A Design of a Quasi-Isochronous Storage Ring

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Abstract

Isochronous electron storage rings may offer advantages for future high luminosity meson facotries. A Quasi-isochronous lattice based on the design principle of flexible γ_T lattice is studied. The emittance and chromatic properties of such a lattice are studied. Applications of this design technique for electron storage rings will be discussed.

1 INTRODUCTION

Shorten the bunch length is an effective way of increaing the brightness and luminosity of the electron storage rings. A possiblility of reducing the bunch length is to operate the electron storage ring near to the transition energy, where nonlinear synchrotron motion [1-3] determines the equilibrium geometry of the bunch distribution.

Driven by the demand of high energy physics experiments and a possibility of high brightness synchrotrons or damping rings, UCLA group has proposed a quasi-isochronous (QI) Φ-factory design by using reverse bends in the lattice, which will result large dispersion function values.

On the other hand, the QI can easily be achieved from the flexible γ_T design principle. Teng [4] had advanced the flexibible γ_T lattice design by introducing the π insertion. The combination of the FODO cells with π insertion was applied successfully and studied extensively by Trbojevic et al. [5] The transverse beam dynamics of these lattice is well understood. In this paper, we apply the design principle to achieve QI condition without using the reverse bend. Section 2 reviews the basic module of the flexible γ_T lattice. Section 3 discusses the QI condition. Section 4 evaluate the emittance of the electron storage ring. The conclusion is given in section 5.

2 THE BASIC MODULE

The basic module for a variable γ_T lattice is given by [5]

$$M_a\left\{\frac{1}{2}Q_F B Q_D B \frac{1}{2}Q_F\right\} M_b \left\{Q_{F_1} O_1 Q_{D_2} O_2\right\} M_c + \text{ref. sym.}$$

where $M_{a,b,c}$ are marker locations, Q's are quadrupoles, O's are drift spaces, and B's stand for dipoles. The horizontal betatron transfer matrix of the FODO cell is given by

$$M_{\rm FODO} = \begin{pmatrix} \cos\mu & \beta_F\sin\mu & D_F(1-\cos\mu) \\ -\frac{1}{\beta_F}\sin\mu & \cos\mu & \frac{D_F}{\beta_F}\sin\mu \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where μ is the horizontal phase advance in the FODO cell. We have assumed symmetry in the Courant-Snyder parameters at the center of the focusing quadrupole, i.e., $\beta'_F = 0$ and $D'_F = 0$.

In the thin-lens approximation with equal focusing and defocusing strengths, the Courant-Snyder parameters are given by $\sin\frac{\mu}{2} = \frac{L_F}{2f}$, $\beta_F = \frac{2L_F(1+\sin\frac{1}{2}\mu)}{\sin\mu}$, $D_F = \frac{L_F\theta(2+\sin\frac{1}{2}\mu)}{2\sin^2\frac{1}{2}\mu}$, where L_F is the length of the half FODO cell, f is the focal length of quadrupoles in the FODO cell, and θ is the bending angle of the dipole B. However it is worth pointing out that the applicability of Eq. (1) is not limited to thin-lens approximation. In the normal FODO lattice, the dispersion function is assumed to be periodic in each FODO cell. In this case, the dispersion function at the center of the focusing quadrupole is D_F with $D'_F = 0$.

Since the momentum compaction factor is given by

$$\alpha = \frac{1}{\gamma_T^2} = \frac{1}{L_m}\sum_i D_i\theta_i, \quad (2)$$

the flexible γ_T lattice can be achieved by prescribing the dispersion function at the beginning of the FODO cell with values D_a and D'_a. The value $D'_a = 0$ is usually chosen for convenience in the lattice function matching.

