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Particle Acceleration in Extremely Strong Electromagnetic Wave Fields

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1. Particle Motion in Plane Monochromatic Wave Fields

The dynamics of electrically charged particles in electromagnetic wave fields is of relevance for a large variety of physical phenomena. Therefore this topic is dealt with more or less extensively in many text books on classical electrodynamics.

The present paper can be seen in the context of recent work on the origin of high energy cosmic ray particles [1,2,3]. Plane-wave formalism without radiation reaction based on the *Lorentz* equation is adequate for the description of particle dynamics if the wave amplitude is of moderate strength. Particle motion then is periodic in velocity space so that there is an upper limit to particle energy.

We have shown that plane-wave formalism without radiation reaction also is a powerfull tool to define and to calculate important features of particle dynamics in spherical wave fields. Among these features are the acceleration boundary [4], the plasma border [5] and the $1/r_0^2$ - law of asymptotic energy [1].

But in extremely strong wave fields the influence of radiation forces has to be taken into account. Motion under the influence of radiation reaction no longer is periodic in velocity space. Particles, at least in principle, can achieve unlimited values of energy [6-15]. This mechanism, therefore, may be relevant for the understanding of cosmic particle acceleration to extremely high energies as, for example, in cosmic jets associated with rotating magnetized configurations. One may also think of man made jets constituted of laser beams connecting natural or artificial satellites [16-18].

It is the intention of this paper to present and to discuss some results, which may be of interest in the above mentioned context.

2. Equations of Motion

Momentum transfer between a particle of mass m and electric charge e and all other electromagnetically charged particles around, represented by an external electromagnetic field and described by a field tensor F_{ik} may, in classical electrodynamics, be described by

(1) $$du_i/d\tau = \eta_0 F_{jk} u^k + \tau_0 G_{jk} u^k$$

with $\eta_0 = e/mc$.

The first term on the right side of this equation of motion is constructed from the *Lorentz* derivative $u^L_j = \eta_0 F_{jk} u^k$ of the particle velocity u_j, representing the *Lorentz* force. The *Lorentz* term contains the first order contributions in the interaction constant e to the total force acting on the particle under consideration.

In the second term on the right side represents the radiation force. $\tau_0 = 2e^2/3mc^3$ is the radiation constant and G_{jk} is the radiation force tensor.

Both, the field tensor F_{jk} , as well as the radiation force tensor G_{jk} are antisymmetric tensors thus allowing for the conservation of the norm of particle velocity $u_j u^j = c^2$ or, as one may take it, for the particle to stay on its mass shell $p_j p^j = m^2 c^2$.

Dirac in his early work [19] has suggested

(2) $$G_{jk} = ([d^2u_j / d\tau^2] u_k - u_j [d^2u_k / d\tau^2]) / c^2 .$$

The *Lorentz-Dirac* equation (L-D equation), unfortunately, also describes run-away solutions: For vanishing external fields, $F_{jk} = 0$, by multiplication with $du_j / c^2 d\tau$ equation (1) with (2) reduces to

(3) $$d \{(du_j/d\tau)(du^j/d\tau)\} / d\tau = 2 \{(du_j/d\tau)(du^j/d\tau)\} / \tau_0$$

with the (unphysical) solution

(4) $$d \log \gamma / d\tau = (d \log \gamma / d\tau)_{\tau=0} \exp(\tau / \tau_0).$$

These difficulties have widely been discussed in literature. Obviously, they arise, since in the radiation force tensor (2) the kinematic acceleration $du_j / d\tau$ has been introduced instead of the *Lorentz* acceleration $u^L_j = \eta_0 F_{jk} u^k$. But there are strong arguments in favour of the

Lorentz accelaration, because in self-consistent *Maxwell* theory the only forces available are the electromagnetic forces.

Therefore, instead of (2), one may introduce the radiation force tensor

$$(5) \qquad G_{jk} = (u^{LL}_j u_k - u_j u^{LL}_k) / c^2$$

constructed from the second *Lorentz* derivative $u^{LL}_j = \eta_0^2 F_{jk} F^{kl} u_l$. The radiation force constructed with the help of the radiation force tensor (5) contains fourth order contributions in the interaction constant e to the total force acting on the particle.

The second part of this radiation force, $\tau_0 u^{Lk} u^L_k u_j / c^2$, the *Larmor* term, can be deduced through *Lorentz* transformation from *Larmor*'s radiation formula for the rate of energy at which the particle emits electromagnetic waves in the momentary rest frame MRS, $P = (\tau_0 / m)(e \mathbf{E})_{MRS}^2$, where **E** is the electric vector.

