Global Tuning Knobs for the SLC Final Focus.*

N. J. Walker, J. Irwin, M. Woodley
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

Abstract

The beam phase space at the exit of a given transport line generally depends on the incoming beam conditions, and thus in order to adjust the beam parameters at the exit of the line requires a prior knowledge of the initial beam parameters. The same is generally true for final focus systems. A tuning algorithm for β matching the SLC final focus is reported here in which no prior knowledge of the exact incoming phase space is required. Only a single beam size diagnostic located at either the interaction point (IP) or an image of the IP is required, together with a knowledge of the linear lattice from the quadrupoles to the tuning point. The algorithm is presented within the Lie Algebra framework. Although the algorithm is presented here is specific to linear collider final focus systems, the technique is generally applicable to any beamline.

I. INTRODUCTION.

The SLC final focus consists of two telescopes separated by a chromatic correction section (CCS)[1]. The demagnification of the final telescope (FT) is 5:1 in both planes, and approximately 8:1 horizontally and 3:1 vertically in the upper telescope (UT). Adjusting the focus at the interaction point (IP) is critical to achieving the maximum luminosity: the beam waist ($\alpha=0$) must be at the IP, and the correct β function (β^*) is required to give the minimum possible beam size[2]. Given the dynamic range of the incoming beam conditions (especially emittance), it is necessary to be able to efficiently tune the final focus to acheve the required conditions at the IP. In general, beam matching in transport lines requires a prior knowledge of the initial beam phase space. These measurements are then given to a non-linear fitting algorithm that adjusts the quadrupoles in the line to achieve the desired results. In the following sections, an algorithm for achieving the correct β match in the SLC final focus is presented that requires only a single beam size diagnostic (wire scanner), and no prior knowledge of the incoming beam conditions. It is proposed to implement such an algorithm as part of the SLC final focus upgrade[3], where all the tuning will take place in the UT. To facilitate this, a new wire scanner will be positioned at the IP image point at the entrance to the CCS (exit of the UT). Although the algorithm is presented within the framework of the SLC final focus, the technique is generally applicable to any beamline.

II. β MATCHING ALGORITHM.

Figure 1 shows schematically the proposed tuning algorithm. For simplicity, only one plane will be considered. The

initial adjustment (figure 1a) brings the beam waist to the wire ($\alpha=0$). This is achieved by applying an effective drift matrix:

$$R_d = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}. \qquad (1)$$

In practice, the parameter s is scanned until a minimum in the beam size at the wire scanner is achieved. The next stage is to correct the beam size, *i.e.* adjust the β function (figure 1b), which is simply the application of a *magnifying* matrix of the form

$$R_m = \begin{pmatrix} m & 0 \\ 0 & \dfrac{1}{m} \end{pmatrix}. \qquad (2)$$

In this case, the parameter m can either be scanned until the desired beam size at the wire is achieved, or it can be calculated and applied from the knowledge of the initial beam size at the wire. If there is no coupling in the beam (see section III), the beam is now matched to give the desired image at the IP (assuming that the demagnification and phase advance of the CCS and final telescope are correctly set).

The problem is now reduced to designing orthogonal control over the two parameters s and m (four when considering both X and Y planes). In the 2×2 matrices given in (1) and (2), there are three independent parameters; thus to generate the desired form of the matrix requires three (six) independent quadrupoles. It may at first seem strange that three independent variables are required to match two parameters (α and β); however the phase advance is also implicitly constrained, giving a total of three parameters. Because of the phase advance constraint, the solution arrived at may not be the optimum with respect to magnet strengths. A phase knob can be constructed which adjusts only the phase advance and leaves the β function unchanged and the waist at the wire. The corresponding matrix has the form

$$R_\phi = \begin{pmatrix} \sqrt{\beta} & 0 \\ 0 & \dfrac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \dfrac{1}{\sqrt{\beta}} & 0 \\ 0 & \sqrt{\beta} \end{pmatrix}. \qquad (3)$$

which is more complicated than either (1) or (2), and requires a prior knowledge of the β function. The single parameter ϕ can be adjusted to relieve magnet power supplies which are at their maximum strength, without losing the β match.

III. CONSTRUCTION OF TUNING KNOBS.

To construct the orthogonal *knobs* to adjust the matching parameters, a perturbation technique is employed. The three matching quads need to be adjusted in such a way as to produce either a pure s, m, or ϕ. An elegant method for calculating

