Experimental Results of the Betatron Sum Resonance *

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Abstract

The experimental observations of motion near the betatron sum resonance, $\nu_x + 2\nu_z$=13, are presented. A fast quadrupole (Panofsky-style ferrite picture-frame magnet with a pulsed power supplier) producing a betatron tune shift of the order of 0.03 at rise time of 1 μs was used. This quadrupole was used to produce betatron tunes which jumped past and then crossed back through a betatron sum resonance line. The beam response as function of initial betatron amplitudes were recorded turn by turn. The correlated growth of the action variables, J_x and J_z, was observed. The phase space plots in the resonance frame reveal the features of particle motion near the nonlinear sum resonance region.

I. INTRODUCTION

For particle motion in a circular accelerator, the betatron oscillation $x(s)$ and $z(s)$, around the closed orbit is given by the Hill's equation [1]:

$$\frac{d^2x}{ds^2} + K_x(s)x = \frac{\Delta B_z}{B\rho}; \quad \frac{d^2z}{ds^2} + K_z(s)z = -\frac{\Delta B_x}{B\rho}, \qquad (1)$$

with

$$\Delta B_z + i\Delta B_x = B_o \sum_{n\geq 2}(b_n + ia_n)(x+iz)^n,$$

where b_n and a_n are the normal and the skew multipole components, respectively. The functions $K_x(s)$, $K_z(s)$ are the functions of the quadrupole strength, $B\rho = p/e$ is the magnetic rigidity, and s is the particle longitudinal coordinate, which advances from 0 to C, the circumference, as the particle completes one revolution of the cyclic accelerator. Both $K_{x,z}(s)$ and the anharmonic term $\frac{\Delta B_{x,z}}{B\rho}$ are periodic functions of s with period C. The number of oscillation periods in one revolution are betatron tunes, ν_x and ν_z, which can be adjusted by varying the quadrupole strength $K_{x,z}(s)$. The anharmonic term $\frac{\Delta B_{x,z}}{B\rho}$, which arise from higher-order multipoles, is normally small. However when the condition, $m\nu_x + n\nu_z = l$, with m, n, l as integers is satisfied, the particle can encounter coherent perturbative kicks from the multipoles. These nonlinear resonances usually lead to beam diffusion, halo, and beam loss. The experimental studies of the betatron sum resonances will aid in the understanding of these nonlinear effects.

Based on the theory of betatron motion [1], the betatron amplitude of particle motion is given by,

$$x = \sqrt{2\beta_x J_x}\cos(\phi_x); \quad z = \sqrt{2\beta_z J_z}\cos(\phi_z) \qquad (2)$$

with J_x, J_z as invariant actions for unperturbed motion. The Hamiltonian based on perturbation expansion of action-angle variables, and assuming a single dominant resonance can be written as [2]

$$H \approx H_o(J_x, J_z) + gJ_x^{\frac{|m|}{2}} J_z^{\frac{|n|}{2}} \cos(m\phi_x + n\phi_z - l\theta + \chi)$$

where g, χ are determined by the nonlinear multipole denoted by (m, n). Using the generating function, [2]

$$F_2(\phi_x, \phi_z, J_1, J_2) = J_1(m\phi_x + n\phi_z - l\theta + \chi) + J_2\phi_z,$$

the Hamiltonian in the resonance frame, with the new coordinates $\phi_1 = (m\phi_x + n\phi_z - l\theta + \chi)$, $\phi_2 = \phi_z$, $J_1 = \frac{J_x}{m}$, and $J_2 = J_z - \frac{n}{m}J_x$, is given by

$$\tilde{H} = \tilde{H}_o(J_1, J_2) + g(mJ_1)^{\frac{|m|}{2}}(nJ_1 + J_2)^{\frac{|n|}{2}}\cos(\phi_1).$$

This new Hamiltonian is independent of θ and ϕ_2, thus $\tilde{H}$ and J_2 are the constants of motion and with the boundary conditions they should determine particle orbit completely. For the resonance given by the condition $\nu_x + 2\nu_z = 13$, the constant of motion J_2 is $J_z - 2J_x$, which implies the motion is unbounded and that this resonance is unstable. In the phase space plot $(\sqrt{J_x}\cos\phi_1, \sqrt{J_x}\sin\phi_1)$, particle motion which are outside of the stop-band of the sum resonance will form a circle and reach outward when they are on the resonance. The experimental results for motion near the betatron sum resonance at $\nu_x + 2\nu_z = 13$ are presented in this paper. The Sec. II describes the experimental method, the data and analysis are presented in Sec. III, and Sec. IV is the summary.

