MATCH 1.0 – The Program for Analytical Matching of Insertion

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Abstract

An algorithm and a program for investigation of a matched straight insertion has been proposed. Formulae have been derived for analytic computation of the insertion parameters in a "thick" lenses aproximation. The results of matching of the synchrotron straight section are presented.

I. INTRODUCTION

Beam Matching

For the beam transport of charged particles, there is the need to solve often an inverse problem of beam dynamics, such as the matching of phase volumes of the beam [3-6]. We assume the detection of parameters for the elements of the magnetic optics (such as geometry, fields) of a facility realizing the beam transport with a phase volume of $V_1 = (x_1,\ x_1',\ y_1,\ y_1')$ from point s_1 (beginning of the mathing section of the transport channel, see fig.1) to point s_2 (end of the mathing section) with a phase volume of $V_2 = (x_2,\ x_2',\ y_2,\ y_2')$ to be the solution of this matching problem. We shall assume the betatron functions α_1^x, β_1^x, α_1^y, β_1^y for horisontal and vertical motion at the beginning of the matching section and α_2^x, β_2^x, α_2^y, β_2^y at the end to be well-known quantities. Under certain circumstances, only some of above mentioned quantities or their combinations, such as $\sqrt{\beta_2^x/\beta_1^x}$, $\sqrt{\beta_2^y/\beta_1^y}$, $\alpha_1^x = \alpha_1^y = 0$, etc. might be known, whereas the remaining quantities will be arbitrary ones. In order to solve the matching problem, it is necessary to establish the required phase leading on the matching section $(s_1,\ s_2)$, to dispose of applicable chromaticity or even to match the dispersion function $D_{x,y}$ $D'_{x,y}$ totally, or sometimes we must take care of the matching of direction and polarization degree of the beam and, possibly, of other characteristics.

Optimization

The problem of optimizing parameters of such a facility is connected closely to the matching problem. Its solution will be carried out by taking into account the limitations on all possible values of the parameters for elements of magnetic optics of the projected facility.

Sometimes, it is impossible or objectionable during the projection period to have lengths of lenses and single gaps different from those, which were selected during preliminary designing. In experiments with colliding beams [7] for instance, one strives to provide for an interaction point: the cross-over in some free gaps, where $\beta_{min}^{x,y}$ does not surpass a certain taken value. The solution of the optimization problem as a whole tends to increase the planning and operation economy of the transport channel under investigation.

Usually (see ref.[8,9]), the matching is not separated from the general optimization problem and their solution occurs commonly and simultaneously. In the proposed method, the matching and optimization functions were separated. The optimization was carried out on the basis of a transport channel, which was developped and matched already. In order to solve the matching problems, we propose analytical expressions, which will be compiled below.

II. MATCHING METHOD

We shall present a solution of the matching problems in linear approximation by means of a minimum number of magneto-optical elements: a doublet of "thick" magnetic quadrupole lenses. For the chosen matching section (see fig.1), we shall whrite the matrix equation of matching as follows:

$$\hat{L}_3 \cdot \hat{F} \cdot \hat{L}_2 \cdot \hat{D} \cdot \hat{L}_1 = \hat{M}(2/1), \qquad (1)$$

where $\hat{L}$, $\hat{F}$, $\hat{D}$ are matrices of linear optics of 2×2 size, the shape of which is well-known, ref.[3,10]. For instance, the matrix elements (m.e.) $L_{12}^x = L_{12}^y = L$, $F_{11}^y = \varphi_F$, $D_{12}^y = (1/k_D) \cdot \sin\varphi_D$, $F_{11}^x = \cos\varphi_F$, $\varphi_F = k_F \cdot l_F$, $k_F = \sqrt{G_F/B_\rho}$; G_F is the gradient and l_F is the effective length of the lens; L indicates the length of a free path, and B_ρ– the magnetic particle rigidity of transported particles. $\hat{M}(2/1)$ can be expressed (see e.g. [10]) by betatron functions $\alpha_1^{x,y}$, $\beta_1^{x,y}$, $\alpha_2^{x,y}$, $\beta_2^{x,y}$ and phase leading $\Delta\psi^{x,y} = \psi^{x,y}(s_2) - \psi^{x,y}(s_1)$:

