_{x,z}}{2}, \quad \psi'_{x,z} = \nu_{x,z}, \\
I_s = \frac{pR_0\nu_s\varphi_s^2}{2\alpha} = \frac{pJ_s}{2}, \quad \psi'_s = -\nu_s.
\end{gathered} \tag{4}$$

The hamiltonian part of equations of motion of a perturbed particle is generated by the following Hamiltonian

$$\begin{gathered}
H = \nu_x J_x + \nu_z J_z - \nu_s J_s + U_{bb} - W\Delta(s)\sin(\Phi), \\
W = W_0\frac{2x}{k_1 r}J_1(k_1 r), \quad W_0 = \frac{2eVR_0}{E},
\end{gathered} \tag{5}$$

where U_{bb} describes the beam-beam interaction. Provided that the dispersion function in the cavities is zero, and the betatron phase advance between the tilting and restoring cavities is π, the Hamiltonian in Eq.(5) predicts only resonances due to the beam-beam interaction. We assume that the working point of the ring is chosen outside the stopbands of the beam-beam instability. Then, the term U_{bb} describes only the beam-beam tune shifts. For a bunch with a very flat, Gaussian distribution in transverse coordinataes the tune shift of the horizontal betatron oscillations is

$$\Delta\nu_x = \xi_x\frac{1-\exp(-J_x/2\epsilon_x)}{J_x/2\epsilon_x}, \quad \xi_x = \frac{Ne^2}{2\pi E\epsilon_x}. \tag{6}$$

Assuming $r \ll b$ and using a Taylor expansion of the Bessel function, we write

$$\delta W = W - W_0 x = -W_0 x\left(1.83\frac{r^2}{b^2} - 1.121\frac{r^4}{b^4} + \dots\right). \tag{7}$$

The lowest synchrobetatron resonances due to the residual dispersion are made by $\delta W_1\Delta(s)\sin(\Phi)$, where

$$\delta W_1 = -1.83W_0\eta\frac{\Delta p}{p}\frac{3x_b^2 + z^2}{b^2}. \tag{8}$$

These resonances correspond to combinations $2\nu_x + m_s\nu_s = n - \Omega/\omega_0$ and $2\nu_z + m_s\nu_s = n - \Omega/\omega_0$, where $m_s = 2l$, l

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and n are integers. If we take $\Omega/\omega_0 = h$, the averaging of the Hamiltonian (5) near, for instance, the resonance $2\nu_x = m_s\nu_s$ results in

$$H = H_0 + \xi_x F + \Lambda|\eta m_s J_{m_s}(h\varphi_s)|J_x\cos\chi,$$
$$H_0 = \nu_x J_x - \nu_s J_s, \quad F = 2\epsilon_x \int_0^{J_x/2\epsilon_x} dt\frac{1-e^{-t}}{t}, \tag{9}$$
$$\Lambda = 5.5\frac{eV\nu_s\beta_{crab}}{\pi b^2 Eh\alpha}, \quad \chi = 2\psi_x - m_s\psi_s.$$

Due to $J_m(x) \simeq x^m/(2^m m!)$, when $x < 1$, this instability mainly affects the particles with large amplitudes of synchrotron oscillations.

Since H in Eq.(9) depends only on one phase variable χ, we can use an additional integral of motion $C = (J_s/m_s) - (J_x/2)$ to reduce the study of a 4-dimensional problem described by H to the study of an equivalent two-dimensional problem, described by the Hamiltonian $H'(J_x,\chi) = H[J_x, m_s(J_x/2+C),\chi]$. The character of the trajectories in the phase-space (J_x,χ) can be figured out inspecting the behaviour of the curves $H^{\pm}(J_x) = H'(J_x, \cos\chi = \pm 1)$. The oscillations are stable, if the horizontal line $H' = const$ starting, for instance, from the curve $H^+(J)$ crosses the curve $H^-(J)$, or crosses the curve $H^+(J)$ again. Otherwise, the Hamiltonian $H'(J_x,\chi)$ describes unstable oscillations(see, for instance, Ref.[6]).

From Eq.(9), we obtain ($\Delta = \nu_x - m_s\nu_s/2$)

$$H^{\pm} = \Delta \pm \Lambda|\eta m_s J_{m_s}(h\varphi_s)| + \xi_x F,$$
$$\varphi_s^2 = (\varphi_s)^2_{in} + \frac{\alpha\epsilon_x m_s}{2R_0\nu_s}\left(\frac{J_x - J_{xin}}{\epsilon_x}\right), \tag{10}$$

where φ_{sin} is initial amplitude of synchrotron oscillations and J_{xin} initial value of J_x. As far as $F \simeq 2\epsilon_x\ln(J_x/\epsilon_x)$, when $J_x \gg \epsilon_x$, the Hamiltonians $H^{\pm}$ describe unstable oscillations provided that

