Procedure for Determining Quadrupole and BPM Offset Values in Storage Rings*

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Abstract

One of the most elusive problems in storage-ring commissioning has historically been the determination of quadrupole and BPM offset values. We present a simple linear solution based on the principle that the element offset values are independent of lattice configuration.

I. INTRODUCTION

The conversion of SPEAR from a collider facility to a synchrotron radiation source lead to an increased emphasis on understanding the absolute beam orbit in the storage ring. The new goal, of course, is to steer the photon beams down the beamlines with minimum electron-beam offset in the quadrupoles and sextupoles, and minimum corrector strengths [1]. This condition requires both precision quadrupole alignment and minimum DC readback errors on the beam position monitors (BPMs).

At present, a number of quadrupoles in SPEAR are known to be misaligned by several mm horizontally, and the estimated BPM readback offsets are in some cases also several mm. The combined errors complicate both beamline steering and analysis of the electron beam orbit. For this reason, we have formulated a general procedure for determining quadrupole and BPM offset values in storage rings.

II. THEORY

The first step of any beam-based alignment procedure is experimental verification of the first-order optics model [2]. In this context, the model refers to quadrupole and corrector strengths, and BPM linearity factors.

Once the model is established, the component of the closed orbit distortion (COD) induced by correctors can be computed,

$$x_c^i = C^{ij}\Delta\theta^j \tag{1}$$

where C^{ij} is the corrector response matrix (units mm/mrad), $\Delta\theta^i$ are the corrector strengths, and x_c^i is the orbit displacement evaluated at each BPM. The model can also be used to predict the COD induced by quadrupole displacements,

$$x_q^i = Q^{ij}\Delta x_q^j, \tag{2}$$

where Q^{ij} has units (mm/mm) and Δx_q^j is the column vector of quadrupole misalignments.

In general, the errors could be due to quadrupole angle errors, bend roll errors, etc., or sector errors where a group of magnets is mounted to a common (misaligned) support. For this analysis, we assume the kicks generating COD to emanate from offset errors at the quadrupoles.

Superposition of (1) and (2) yields the total COD:

$$x_{cod}^i = x_c^i + x_q^i = C^{ij}\Delta\theta^j + Q^{ij}\Delta x_q^j. \tag{3}$$

Isolating the quadrupole contribution,

$$Q^{ij}\Delta x_q^j = x_{cod}^i - C^{ij}\Delta\theta^j, \tag{4}$$

we can solve for the quadrupole offset vector Δx_q^j using standard techniques:

1. Q-Matrix Inversion (e.g., Singular Value Decomposition [3])
2. Most Effective Quadrupole (MICADO [4])
3. GOLD Method (Piecewise Solution [5])

In SPEAR, however, the problem is complicated by constant but unknown BPM readback errors, and an unknown energy offset of the beam. Thus, we have,

$$x_{cod}^i + \Delta x_b^i = C^{ij}\Delta\theta^j + Q^{ij}\Delta x_q^j + \eta^i\frac{\Delta p}{p}, \tag{5}$$

where Δx_b^i is the column vector of BPM readback errors, and η^i is the dispersion function evaluated at each BPM.

In matrix form, the COD equation reads

$$\vec{x}_{cod} - \vec{C}\Delta\vec{\theta} = \left[-\vec{I}:\vec{Q}:\vec{\eta}\right]\left[\Delta\vec{x}_b:\Delta\vec{x}_q:\frac{\Delta p}{p}\right]^T \tag{6}$$

where I is the identity matrix, and the colons indicate partitioning of vectors and matrices. With the set of unknowns expanded to $\vec{S} = \left\{\Delta\vec{x}_b:\Delta\vec{x}_q:\Delta p/p\right\}$, use and interpretation of techniques 1–3 outlined above is complicated.