Thus, in the MRS, the *Larmor* term characterizes the particle as a source of electromagnetic waves while, at the same time, the first part of the radiation force characterizes the particle as a sink of electromagnetic waves. This characterization corresponds to the fact that the second part of the radiation force, the *Larmor* term, can be related to a retarded *Green's* function, while the first part of the radiation force can be related to an advanced *Green's* function, as has already been considered by Dirac [19].

In the MRS, non relativistic kinematics and dynamics applies: Under the action of a force **F** (any force) during some interval of the time coordinate δt a momentum $\mathbf{F}\,\delta t = \delta \mathbf{p}$ is transferred to the particle. But in the same time interval δt the energy transferred to the particle vanishes: $(\mathbf{F}, \delta \mathbf{x}) = (\mathbf{F}, \mathbf{v})\, \delta t = \delta T = 0$.

Correspondingly, the zeroth component of the radiation force (as of any force) vanishes in the MRS: the particle can be said to function as a relay for wave energy, which is emitted at the same rate as it is absorbed, it actually mediates the transfer of energy from incoming to outgoing waves.

In the MRS, the equation of motion (1) with (5) may be written

$$(6) \qquad d\mathbf{v} / dt = \eta_0 \mathbf{E} + \tau_0\, \eta_0^2 [\mathbf{E}, \mathbf{H}],$$

where **v** is the velocity vector and **H** is the magnetic field vector. The first term on the right side of this equation of motion represents the static *Coulomb* force, which is proportional to the electric vector, while the last term, which is proportional to *Poynting*'s vector, represents the radiation force (often referred to as radiation pressure).

Instead of (5), *L.D. Landau and E.M. Lifschitz* [20] have suggested the radiation force tensor

$$(7) \qquad G_{jk} = \eta_0 (dF_{jk} / d\tau) + (u^{LL}_j u_k - u_j u^{LL}_k) / c^2,$$

leading to the *Lorentz-Dirac-Landau* equation (L-D-L equation)

$$(8) \qquad du_j / d\tau = \eta_0 F_{jk} u^k + \eta_0 \tau_0 (dF_{jk} / d\tau) u^k + \tau_0 (u^{LL}_j u_k - u_j u^{LL}_k) u^k / c^2,$$

which incorporates an additonal third oder contribution in the interaction constant e, $\eta_0 \tau_0 (dF_{jk} / d\tau) u^k$.

In the MRS, the L-D-L equation has the form

$$(9) \qquad d\mathbf{v} / dt = \eta_0 \mathbf{E} + \tau_0 \eta_0 d\mathbf{E} / dt + \tau_0\, \eta_0^2 [\mathbf{E}, \mathbf{H}].$$

In general, within a given inertial frame of reference S, the particle is expected to loose energy through the emission of electromagnetic waves and, at the same time, to gain energy through the absorption of electromagnetic waves in addition to the change of energy caused through the work done by the *Lorentz* force.

In many examples the radiative loss of energy is known to exceed the radiative gain of energy, as e.g. for particle motion within a plane perpendicular to an external homogeneous, constant magnetic field when the particle moves in on the narrowing windings of a spiral, while dissipating energy in the form of synchrotron radiation.

In other examples the radiative loss and gain of energy compensate, as for particle motion on a straight line parallel to an external homogeneous, constant electric field. While dissipating energy in the form of electromagnetic waves through longitudinal acceleration the increase of kinetic particle energy through the work done by the *Coulomb* force occurs at the same rate it would do without radiation being produced.

Still there are examples in which the radiative gain of energy exceeds the radiative loss of energy. This is the case, e.g., for particle motion in an external plane, monochromatic wave field. Actually this can already be expected to happen from the appearance of the *Poynting* vector in the equation of motion (1) with (5) or with (7) within the MRS.

3. Asymptotic Behaviour of Energy Development

In view of possible applications to cosmic particle acceleration we have studied the asymptotic behaviour of solutions of the L-D-L equation for sufficiently large values of particle phase $\Phi = \mathbf{x}^0 - \mathbf{x}^1$, where $\mathbf{x}^0 = x^0 / r_L$ with $x^0 = ct$ is the dimensionless time coordinate, $r_L = c/\omega = \lambda / 2\pi$ is the light radius and λ is the wave length, while $\mathbf{x}^1 = x^1 / r_L$ is the spatial, cartesian coordinate in the direction of wave propagation. The conditions for the asymptotic regime are

(10) $\Phi >> 1$

and

(11) $w_0 \, l_0 \, f_0^2 \, \Phi / 2 >> 1$,

where $w = d\Phi / ds$ with $(ds)^2 = dx^i \, dx_i$ and the initial value $w_0 = w (\Phi_0)$. Especially, $w_0 = 1$ for particles initially at rest. $l_0 = c \, \tau_0 / r_L$ is the dimensionless radiation constant and $f_0 = (e / mc^2) \, r_L \, E_0$ is the dimensionless field amplitude.

