Single Beam Effects due to Errors in Crab Compensation*

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Abstract

Errors in a crab compensation scheme such as betatron phase advance errors will lead to shape and orientation changes of a bunch. These changes can be computed in a systematic manner.

Introduction

The necessity of using a large number of bunches in the B–factories envisioned for the future necessarily aggravates the problem of separating the beams at the parasitic crossing points near the interaction point (IP). A possible solution is to use a crossing angle. The induced synchro-betatron coupling of an angle crossing can be completely eliminated in theory with 'crab' compensation in which the bunches of each beam would be tilted at the IP by an angle $\theta_{crb} = \theta_{cross}/2$ where θ_{cross} is the full crossing angle. In terms of the beam–beam interaction, the problems with a crab compensation scheme for a crossing angle are therefore caused by imperfections in the compensation. It is therefore important to understand the effect that crabbing errors will have on the beam in order to set tolerances on allowable errors. These errors can come from various sources. Possibilities include errors in the betatron phase advances from the crab cavities to the IP, the finite wavelength of the crab cavity RF, etc. It is the purpose of this paper to show how the effect of these errors can be analyzed.

In general, the compensation errors can be divided into three groups: The first group comprises all zeroth order effects where the effect of the error on an individual particle is independent of the particle's coordinates. Errors in this group can result in transverse offsets between opposing bunches at the IP and/or longitudinal offsets of the IP itself. The second group comprises all first order effects where the effect of a compensation error on an individual particle is linear in the particle's coordinates. These errors result in crab angle errors and/or changes in the bunch's width and length. The final group comprises all higher order effects. These errors will cause distortion of the Gaussian shape of the bunch.

Several different types of crab compensation schemes have been proposed. In this paper 'transverse crabbing' — in which two RF cavities are used to give a time-varying sideways kick to a bunch as it passes through either cavity — will be considered[1]. An alternate scheme calls for using dispersion at the RF accelerating cavities to give the correct crab angle at the IP[2]. In any case, the general analytic technique outlined in this paper will be applicable.

Zeroth Order Errors

Errors in a Crab cavity's RF phase (timing errors) will produce zeroth order effects. A phase error in one cavity will add a constant horizontal kick to the beam creating an orbit bump. At the IP the displacement Δx_{ip} due to a timing error Δt_{RF} in one cavity is [1]

$$\Delta x_{ip} = \frac{c\,\Delta t_{RF}}{2}\tan(\theta_{crb})\,. \qquad (1)$$

First Order Errors

First order errors in crab compensation lead to changes in the crabbing angle θ_{crb} and aspect ratio r_σ which is the ratio of the beam height (minor axis) to beam length (major axis). In order to be able to make the calculation of θ_{crb} and r_σ at the IP one needs to know two quantities[3]; namely, the one turn transfer matrix from IP to IP, $\mathbf{T}_{ip\to ip}$, and the ratio of the emittances of normal modes $r_\epsilon \equiv \epsilon_a/\epsilon_b$. For the purposes of the analysis one need only consider the degrees of freedom in the plane in which the beam is being tilted. $\mathbf{T}_{ip\to ip}$ will then be a 4 x 4 matrix. Using the idealized ring shown in Figure 1 $\mathbf{T}_{ip\to ip}$ can be constructed as the product of seven matrices

$$\mathbf{T}_{ip\to ip} = \mathbf{T}_{c1\to ip}\cdot\mathbf{T}_{k1}\cdot\mathbf{T}_{arc}\cdot\mathbf{T}_{k2}\cdot\mathbf{T}_{ip\to c2}\,, \qquad (2)$$

where $\mathbf{T}_{c1\to ip}$ and $\mathbf{T}_{ip\to c2}$ are transport matrices between the crab cavities and the IP, $\mathbf{T}_{arc}$ is the transport matrix through the arc, and $\mathbf{T}_{k1}$ and $\mathbf{T}_{k2}$ are the kick matrices for crab cavities labeled C1 and C2 respectively.