Depending on the initial dispersion value at marker M_a, the dispersion function at marker M_b is given by

$$D_b = D_F - (D_F - D_a)\cos\mu\ , \quad D'_b = \frac{D_F - D_a}{\beta_b}\sin\mu\ , \quad (3)$$

where β_b is the betatron amplitude function at marker M_b with $\beta_b = \beta_F$. In the matching section (assuming that there is no dipole or negligible dipole contribution to dispersion), the dispersion action is invariant, i.e.,

$$2J_c = 2J_b = \frac{D_b^2}{\beta_b} + \beta_b D_b'^2 = 2J_F[1-2(1-\zeta)\cos\mu + (1-\zeta)^2]\ ,$$

with $\zeta = \frac{D_a}{D_F}$ as the ratio of the desired dispersion at marker M_a of the FODO cell and J_F is the dispersion action for the regular FODO cell at the focusing quadrupole location.

The dispersion functions and other Courant-Snyder parameters are then matched at the symmetry point at marker M_c with a doublet (or triplet). The betatron transfer matrix is given by

$$M_{b\to c} = \begin{pmatrix} \sqrt{\frac{\beta_c}{\beta_b}}\cos\psi & \sqrt{\beta_b\beta_c}\sin\psi & 0 \\ -\frac{1}{\sqrt{\beta_b\beta_c}}\sin\psi & \sqrt{\frac{\beta_b}{\beta_c}}\cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

where we have also assumed a symmetry condition at marker M_c for the Courant-Snyder parameters, i.e., $\beta_b' = 0$ and $\beta_c' = 0$. Here, β_b and β_c are the betatron amplitudes at, respectively, markers M_b and M_c, while ψ is the betatron phase advance between markers M_b and M_c.

The required dispersion matching condition at marker M_c is $D_c' = 0$. Using Eq. (3), we obtain then

$$\tan\psi = \frac{(1-\zeta)\sin\mu}{1-(1-\zeta)\cos\mu}\,. \tag{5}$$

This means that the phase advance of the matching section is not a free parameter, but is determined completely by the initial dispersion value D_a at marker M_a and the phase advance of the FODO cell. This condition is independent of whether we use a doublet or a triplet for the betatron-parameter matching. However, it is preferable to use a doublet matching section. The total phase advance of the whole basic module is then given by $2(\mu+\psi)$, which is a function of only the desired dispersion function at marker M_a and the phase advance μ in the FODO cell.

Quadrupoles Q_{F_1} and Q_{D_2} in the matching section are then adjusted to achieve the required phase advance ψ given by Eq. (5). A low betatron amplitude function at marker M_c is desired so that D_C will be small. Care should also be taken in the arrangement and choices of quadrupoles Q_{F_1} and Q_{D_2} in order to achieve reasonably small vertical Courant-Snyder parameters. Then, the matching becomes relatively simple. [5]

The dispersion values at the midpoints of dipoles in the FODO cell are given by $D_{B_1} = D_a(1-\frac{1}{2}\sin\frac{1}{2}\mu)$, $D_{B_2} = D_a(1-\frac{1}{2}\sin\frac{1}{2}\mu) + (D_F-D_a)\sin^2\frac{1}{2}\mu$. In the thin-element approximation, the momentum compaction becomes

$$\alpha = \frac{1}{L_m}\sum_{\text{modules}}(D_{B_1}+D_{B_2})\theta\;, \tag{6}$$

where θ is the bending angle of each dipole and L_m is the length of the half-module. In comparison with the momentum compaction factor of lattice made from conventional FODO cells, we obtain

$$\frac{\alpha}{\alpha_{\text{FODO}}} = \frac{L_F}{L_m}\left[\zeta + (1-\zeta)\frac{\sin^2\frac{1}{2}\mu}{2(1-\frac{1}{2}\sin\frac{1}{2}\mu)}\right]\;, \tag{7}$$

which agrees well [5] with that obtained from realistic lattice design by using the MAD or the SYNCH programs. Note here that the momentum compaction factor is a linear function of the initial dispersion function if the module length is a constant. Although the thin-lens approximation has been used for the quadrupoles and dipoles, it is easy to see that this linear relationship is exact even for thick elements.