Under these premises

(12) $\gamma \to \mathbf{u}^1 \to l_0 \, (f_0^2 / 4) \, [1 + f_0^2 \, (\sin^2\alpha \, \sin^2\Phi + \cos^2\alpha \, \cos^2\Phi)] \, \Phi$,

where α ist the polarization parameter with $\alpha = 0$ for linear polarization and $\alpha = \pm \pi / 4$ for circular polarization, furthermore

(13) $\mathbf{x}^1 \to (1/24) \, l_0^2 \, f_0^4 \, [1 + f_0^2 / 2] \, \Phi^3 + (1/32) \, l_0^2 \, f_0^6 \, [\Phi^2 \sin(2\Phi) + \Phi \cos(2\Phi)] \cos(2\alpha)$.

Therefore, in the special case of circular polarization,

(14) $\gamma \to \mathbf{u}^1 \to l_0 \, (f_0^2 / 4) [1 + f_0^2 / 2] \, \Phi$

and

(15) $\mathbf{x}^1 \to (1 / 24) \, l_0^2 \, f_0^4 \, [1 + f_0^2 / 2] \, \Phi^3 + \mathrm{const}$

so that

(16) $\gamma \to \mathbf{u}^1 \to (3^{1/3}/2) \, l_0^{1/3} \, f_0^{2/3} \, [1 + f_0^2/2]^{2/3} \, (\mathbf{x}^1)^{1/3}$.

Obviously, there are the following two regimes which can be distinguished

(17) $\gamma \to \mathbf{u}^1 \to (3^{1/3}/2) \, l_0^{1/3} \, f_0^{2/3} \, (\mathbf{x}^1)^{1/3}$, for $f_0^2 << 1$

and

(18) $\gamma \to \mathbf{u}^1 \to (3^{1/3}/2^{5/3}) \, l_0^{1/3} \, f_0^2 \, (\mathbf{x}^1)^{1/3} = (4\pi)^{-4/3} \, e^{8/3} \, m^{-7/3} \, c^{-3} \, H_0^2 \, t^{1/3} \, \nu^{-4/3}$, for $f_0^2 >> 1$.

4. References

[1] K.O. Thielheim
Fundamentals of Cosmic Physics, **13**, 357-399 (1989)

[2] K.O. Thielheim
in: P.R. Blake and H.F. Nash, eds.
Proc. 12th ECRS, Nottingham
Nucl. Phys. B (Proc. Suppl.), **228**, 60-79 (1991)

[3] K.O. Thielheim
in: P. Bosetti, ed.
Trends in Astroparticle Physics
Proc. 2nd Int. Symp., Aachen
Teubner Verlag, in press (1992)

[4] K.O. Thielheim
J. Phys. A: Math. Gen., **20**, L203-L206 (1987)

[5] K.O. Thielheim
in: N.M. Shapiro and J.P. Wefel, eds.
NATO ASI series **27O**, 523-536 (1989)

[6] J.J. Sanderson
Phys. Lett., **18**, 114-115 (1965)

[7] T.W.T. Kibble
Phys. Lett., **20**, 627-628 (1966)

[8] E.S. Sarachik and G.T. Schappert
Phys. Rev. D, **1**, 2738-2753 (1970)

[9] A.D. Steiger and C.H. Woods
Phys. Rev. D, **6**, 1468-1473 (1972)

[10] H. Heintzmann and M.Grewing
Z. Physik, **251**, 77-86, (1972)

[11] M. Grewing, E. Schrüfer and H.Heintzmann
Z. Physik, **260**, 375-384 (1973)

[12] J. Krüger and M. Bovyn
J. Phys. A: Math. Gen., **9**, 1841-1846 (1976)

[13] H. Stöckel
Fortschr. d. Physik, **24**, 417-475 (1976)

[14] D.M. Fradkin
Phys. Rev. Lett., **42**, 1209-1212 (1979)

[15] D.M. Fradkin
Phys. Rev. D, **22**, 1018 - 1023 (1990)

[16] K.O. Thielheim
in: W. Scandale, G. Turchetti, eds.
Proc. Int. Symp., Capri
World Scientific, 255-269 (1990)

[17] K.O. Thielheim
in: P. Grieder, ed.
Proc. 13th ECRS, Genf
in press (1992)

[18] K.O. Thielheim
Ap. J., May (1993)

[19] P.M.A. Dirac
Proc. Roy. Soc. (London) A, **167**, 148-169 (1938)

[20] L.D. Landau and E.M. Lifschitz
Klassische Feldtheorie
Akademie-Verlag, Berlin (1981)