The matrices on the right hand side of equation (2) are parametrized using a set of input parameters. The crabbing is taken to be in the horizontal plane with z being the longitudinal axis and x being the horizontal axis. For the purposes of this paper, the input parameters are denoted with a tilde to distinguish them from calculated quantities. For example, the input parameter $\widetilde{\beta}_x(ip)$ is used in the construction of the matrices $\mathbf{T}_{ip\to c2}$, and $\mathbf{T}_{c1\to ip}$ (see below) but once $\mathbf{T}_{ip\to ip}$ is specified one can explicitly calculate the value of beta at the IP, $\beta_x(ip)$. Only in the case where there are no crab compensation errors will one be assured that $\beta_x(ip)$ will be equal to $\widetilde{\beta}_x(ip)$. Explicitly, the matrices were constructed as follows: The transport matrices $\mathbf{T}_{c1\to ip}$, $\mathbf{T}_{arc}$, and $\mathbf{T}_{ip\to c2}$ are assumed to have the

*Work supported by the National Science Foundation

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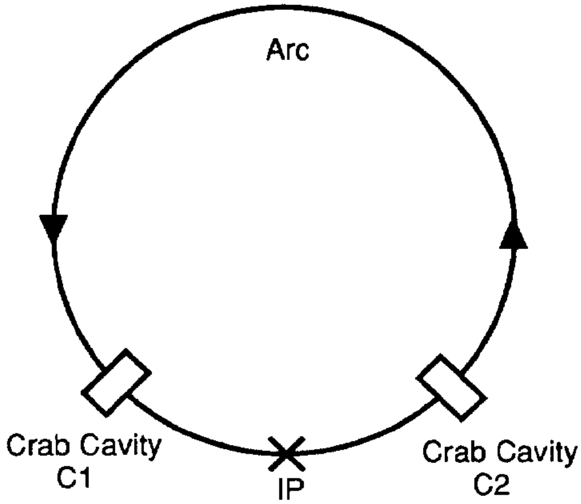

Figure 1: Model ring with 2 crab cavities.

general form:

$$\mathbf{T}_{s1\to s2} = \mathbf{T}_{\eta(s2)} \begin{pmatrix} \mathbf{R}_x & 0 \\ 0 & \mathbf{R}_z \end{pmatrix} \mathbf{T}^{-1}_{\eta(s1)} , \quad (3)$$

where $\mathbf{T}_{\eta(s)}$ is used for putting in the dispersion η and dispersion derivative η' at a given point s:

$$\mathbf{T}_{\eta(s)} = \begin{pmatrix} 1 & 0 & 0 & \widetilde{\eta}(s) \\ 0 & 1 & 0 & \widetilde{\eta}'(s) \\ -\widetilde{\eta}'(s) & \widetilde{\eta}(s) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \quad (4)$$

and $\mathbf{R}_x$ is the 2 x 2 rotation matrix

$$\mathbf{R}_x = \begin{pmatrix} \sqrt{\frac{\widetilde{\beta}_x(s2)}{\widetilde{\beta}_x(s1)}}\cos(2\pi\widetilde{Q}_x) & \sqrt{\widetilde{\beta}_x(s1)\widetilde{\beta}_x(s2)}\sin(2\pi\widetilde{Q}_x) \\ \frac{-\sin(2\pi\widetilde{Q}_x)}{\sqrt{\widetilde{\beta}_x(s1)\widetilde{\beta}_x(s2)}} & \sqrt{\frac{\widetilde{\beta}_x(s1)}{\widetilde{\beta}_x(s2)}}\cos(2\pi\widetilde{Q}_x) \end{pmatrix} \quad (5)$$

with $\widetilde{Q}_x$ being the horizontal phase advance between $s1$ and $s2$. For $\mathbf{T}_{arc}$ the matrix $\mathbf{R}_z$ has the same form of equation (5) with $Q = Q_s$, the synchrotron tune, and $\beta(s1) = \beta(s2) = \sqrt{\sigma_z/\sigma_{p_z}}$ with $\sigma_{p_z} \equiv \sigma_E/E$. For $\mathbf{T}_{c1\to ip}$ and $\mathbf{T}_{ip\to c2}$, $\mathbf{R}_z$ is given by

$$\mathbf{R}_z = \begin{pmatrix} 1 & -l \\ 0 & 1 \end{pmatrix} , \quad (6)$$

where l in the above equation can be thought of as the local momentum compaction factor. $l = l_{c1\to ip}$ for $\mathbf{T}_{c1\to ip}$, $= l_{ip\to c2}$ for $\mathbf{T}_{ip\to c2}$.