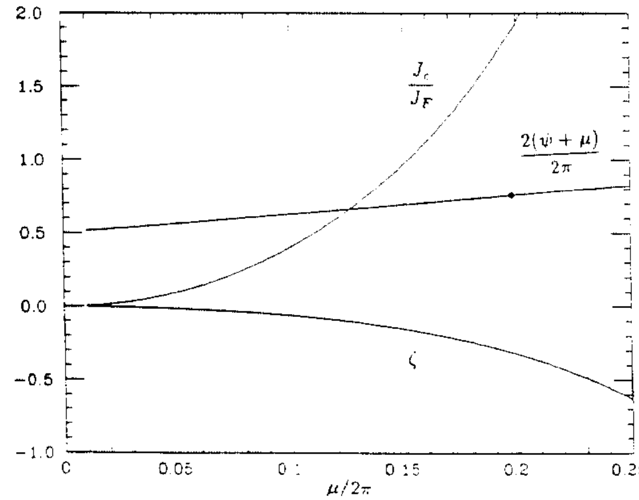

Figure 1: The dispersion function parameter ζ at M_a, the total phase advance of the QI module and the dispersion action in the straight section are plotted as a function of the phase advance of the FODO cell component

3 QUASI-ISOCHRONISM CONDITION

A basic module with isochronous condition is given by the condition $\alpha = 0$. Using Eq. (7), we obtain

$$\zeta = -\frac{\sin^2\frac{\mu}{2}}{2-\sin\frac{\mu}{2}-\sin^2\frac{\mu}{2}}, \tag{8}$$

for the basic module. The parameter ζ, the phase advance, $2(\psi+\mu)$, and $\frac{J_c}{J_F}$ of the isochronous module is shown in Fig. 1 as a function of μ of the FODO cell. Thus the accelerator made of modules which satisfies the above condition will be a QI storage ring. These QI storage rings are tunable by adjusting the ζ through the phase advance μ of the FODO cell.

A special class of these QI storage rings are composed with module having odd multiple of the 90° betatron phase advance, which is usually prefered by beam dynamics consideration. The 270° phase advance QI module can be achieved with the condition $\mu = 69°$ shown as a dot in Fig. 1. Such module have the dispersion action in the straight section given by $J_c \approx 1.75 J_F$. The maximum positive dispersion function at the symmetry point, M_c is then given by

$$D_c \approx \sqrt{1.75\frac{\beta_c}{\beta_F}|D_a|} \approx 0.38\sqrt{\frac{\beta_c}{\beta_F}}|D_F(69°)|$$

Thus the dispersion function of such a module is smaller than the corresponding FODO cell at 90° of phase advance.

4 EMITTANCE OF ISOCHRONOUS CELLS

The emittance of electron storage ring is an important quantity in the performance of the electron storage ring

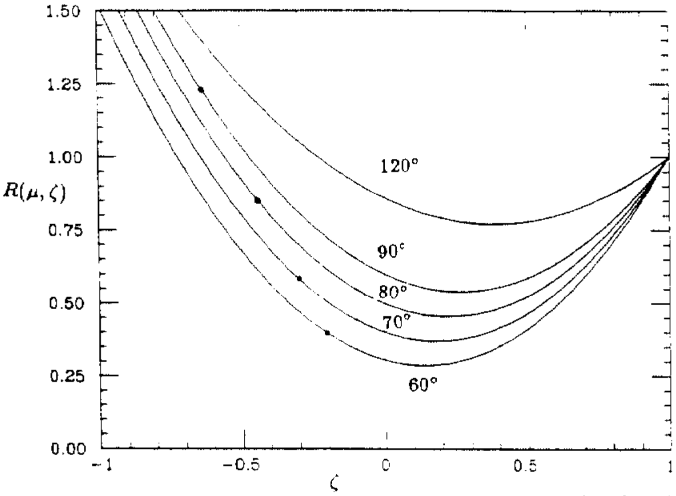

Figure 2: The ratio of the average H-function in the dipole is plotted as a function of ζ, the initial dispersion function at QF location for phase advance $\mu = 60°, 70°, 80°, 90°$ and $120°$. Here a QI lattice is given by a relation between ζ and μ shown in Fig. 1