II. EXPERIMENTAL METHOD

This experiment was performed using Indiana University Cyclotron Facility (IUCF) cooler ring which is hexagonal with a circumference of 86.82 m. A 45 MeV proton

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beam injected, electron cooled and stored in a 10 second cycle. The stored beam is consisted of a single bunch with typically 3×10^8 protons and the bunch length of about 5.4 m(or 60 ns) spread full width at half maximum (FWHM). The revolution period in the accelerator was 969 ns with bunching produced by operating a rf cavity with frequency 1.03168 MHz with a harmonic number h of 1. The stability of the horizontal and vertical closed orbits were estimated to be better than 0.2 mm FWHM, respectively. To excite a betatron oscillation, a pulsed deflecting magnets having a time width of 600 ns FWHM were used. In this measurement, the beam was kicked by the deflecting magnets in both the horizontal and vertical directions simultaneously. In order to reach the betatron sum resonance, a betatron tune jump quadrupole, which is a Panofsky-style ferrite picture-frame magnet with pulsed power supply, was used [3]. This quadrupole is capable of producing a tune shift of the order of 0.03 with a rise time of 1 μs. The tune shift then decreases exponentially with a time constant of 3 *ms* due to the time constant of the pulsed power supply. The quadrupole was used to jump the betatron tunes onto the sum resonance or alternatively the tunes jumped over the sum resonance line which then cross through the resonance due to the pulsed power supply used for the fast quadrupole. The tune diagram for IUCF cooler ring is shown in Fig. 1, where the points A(3.7758,4.6374) and B(3.7988,4.5829) refer to the tunes before and after the tune jump quadrupole were turned on, respectively. After the tunes were jumped (with a rise time of 1μs), the tunes change from B to A crossing the resonance line due to the pulsed power supplier. The relative time between the kick

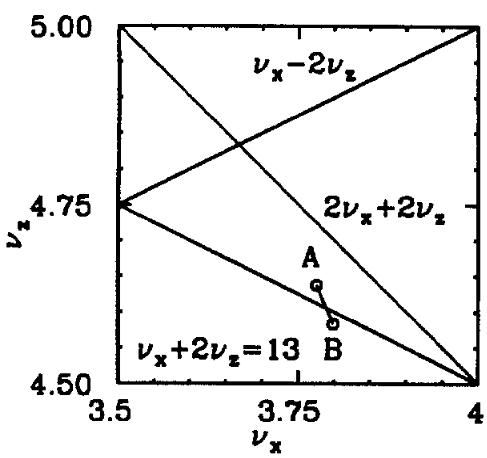

Fig. 1 Tune diagram

and the tune jump was adjustable. The data presented here was obtained for this case in which the kick magnet and the tune jump quadrupole had been turned on at the same time. The horizontal and vertical betatron motion of the beam centroid was tracked using four beam-position monitors (BPM's), two for each direction. The right (R) and sum (Σ) signal from each BPM were recorded turn by turn (up to 16000 turns) using our digitizing system [4]. The normalized position then was deduced from the relation $\frac{2R-\Sigma}{\Sigma}$. The position calibration for the BPM was determined by calibrating a BPM identical to those used in the experiment and with amplifiers that were carefully matched to have the same gain as those used for the experiment, against a nearby wire scanner. The position signal at two different positions around the ring is used to locate the beam in the phase space x-p_x.

III. DATA AND ANALYSIS

The measured beam position spectra, $x_1(n)$ and $z_1(n)$, around closed orbit are shown in Fig. 2 as functions of

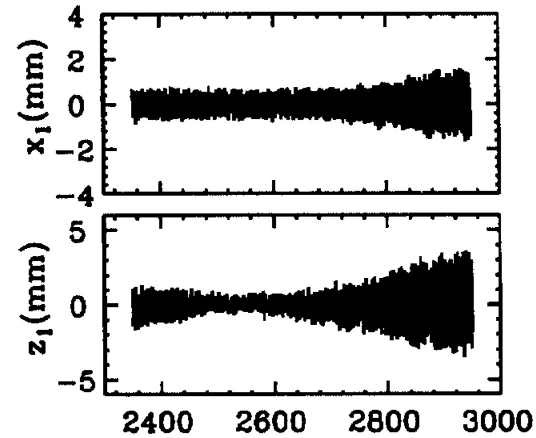

Fig. 2 Betatron oscillation amplitudes as function of turns

turns. The data shown are the betatron oscillation amplitudes from the turn numbers 2350 to 2950, which is in the time period 2.28 to 2.86 *ms* after the kick and the tune jump quadrupole were turned on. The measured betatron oscillation amplitudes reach a maximum near the turn number 2950 and these data after it is not shown here. The normalized momentum p_{x1} and p_{z1} which are defined as the conjugate variables of those in Eq. (2) can be evaluated from the measured quantities (x_1, x_2) and (z_1, z_2), respectively. The p_{x1} becomes [4]

$$p_{x1} = -x_1 cot\phi_{12} + \frac{\sqrt{\beta_{x1}/\beta_{x2}}}{\sin\phi_{12}} x_2, \tag{3}$$

where the ϕ_{12} is the betatron phase advance between the two BPM's. The equation for a circle in x_1, p_{x1} is an equation for an ellipse in the variables x_1 and x_2, given by

$$\left(-x_1 cot\phi_{12} + \frac{\sqrt{\beta_{x1}/\beta_{x2}}}{\sin\phi_{12}} x_2\right)^2 + x_1^2 = 2\beta_{x1} J. \tag{4}$$