$$\hat{M}(2/1) = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \qquad (2)$$

where

$$M_{11} = \sqrt{\beta_2/\beta_1}(cos\Delta\psi + \alpha_1 \cdot sin\Delta\psi)$$
$$M_{12} = \sqrt{\beta_1 \cdot \beta_2} sin\Delta\psi$$
$$M_{21} = -\frac{(1+\alpha_1\alpha_2) \cdot sin\Delta\psi + (\alpha_2 - \alpha_1) \cdot cos\Delta\psi}{\sqrt{\beta_1 \cdot \beta_2}}$$
$$M_{22} = \sqrt{\frac{\beta_1}{\beta_2}}(cos\Delta\psi - \alpha_2 \cdot sin\Delta\psi)$$

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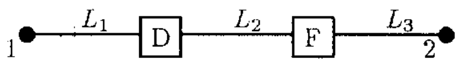

Fig.1 Section of matching by lens doublet.

After having determined the m.e. in (1), we can write eq. (1) as a system of 6 equations:

$$\begin{cases} T^x_{11} = M^x_{11} \quad T^x_{21} = M^x_{21} \quad T^x_{22} = M^x_{22} \\ T^y_{11} = M^y_{11} \quad T^y_{21} = M^y_{21} \quad T^y_{22} = M^y_{22} \end{cases} \qquad (3)$$

where the matrix T is the result of the matrix product on the left side of (1). We should note that the equation of $T_{12} = M_{12}$ is fulfilled automatically due to the Lousville theorema on the concervation of the phase volume: $det|T| = det|M| = 1$. The problem of matching phase volumes of the transported beam is solved usually by nonlinear programming methods [8,9]. If we have written the matching equation system of (1), which is transcendental, we are able to proceed with the following stage: its solution by direct numerical methods. We are able to reduce the composed matching equation system (3) in the present paper by means of successive analitical transformations to one equation with one unknown quantity, that is $\Delta\psi^x$. After having solved this equation on a computer, we can write the characteristics of the matching section: the lengths of fieldless paths, the effective lengths and the gradients of magnetic lenses analitically as a function of the chosen vector of the parameters: $\bar{P} = (\alpha^x_1,\ \beta^x_1,\ \alpha^x_2,\ \beta^x_2,\ \alpha^y_1,\ \beta^y_1,\ \alpha^y_2,\ \beta^y_2,\ \varphi_F,\ \varphi_D,\ \Delta\psi^y)$ and the free parameter $\Delta\psi^x$.

We shall introduce a series of notations, which are helpful for solving the equation system (3):

$$\Delta_{12} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21},$$

$$\Delta'_{12} = b_{11} \cdot L_3 \cdot b_{12} \equiv (a_{22} \cdot a_{13} - a_{12} \cdot a_{23}) +$$

$$-L_3 \cdot (a_{22} \cdot a_{14} - a_{12} \cdot a_{12} \cdot a_{24}),$$

$$\Delta''_{12} = b_{21} + L_3 \cdot b_{22} \equiv (a_{11} \cdot a_{23} - a_{21} \cdot a_{13}) +$$

$$+ L_3 \cdot (a_{11} \cdot a_{24} - a_{21} \cdot a_{14}),$$

$$\Delta_{34} = a_{31} \cdot a_{42} - a_{41} \cdot a_{32},$$

$$b_{31} + b_{32}/k_F) \equiv (a_{33} \cdot a_{42} - a_{32} \cdot a_{43}) +$$

$$+(1/k_F) \cdot (a_{34} \cdot a_{42} - a_{44} \cdot a_{32}),$$

$$b_{41} + b_{42}/k_F \equiv (a_{31} \cdot a_{43} - a_{33} \cdot a_{41}) +$$

$$+(1/k_F) \cdot (a_{31} \cdot a_{44} - a_{34} \cdot a_{41}),$$

$$\Delta = \Delta_{12} \cdot \Delta_{34} \cdot (b_{22} \cdot b_{32} - b_{12} \cdot b_{42}),$$