$$|\Delta| \le \Delta_{th} = \Lambda|\eta m_s J_{m_s}(h\varphi_s)|. \tag{11}$$

In the colliding beam mode the instability at small amplitudes is suppressed by a nonlinearity of the beam-beam kick. At large amplitudes of betatron oscillations the beam-beam nonlinearity becomes too weak to suppress the instability. The balance between excitation and suppression effects determines the dynamic aperture of the ring. As can be seen from Fig.1, on exact resonance $2\nu_x = 2\nu_s$ the instability limits dynamic aperture of tail particles at $a_x \simeq 20\div30\sigma_x$. However since the value Δ_{th} indeed is very small ($\Delta_{th} = 10^{-4}$), this resonance is very narrow and can be easily avoided by a small variation of either ν_x, or ν_s.

For KEK B-factory $(\beta_{crab})_x \simeq 2(\beta_{crab})_z$ [2] and, therefore the strengths of the vertical synchrobetatron resonances are 6 times smaller then the strengths of the corresponding horizontal resonances. However, since this instability determines vertical dynamic aperture in terms of σ_z, due to small aspect ratio ($\sigma_z \ll \sigma_x$) the limitation of the vertical dynamic aperture due to this instability can be more severe.

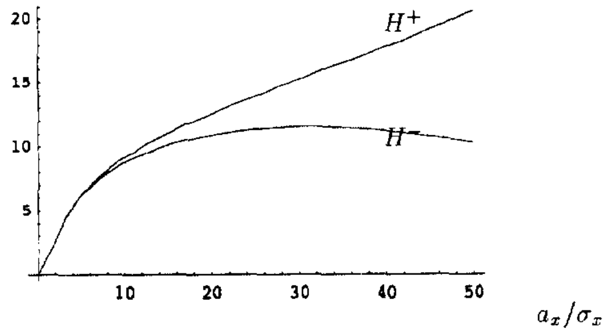

Figure 1: Hamiltonians $H^{\pm}$; $\eta = 10cm$, $m_s = 2$, $A_s = 20\sigma_s$, $a_{xin} = 20\sigma_x$, $\Delta = 0$.

III. CHROMATIC DISTORTIONS

Stronger perturbations can be caused by the dependence of the phase advance of the horizontal betatron oscillations

$$\Delta\psi = \int_{-L/2}^{L/2}\frac{ds}{\beta(s)} \tag{12}$$

on the particle momentum $\beta(s,\Delta p) \simeq \beta(s)(1+\zeta\Delta p/p)$, where $\zeta = (\partial\ln\beta/\partial\ln p)$. Assuming that $|\zeta\Delta p/p| \ll 1$ and that $\zeta = const$ between the tilting and restoring cavities, the additional phase advance ($\Delta\psi = \pi + \delta\psi$) becomes $\delta\psi \simeq -\pi\zeta(\Delta p/p)$. Then, a combination $W\Delta(s)\sin(\Phi)$ excites the following set of resonances: $\nu_x + m_s\nu_s = n$, $3\nu_x + m_s\nu_s = n$, $\ldots$, $m_s = 2l$. The lowest family ($\nu_x + m_s\nu_s = n$) is described by the perturbation ($\vartheta = \sqrt{\beta_{crab}/\epsilon_x}$)

$$\delta H = \Lambda_1\sqrt{\frac{J_x}{\epsilon_x}}\cos(\chi), \quad \Lambda_1 = \zeta\frac{eVm_s\nu_s|J_{m_s}(h\varphi_s)|}{Eh\alpha\vartheta}. \tag{13}$$

Due to $\delta H \sim \sqrt{J_x}$, this perturbation can open Hamiltonians $H^{\pm}$ for resonant particles ($\Delta = \nu_x + m_s\nu_s - n < 0$) in the region $J_x \sim \epsilon_x$ (see Fig.2). Fig.3 shows examples of the trajectories for such particles in the slow phase-space ($x = \sqrt{J_x/\epsilon_x}\cos\chi$ and $p = -\sqrt{J_x/\epsilon_x}\sin\chi$). These

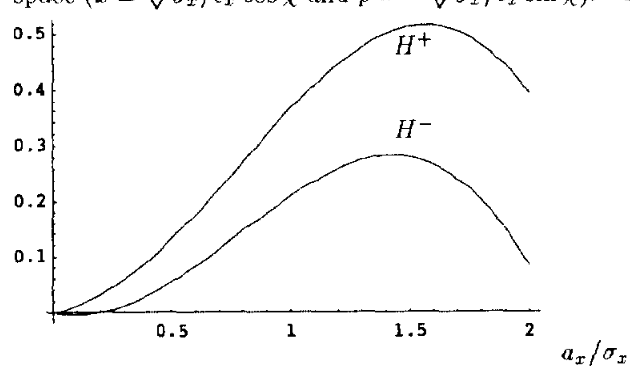

Figure 2: Hamiltonians $H^{\pm}$; $\zeta = 1$, $m_s = 2$, $A_s = 5\sigma_s$, $a_{xin} = \sigma_x$, $\Delta = -.6\xi$, $\xi = .05$.