design. For the isochrounous storage ring, the emittacne is given by

$$\epsilon = C_q \frac{\gamma^2 \langle H \rangle}{J_x \rho}$$

where $C_q = 3.84 \times 10^{-13}$ m and the dispersion action, H, is given by $H = \frac{1}{\beta_x}[D_x^2 + (\beta_x D_x' - \frac{\beta_x'}{2} D_x)^2]$. Using the thin lense approximation by dividing each dipole in FODO cell into two sections, we obtain

$$\frac{D_{B1}}{D_F} = \zeta(1 - \frac{1}{2}\sin\frac{\mu}{2}) + \frac{1}{4}\frac{\sin^2\frac{\mu}{2}}{2+\sin\frac{\mu}{2}},$$

$$\frac{D_{B1}' L}{D_F} = -\zeta \sin\frac{\mu}{2} + \frac{\sin^2\frac{\mu}{2}}{2+\sin\frac{\mu}{2}},$$

$$\frac{D_{B2}}{D_F} = \zeta(1 - \frac{1}{2}\sin\frac{\mu}{2} - \sin^2\frac{\mu}{2}) + \frac{\sin^2\frac{\mu}{2}}{2+\sin\frac{\mu}{2}}(\frac{9}{4} + \sin\frac{\mu}{2}),$$

$$\frac{D_{B2}' L}{D_F} = \zeta \sin\frac{\mu}{2}(1 - 2\sin\frac{\mu}{2}) + \frac{\sin^2\frac{\mu}{2}}{2+\sin\frac{\mu}{2}}(3 + 2\sin\frac{\mu}{2}),$$

$$\beta_{B1} = \beta_{B2} = \frac{L}{\sin\mu}(2 - \sin^2\frac{\mu}{2}), \quad \frac{\beta_{B1}'}{2} = -\frac{\beta_{B2}'}{2} = -\frac{1}{\cos\frac{\mu}{2}}.$$

The H-function can then be expressed as

$$\langle H \rangle = \rho\theta^3 F_{FODO}(\mu) R(\mu,\zeta), \quad (9)$$

where

$$F_{FODO} \approx \frac{1 - \frac{3}{4}\sin^2\frac{\mu}{2}}{\sin^3\frac{\mu}{2}\cos\frac{\mu}{2}},$$

in the thin lense approximation. By definition, we have $R(\mu, \zeta = 1) = 1$. The ratio function R of the variable γ_T lattice is shown in Fig. 2, where the QI lattice corresponds to points marked on the figure. The particularly interesting QI lattice with $\zeta \approx -0.3$ and $\mu = 70°$ has an emittacne of about 60% of that of the corresponding regular FODO cell lattice. Since the emittance of FODO cell lattice is proportional to $\frac{1}{\mu^3}$, the QI lattice at the phase advance of 70° is equivalent to that of a regular 90° FODO cell lattice. Although the emittance of a QI lattice is still about two orders of magnitude larger than the minimum emittance Chasman-Green lattice with $\langle H \rangle|_{MBCG} = \frac{1}{4\sqrt{15}}\rho\theta^3$, [6] the QI lattice has the simplicity of tunability of the momentum compaction factor α.

5 CONCLUSION

Lattice, which is made of QI modules are QI lattice. For a low energy accelerator, such as the Φ-factory, with $B\rho \approx 1.7$ Tm, race track with two module can be considered. Such lattice does not have dispersion free straight sections. For higher energy accelerators, dispersion free straight sections discussed in ref. [5] can be incoorporated into the lattice.

We have made an extension of the flexible γ_T lattice to the regime of the quasi-isochronism. We found that the resulting emittance is better than that of the corresponding FODO cell lattice. The simplicity of the FODO cell remains to be the key feature of the QI lattice. The longitudinal phase space can however be controlled by the momentum compaction factor in proper balance between the microwave instability and the synchrotron radiation damping. Possible applications of the QI lattices are Φ-factory, small damping ring, or synchrotron radiation sources. It might offer advantages in offering a smaller equilibrium longitudinal phase space area without resorting to high rf voltage consideration.

6 REFERENCES

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