$$\Delta_1 = \Delta_{12} \cdot [\Delta_{12} \cdot (b_{32} \cdot b_{41} - b_{31} \cdot b_{42}) +$$

$$+\Delta_{34} \cdot (b_{11} \cdot b_{42} - b_{21} \cdot b_{32})],$$

$$\Delta_2 = \Delta_{34} \cdot [\Delta_{12} \cdot (b_{12} \cdot b_{41} - b_{22} \cdot b_{31}) +$$

$$+\Delta_{34} \cdot (b_{11} \cdot b_{22} - b_{12} \cdot b_{21})],$$

where

$$\begin{array}{ll}
a_{11} = \tilde{s}_1 \cdot c_2, & a_{12} = \tilde{s}_1 \cdot s_2, \\
a_{21} = -s_1 \cdot \tilde{c}_2, & a_{22} = -s_1 \cdot \tilde{s}_2, \\
a_{31} = \tilde{s}_1 \cdot s_2, & a_{32} = -\tilde{s}_1 \cdot c_2, \\
a_{41} = s_1 \cdot \tilde{s}_2, & a_{42} = s_1 \cdot \tilde{c}_2, \\
a_{51} = -\tilde{c}_1 \cdot s_2, & a_{52} = -\tilde{s}_1 \cdot s_2, \\
a_{61} = c_1 \cdot \tilde{s}_2, & a_{62} = s_1 \cdot \tilde{s}_2, \\
a_{13} = M^x_{11} - \tilde{c}_1 \cdot c_2, & a_{14} = -M^x_{21}, \\
a_{23} = M^y_{11} - c_1 \cdot \tilde{c}_2, & a_{24} = -M^y_{21}, \\
a_{33} = -\tilde{c}_1 \cdot s_2, & a_{34} = -M^x_{21}, \\
a_{43} = c_1 \cdot \tilde{s}_2, & a_{44} = -M^y_{21}, \\
a_{53} = M^x_{22} - \tilde{c}_1 \cdot c_2, & a_{54} = -M^x_{21}, \\
a_{63} = M^y_{22} - c_1 \cdot \tilde{c}_2, & a_{64} = -M^y_{21}, \\
c_1 = cos\varphi_D, & c_2 = cos\varphi_F, \\
\tilde{c}_1 = ch\varphi_D, & \tilde{c}_2 = ch\varphi_F, \\
s_1 = sin\varphi_D & s_2 = sin\varphi_F, \\
\tilde{s}_1 = sh\varphi_D, & \tilde{s}_2 = sh\varphi_F.
\end{array}$$

After a series of transformations and using this notation, we are able to express the unknown values of

$$\begin{cases} k_F = \Delta/\Delta_2, \\ k_D = k_F \cdot \Delta''_{32}/\Delta_{12}, \\ L_3 = \Delta_1/\Delta, \\ L_2 = (1/k_D) \cdot \Delta'_{12}/\Delta_{12}, \\ L_1 = (L_2 \cdot k_F \cdot a_{51} + a_{52} \cdot k_F/k_D - a_{53})/a_{54} \end{cases} \qquad (4)$$

Each of this $(L_1,\ L_2,\ L_3,\ k_F,\ k_D)$ is a function of the parameters $\alpha^x_{1,2},\ \beta^x_{1,2},\ \varphi_F,\ \varphi_D,\ \Delta\psi^y,\ \Delta\psi^x$ (one of which may be free, for instance $\Delta\psi^x$). It follows from solving system (3) by taking into account (4) and above notation that these parameters are coupled by the equation:

$$\Phi \equiv L_2 \cdot_{k_F} \cdot a_{61} + a_{62} \cdot k_F/k_D - a_{63} - L_1 \cdot a_{64} = 0, \quad (5)$$

from which we derive the value of the given free parameter, in this case $\Delta\psi^x$. For the matching section of the beam injection channel into the synchrotron SPIN, which we shall investigate below, the function $\Phi(\Delta\psi^x)$ is plotted in the fig.2, where the solution of eq.(5) corresponding to physical requirements is denoted by the symbol (*). The value of φ_F, φ_D determines the length of lenses.