| (a) | (b) | (c) |

$$R = \begin{pmatrix} 1 & -s \\ 0 & 1 \end{pmatrix}$$

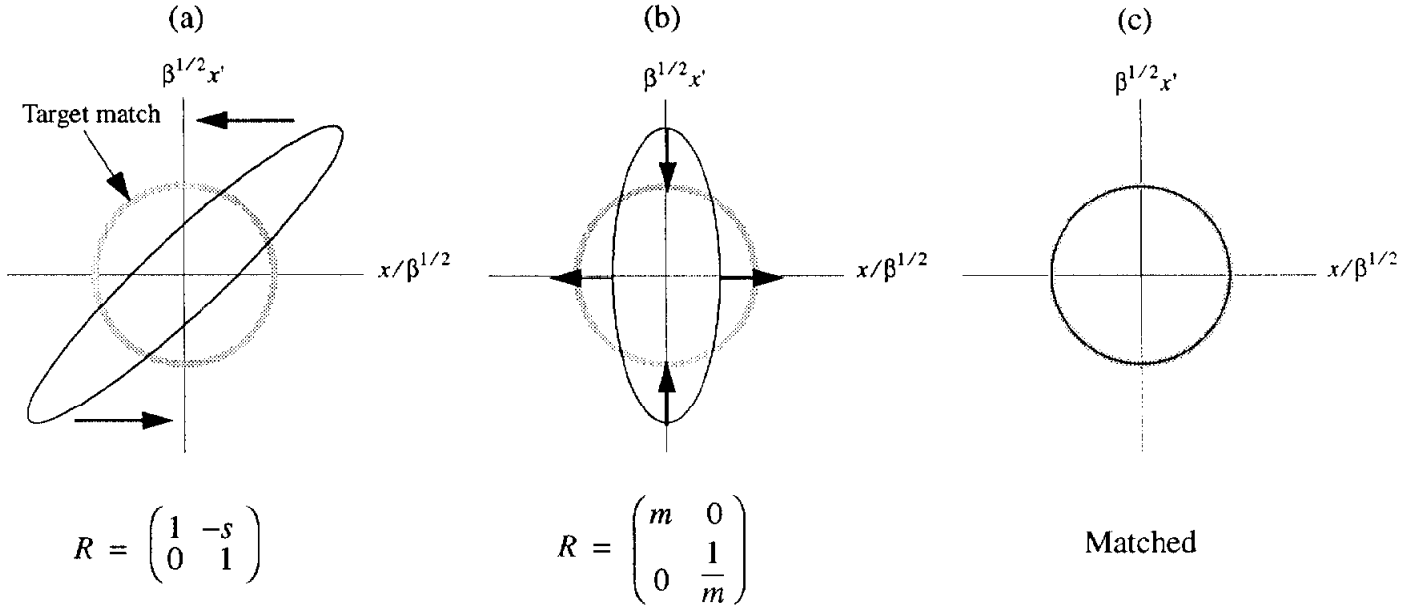$$R = \begin{pmatrix} m & 0 \\ 0 & \dfrac{1}{m} \end{pmatrix}$$

Matched

Figure 1. Figures (a), (b) and (c) represent the tuning stages involved in β matching (the phase space is normalized to the target β). First the waist is brought to the wire scanner by scanning the s parameter (a), then the required matching is achieved by adjusting the m parameter. All operations can be accomplished by measuring the beam size on the wire scanner.

the knobs is to use second-order Hamiltonians to represent the thin lens perturbations to the existing lattice, and then use them as generators in a Lie Algebra[4].

If the phase space coordinates at the wire scanner (matching point) are $x = (x, x', y, y')$, then the total perturbative Hamiltonian due to the three quadrupoles can be expressed as

$$H_{\Delta K} = ax^2 + bxx' + cx'^2, \qquad (4)$$

where the coefficients are functions of the linear lattice (R matrix elements) between the quadrupoles and wire scanner, and the change in quadrupole strengths $\Delta K = (\Delta K_1, \Delta K_2, \Delta K_3)$. To first order, the coefficients in (4) can be expressed as linear functions of the ΔK_i's:

$$a = \frac{1}{2} \sum_{i=1}^{3} \Delta K_i R_{22}(i)^2$$

$$b = -\sum_{i=1}^{3} \Delta K_i R_{12}(i) R_{22}(i)$$

$$c = \frac{1}{2} \sum_{i=1}^{3} \Delta K_i R_{12}(i)^2 \qquad (5)$$

where $R_{pq}(i)$ are the linear Green's functions from the ith quadrupole to the wire scanner. Over some small range, therefore, it is possible to have linear combinations of the quadrupole strengths which give independent control over the coefficients a, b and c.

When exponentiated, the Hamiltonian becomes a generator for a map[4]:

$$e^{:-H_{\Delta K}:}u = u + [-H_{\Delta K}, u] + \frac{1}{2!}[-H_{\Delta K}, [-H_{\Delta K}, u]] + \dots \qquad (6)$$

where u can be any phase space coordinate. To see how the coefficients in $H_{\Delta K}$ relate to the required tuning matrices given in (1), (2) and (3), three cases are considered:

A. $a=0$, $b=0$.

The Hamiltonian given in (4) is now simply cx'^2. Applying (6) to x and x' gives:

$$x \to e^{:-cx'^2:}x = x + 2cx'$$

$$x' \to e^{:-cx'^2:}x' = x' \qquad (7)$$

The equations in (7) can be represented as the matrix

$$\begin{pmatrix} 1 & 2c \\ 0 & 1 \end{pmatrix}, \qquad (8)$$

which is the required drift matrix (1) with $s=2c$.