The crab kick matrix $\mathbf{T}_{k1}$ for crab cavity C1 is given by

$$\mathbf{T}_{k1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \widetilde{V}_{c1}\kappa_t & 0 \\ 0 & 0 & 1 & 0 \\ \widetilde{V}_{c1}\kappa_t & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

with a similar matrix for $\mathbf{T}_{k2}$. In the above equation the normalization constant κ_t is given by

$$\kappa_t = \frac{1}{\sqrt{\widetilde{\beta}_x(crb)\widetilde{\beta}_x(ip)}}\widetilde{\theta}_{crb} . \quad (8)$$

Parameter	Nom. Value	Parameter	Nom. Value
$\widetilde{\beta}_x(ip)$	1.0 m	$\widetilde{\sigma}_z$	0.01 m
$\widetilde{\beta}_x(crb)$	10.0 m	$\widetilde{\sigma}_{p_z}$	$6.5 \cdot 10^{-4}$
$\widetilde{\beta}_x(arc)$	10.0 m	$\widetilde{l}_{c1\to ip}$	0.0 m
$\widetilde{\theta}_{crb}$	0.012 rad	$\widetilde{l}_{ip\to c2}$	0.0 m
$\widetilde{Q}_{c1\to ip}$	0.25	$\widetilde{\eta}(c1)$	0.0 m
$\widetilde{Q}_{ip\to c2}$	0.25	$\widetilde{\eta}(c2)$	0.0 m
$\widetilde{\epsilon}_x$	$1.3 \cdot 10^{-7}$ m	$\widetilde{\eta}(ip)$	0.0 m
$\widetilde{Q}_x$	0.682	$\widetilde{\eta}'(c1)$	0.0 m
$\widetilde{Q}_s$	0.085	$\widetilde{\eta}'(c2)$	0.0 m
$\widetilde{V}_{c1}$	1.0	$\widetilde{\eta}'(ip)$	0.0 m
$\widetilde{V}_{c2}$	1.0		

Table 1: Crab Compensation Input Parameters.

The normalized voltage $\widetilde{V}$ is such that with no errors in the compensation, $\widetilde{V}_{c1} = \widetilde{V}_{c2} = 1$ will result in the actual (*i.e.* calculated) crab angle at the IP, θ_{crb}, being equal to the value of the input parameter $\widetilde{\theta}_{crb}$.

Once $\mathbf{T}_{ip\to ip}$ is specified the calculation of r_ϵ is obtained by assuming that all the emittance is generated uniformly in the arcs as outlined by Sagan[4]. The calculation of θ_{crb} and r_σ then follows from the general analysis for calculating rotation angles given by Orlov and Sagan[3].

Figure 2 shows the effect of varying $\widetilde{Q}_{c1\to ip}$ while keeping the other input parameters at their nominal values as shown in Table 1. The nominal values of the parameters were chosen to be similar to the current design of the Cornell B–factory CESR–B. There is a coupling resonance near $\widetilde{Q}_{c1\to ip} = 0.53$ where the normal mode tunes Q_a and Q_b satisfy $Q_a - Q_b =$ integer. Near the coupling resonance the normal mode emittance ratio r_ϵ blows up and is of order unity and there are large variations in θ_{crb} and r_σ. The sum resonance is near $\widetilde{Q}_{c1\to ip} = 0.70$ where $Q_a + Q_b =$ integer. Near the sum resonance there are also large variations in r_ϵ, θ_{crb} and r_σ. The difference between the sum and coupling resonances is that near the coupling resonance the beam is always stable while near the sum resonance there is a stop band where the one turn matrix displays an instability. In addition to the two mode coupling resonances there are also single mode resonances resonances near $\widetilde{Q}_{c1\to ip} = 0.12$ and near $\widetilde{Q}_{c1\to ip} = 0.62$ when one of the normal mode tunes passes through 0.5 and 1.0 respectively. Within the limits of the linear model used in the analysis, the single mode resonances have little effect upon θ_{crb}, r_σ, or r_ϵ. In fact the half-integer resonance is not detectable on the scale of figure.