curves were calculated taking into account the synchrotron radiation damping and neglecting the variation of φ_s due

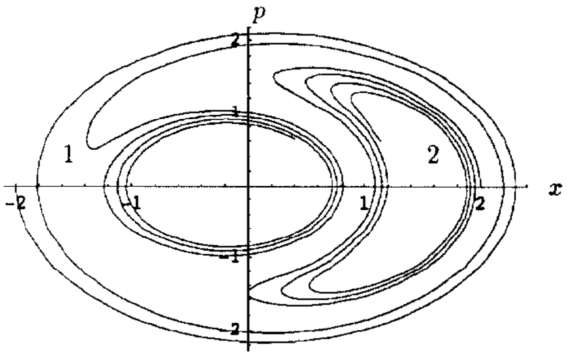

Figure 3: Phase trajectories corresponding to the Hamiltonians $H^{\pm}$ shown in Fig.2; $\lambda_{sr} = 10^{-4}\omega_0$; $p_{in} = 0$; 1. $x_{in} = -2$, 2. $x_{in} = 2$.

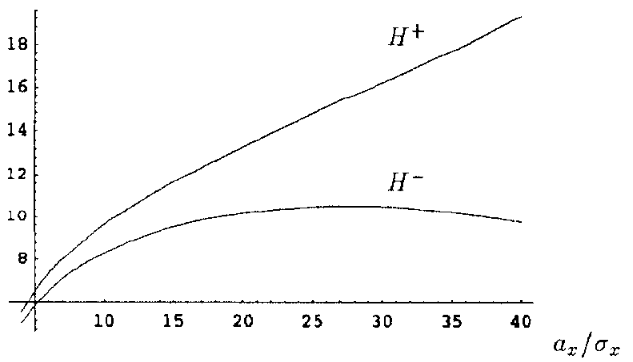

Figure 4: Hamiltonians $H^{\pm}$; $\zeta = 1$, $m_s = 2$, $A_s = 5\sigma_s$, $a_{xin} = 20\sigma_x$, $\Delta = 0$, $\xi = .05$.

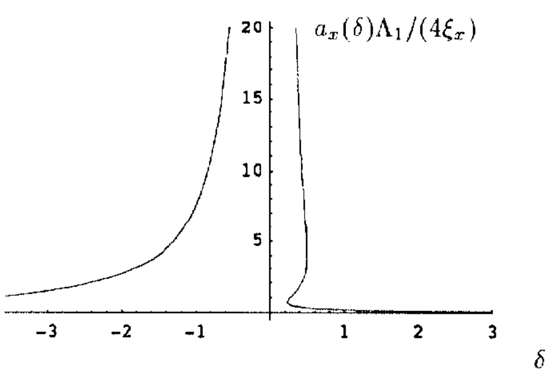

Figure 5: Resonance curve for horizontal oscillations; $\delta = 16\Delta\xi_x/\Lambda_1^2$, $\zeta = 1$, $m_s = 2$.

to the variation of J_x. At large amplitudes ($J_x \gg \epsilon_x$) and $\Delta = 0$ Hamiltonian in Eq.(13) describes instable oscillations (see Fig.4) which can limit the dynamic aperture of the ring. As seen from Fig.5, the width of unstable region is $\Delta < \Lambda_1^2/(16\xi_x)$. For KEK B-factory [2] and $\zeta = 1$ this gives $\Delta < 0.01$.

IV. CONCLUSION

In this paper we showed that without special efforts the perturbations due to residual dispersion in a crab-cavity and chromatic distortions limit the dynamic aperture of the ring, if the working point approaches the lines of the synchrobetatron resonances. Since the value of the dynamic aperture essentially depends on the nonlinearity of the beam-beam deflecting force, one can expect the decrease of the dynamic aperture when ξ decreases (such a decay may occur due to, say, the loss of the bunch intensity). Since the strength of these resonances is proportional to $J_{m_s}(h\varphi_s)$, they mainly disturbe the particles with large amplitudes of synchrotron oscillations ($A_s \gg \sigma_s$).

The perturbation due to residual dispersion in crab-cavities causes rather nerrow resonances, which can be avoided by small variations of tunes.

Chromatic distortions seems to be more dangerous due to the possibility of the excitation of the synchrotron satelites near integer resonance ($\nu_x + m_s\nu_s$). Since these resonances are not suppressed by the synchrotron radiation damping, they must be avoided by the proper choice of the working point in the tune diagram.

In both cases the strengths of resonances are proportional to the ratio $\nu_s/\alpha \propto 1/\sqrt{\alpha}$. This fact can cause an additional limitation on the use of the low-α lattices in the rings with crab-crossing.

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