The values of ϕ_{12} and β_1/β_2 were determined by fitting experimental data, taken where it is known the motion is linear, to this equation of an ellipse. The normalized momentum p_{z1} was evaluated as the same way by replacing the x_1, x_2 by the z_1, z_2. The action variables J_x and J_z

were evaluated by

$$J_x = \frac{p_{x1}^2 + x_1^2}{2\beta_{x1}}, \quad J_z = \frac{p_{z1}^2 + z_1^2}{2\beta_{z1}}. \tag{5}$$

They are plotted, in unit of (π *mm mrad*), in Fig. 3 as a function of the turns. Also in the Fig. 3, the J_z is plotted as function of the J_x. Correlated growth of the action variables J_x, J_z can be seen from this data. The solid line in Fig. 3 is the $J_z - 2J_x = J_2$ relation. The

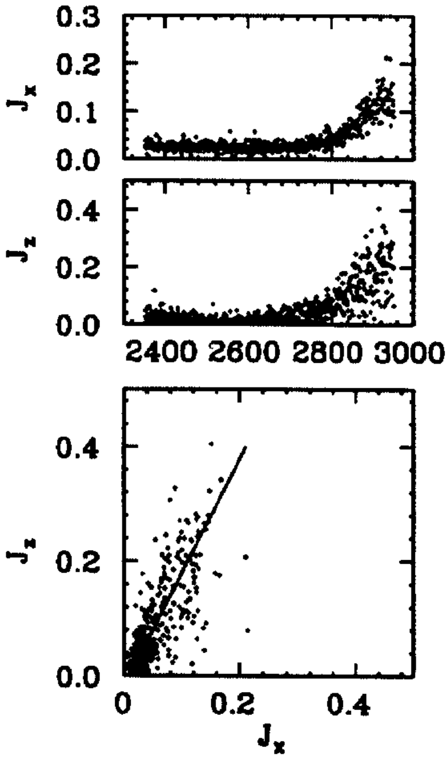

Fig. 3 Action variables J_x and J_z

particle motion is also plotted in phase space ($\sqrt{J_x}\cos\phi_1$, $\sqrt{J_x}\sin\phi_1$), which is shown in Fig. 4. The particle motion at initial 450 turns are denoted by the point in the Fig. 4, and the crosses are refer to the last 150 turns where the particle motion is moving outward.

It is certainly useful to monitor the betatron tunes and it can be an alternative way to check whether the particle motion is on the sum resonance or not. However the conventional method, to do FFT, is not practical because of the nature of the sum resonance and the fast quadrupole which makes the tunes change continously. To measure the tunes, the betatron tunes were tracked using the action angle advance $\Delta\phi$ at each turn, $\nu(n) = \frac{\Delta\phi(n)}{2\pi}$, where the n is the turn number, and a running average (±10 turns) was used in average. The effect of the tune jump quadrupole was shown in Fig. 5. The upper and lower curves are the horizontal tune, ν_x, while the tune jump quadrupole was off and on, respectively, and the solid line is the calculated tune, ν_x, from the parameters of the tune jump quadrupole [3]. The motion of the tune across the resonance line as indicated in Fig. 1 was verified by this tune tracking method.

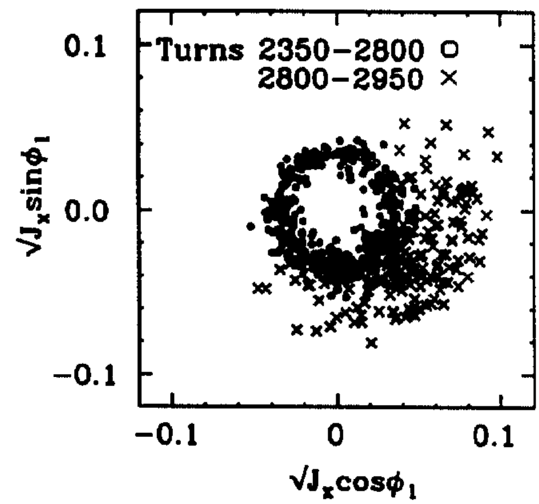

Fig. 4 Phase space plot on resonance frame

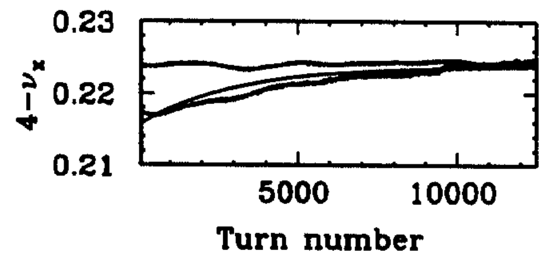

Fig. 5 The effect of the fast quad. on the betatron tune

IV. SUMMARY

The betatron sum resonance has been observed experimentally. The fast quadrupole which can produce a tune shift of 0.03 with a rise time of 1 μs certainly was an important tool for the success of this experiment. The data show the correlated growth of the action variable J_x and J_z. The phase space plot in the resonance frame reveal the expected outward motion when the sum resonance is encountered. Further study of the dynamics at this sum resonance is underway.

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