B. $a=0$, $c=0$.

Now $H_{\Delta K}=bxx'$, and the expansion given in (6) becomes

$$x \to e^{:-bxx':}x = (1 + b + \frac{b^2}{2} + \frac{b^3}{6} + \frac{b^4}{24} + \dots)x = e^b x$$

$$x' \to e^{:-bxx':}x' = (1 - b + \frac{b^2}{2} - \frac{b^3}{6} + \frac{b^4}{24} - \dots)x' = e^{-b}x' \qquad (9)$$

In matrix form, the equations in (9) give the required magnification matrix (2), with $m = e^b$.

C. $a=(1/2)\phi/\beta$, $b=0$, $c=(1/2)\phi\beta$.

The Hamiltonian is now given as:

$$H_{\Delta K} = \frac{1}{2}\frac{\phi}{\beta}x^2 + \frac{1}{2}\phi\beta x'^2. \tag{10}$$

The expansion (7) now generates the phase knob matrix (3), with the corresponding phase change ϕ.

Having equated the coefficients in the Hamiltonian to the desired tuning coefficients s, m and ϕ, it is a simple matter to calculate the required changes in quadrupole strengths to effect the knob. The linear equations given in (5) are only correct over a small range of the ΔK_i, and in order to make large corrections or scans of a given knob, it will be necessary to integrate through the change, *i.e.* take small steps in ΔK_i, recalculating the coefficients at each step. The size of the step has to be found empirically, although in simulations for the SLC tuning scheme, 9 or 10 steps have found to be sufficient. Figure 2 shows simulations of a waist (drift matrix) scan.
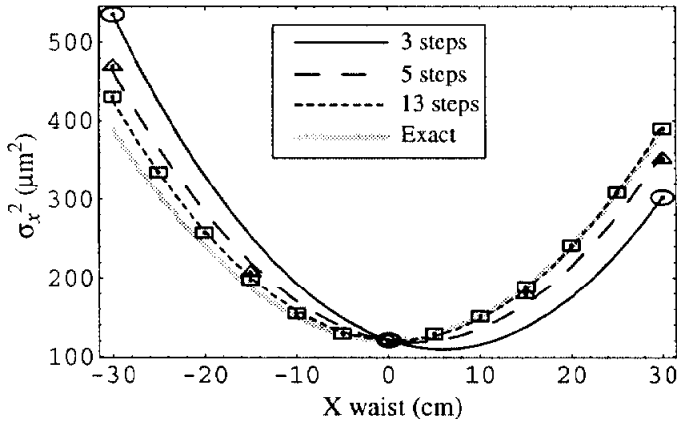

Figure 2. Simulations of an X waist (drift matrix) scan. The solid gray line represents a pure drift, while the other three lines represent scans using different step sizes.

IV. COUPLING CORRECTION.

A coupling correction can be formulated in exactly the same way as the β matching described in section II. In general four skew-quadrupoles are required to independently adjust the four skew parameters in the Hamiltonian:

$$H_{skew} = axy + bxy' + cx'y + dx'y'. \tag{11}$$

The associated coupled (now 4×4) matrix generated by this Hamiltonian is of the form

$$\begin{pmatrix} h & 0 & \zeta c & \zeta d \\ 0 & h & -\zeta a & -\zeta b \\ \zeta b & \zeta d & h & 0 \\ -\zeta a & -\zeta c & 0 & h \end{pmatrix}. \tag{12}$$

where $h=\cos(\theta)$, $\zeta = \sin(\theta)/\theta$, with $\theta^2 = ad-bc$ ($\theta^2 > 0$). In the case of $\theta^2 < 0$, then $h=\cosh(\theta)$, and $\zeta = \sinh(\theta)/\theta$. Because of the flat nature of the beam ($\sigma_x \gg \sigma_y$), only two of these coefficients are of interest in the SLC final focus: the b parameter adjusts the x-y tilt of the beam, while the d parameter increases

the small σ_y proportionally to the horizontal divergence. With the two skew-quadrupoles available in the UT, it is possible to independently control these two coefficients. The remaining coefficients, a and c, will be allowed to vary arbitrarily. Inclusion of the skew correction now necessitates the precise control of eight magnets (six normal- and two skew-quadrupoles), and the coefficients in (5) will in general contain the coupled Green's functions.

V. IMPLEMENTATION.

The beta matching algorithm (with coupling) will be implemented in FORTRAN as part of the SLC control system[5]. This system provides the necessary interface to the readback and control of the magnets which will be used, as well as a standardized user interface consisting of touch panels and displays. In addition, a Correlation Plot facility is provided which allows measurement of beam parameters as a function of magnet strengths[6]. Users will select a desired scan type (*i.e.* X-waist), scan range, and number of incremental steps from a touch panel. The beta matching software will compute quadrupole strengths for each step of the scan and then use the Correlation Plot facility to measure the beam size on a wire scanner as it steps the magnets through their precomputed setpoints. The user will be presented with the results of the scan graphically and will be allowed to select the optimum set of quadrupole strengths.

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