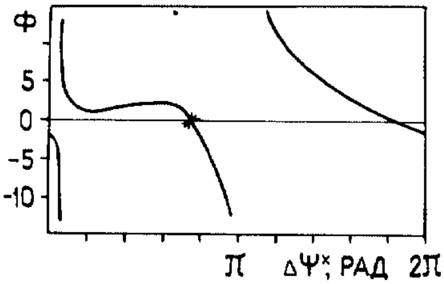

Fig.2 Function $\Phi(\Delta\psi^x)$. (*) denoted the physical solution of $\Phi(\Delta\psi^x) = 0$.

The range of permissible values for these quantities are illustrated in fig.3.

III. PROGRAM MATCH 1.0 and RESULTS

The described matching algorithm was realized as FORTRAN-program MATCH 1.0 on the VAX, IBM, CDC-6500 and IBM PC computers at Dubna JINR. As user's interactive mode with computer, this program allows to look quickly over the spase of the parameters $\vec{P}$ and to select the most economical version of calculated structure.

The matching of the channel [11] for beam injection into the ring of the superconducting synchrotron SPIN [1,2] was carried out by the proposed method for an energy of 0.600 MeV with $B_\rho = 0.111945\ T \cdot m$. The beam envelopes X_{max} and Y_{max} along the injection channel are plotted. We obtained the betatron functions α_1^x, α_1^y and β_1^x, β_1^y by means of MATCH1.0, calculating the betatron motion on the transport section from the entrance into the regular synchrotron structure. At the entrance point $\alpha^x = \alpha^y = 0$, $\beta^x = 2.434\ m$ and $\beta^y = 0.685\ m$. The values of β_2^y, β_2^y and α_2^x, α_2^y were obtained in beam transport matching section. At the beginning of this section

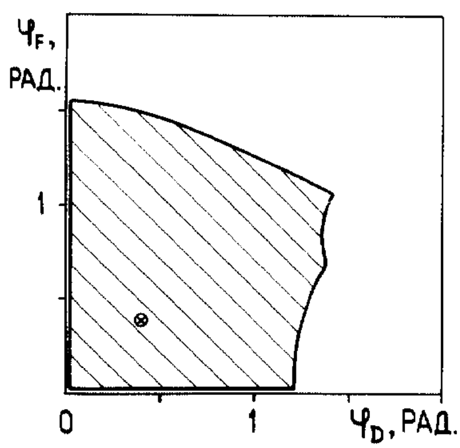

Fig.3 Range of values of the parameters φ_F and φ_D. The symbol ⊗ denoted the chosen value of $\varphi_F = 0.559$ rad and $\varphi_D = 0.488$ rad.

$\beta_x = \beta_y = 1.500\ m$ and $\alpha_x = \alpha_y = 0$. The values of $\beta_{1,2}^{x,y}$ and $\alpha_{1,2}^{x,y}$ which have been obtained at the ends of the matching section follows:

$\beta_1^x = 2.1327$ m $\quad \alpha_1^x = 2.51607$
$\beta_1^y = 1.3168$ m $\quad \alpha_1^y = -2.56981$
$\beta_2^x = 3.5482$ m $\quad \alpha_2^x = 0.38639$
$\beta_2^y = 1.2876$ m $\quad \alpha_2^y = -0.25838$

For the quantities $\varphi_F = 0.559$ rad, $\varphi_D = 0.488$ rad and $\Delta\psi^y = 0.8855$ rad we have $\Delta\psi^x = 2.02296$ rad. Here is $L_1 = 0.5276$ m,

$l_D = 0.1529$ m, $\quad k_D = 3.1919 m^{-1}$, $\quad L_2 = 0.8268$ m,
$l_F = 0.3096$ m, $\quad k_F = 1.8056 m^{-1}$, $\quad L_3 = 0.2096$ m.

IV. CONCLUSION

Analytical expressions have been found, which offer a solution for the matching problem of a doublet of magnetic quadrupole lenses in "thick" lens approximation. For the given beam profile, it is easy to chose the most economical version of the matching structure, which fulfills the given requirements. The mathing of phase volume of a beam, which was injected and captured in the accelerating cycle for the injection channel of the superconducting synchrotron at Dubna JINR, was achieved by means of the given method.

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