One way to find the set of unknowns $\vec{S}$ is the following. By changing quadrupole strengths, we can experimentally generate a linearly independent set of Eq. 6 with different response-matrix coefficients C^{ij}, Q^{ij}, and η^i, and least-squares fit the expanded set of equations to solve for the quadrupole offsets, BPM offsets and energy error.

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The error bars associated with the solution vector $\vec{S}=\{\Delta\bar{x}_b:\Delta\bar{x}_q:\Delta p/p\}$ are the diagonal elements of $[A^TA]^{-1}$, where A is the response matrix

$$A=\begin{bmatrix} -\bar{I} & \overline{Q_1} & \vec{\eta}_1 \\ \cdots & \cdots & \cdots \\ -\bar{I} & \overline{Q}_n & \vec{\eta}_n \end{bmatrix}$$

Note that a set of n measurements based on n different lattice configurations fill the rows of A, that is, Eq. 6 repeated n times to fill the rows of A.

Unfortunately, if we try to determine the entire solution vector $\vec{S}$ for the storage ring in one pass, the error bars are large. Three alternatives are possible:

1. Compute the difference between the COD Eq. 6 evaluated for each new lattice relative to the reference configuration. The result is elimination of BPM offset errors from the solution vector $\vec{S}$. Once the reduced solution vector $\vec{S}=\{\Delta\bar{x}_q:\Delta p/p\}$ is found, computation of the BPM offset errors $\Delta\bar{x}_b$ is straightforward.
2. Solve the set of Eq. 6 for the n configurations simultaneously in a piecewise fashion along sections of the ring, and reconstruct the entire solution from the separate parts. The advantage is reduction of the set of variables, and more control over the fitting procedure.
3. Combination of methods 1 and 2.

III. APPLICATION TO SPEAR

A FORTRAN program (ALIGN) was written to simultaneously solve the set of Eqs. (6) for a multiplicity of lattice configurations in SPEAR. The code structure is straightforward. First, we read the measured COD, corrector strengths, and the response matrices C^{ij}, Q^{ij} and η^i. Next, we subtract the (n-1) orbit Eqs. (6) evaluated with perturbed quadrupole values from the initial reference orbit, form the matrix A, and solve for the quadrupole offset values $\Delta\bar{x}_q$ and energy error $\Delta p/p$. Intrinsic BPM measurement errors can be included in the calculation. In the last step, the quadrupole offsets are held constant, and the fitting procedure is repeated to calculate BPM offsets, with error estimates.

Numerically, we found convergent solutions for test cases using known seeds for quadrupole and BPM offset values in the SPEAR lattice. The solutions had error bars approaching 10 mm which indicated problems with measurement sensitivity (ill-conditioned response matrix A).

Experimentally, the SPEAR data was measured by first moving individual quadrupole family strengths until the tune approached either the integer or 1/3 integer resonance (ν_x=6.820, ν_y=6.720, nominally). The typical excursion in magnet strength was ~1%. Later, pairs of horizontal (or vertical) focussing magnets were moved in opposite directions to obtain up to 15% excursions in strength.

Analysis of the measured data has been limited to piecewise solutions across the collider-interaction regions, where six quadrupole families were varied. The solutions have not converged, however, probably due to the combined effect of small orbit perturbations and inadequate BPM resolution. To increase the measurement sensitivity, one must generate large differences in the beta functions that are used to compute the response-matrix elements C^{ij} and Q^{ij}.

IV. DISCUSSION

This procedure for determining quadrupole and BPM offset values is in some respects similar to the common magnet-shunt technique used to center the beam in optical components which dates back at least to CEA [6]. In the present development, however, the ideas are extended to include use of a beam-calibrated optics model and statistical analysis of the percieved offset errors. Analysis of the true source of errors (i.e., quadrupole offsets versus bend rotations) is extremely complicated and probably not possible for most accelerators. But by determining the most likely locations of kicks and BPM offsets, these points can be checked for error, and a working model of the absolute beam orbit can be defined. The same procedures can be applied to; either circular or linear accelerators.

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