As a practical matter, it is important to understand the variation of θ_{crb} near the operating point as a function of the various parameters. Figures 3 and 4 shows the effect of varying $\widetilde{V}_{c1}$ and $\widetilde{\eta}(C1)$ respectively upon θ_{crb} and r_σ while keeping the other input parameters at their nominal values. From Figure 3 one finds that $\Delta\theta_{crb}/\widetilde{\theta}_{crb} \approx \Delta\widetilde{V}_{c1}/2$. This is what one would naively expect since this change is 1/2 of what one would get if both cavity voltages where

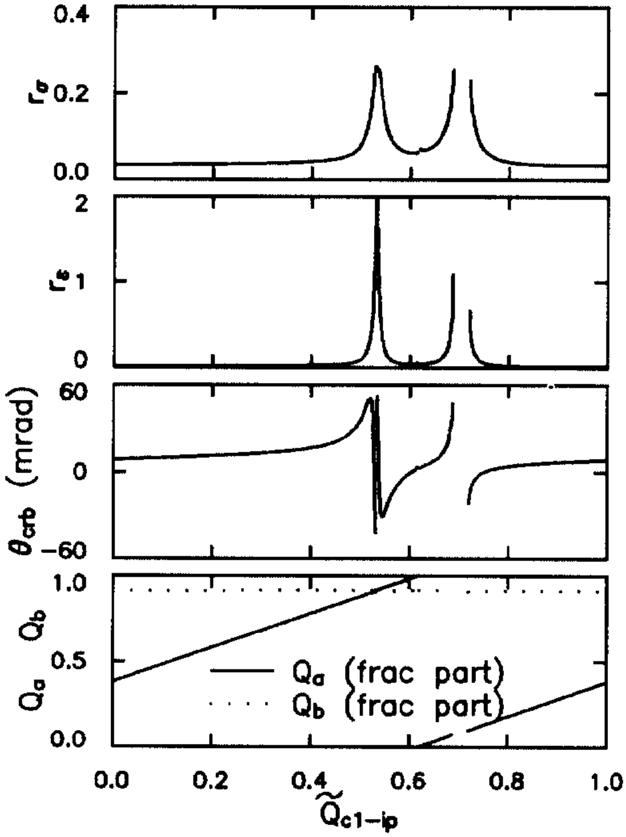

Figure 2: Eigen tunes crab angle, beam aspect ratio and normal mode emittance ratio verses $\widetilde{Q}_{c1\rightarrow ip}$.

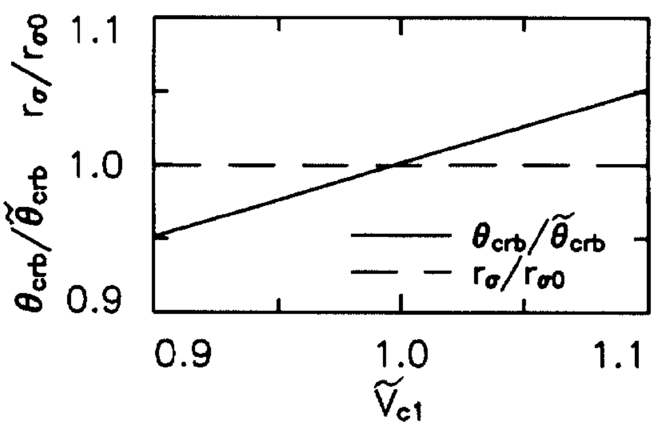

Figure 3: Crab angle and beam aspect ratio as a function of voltage in crab cavity C1.

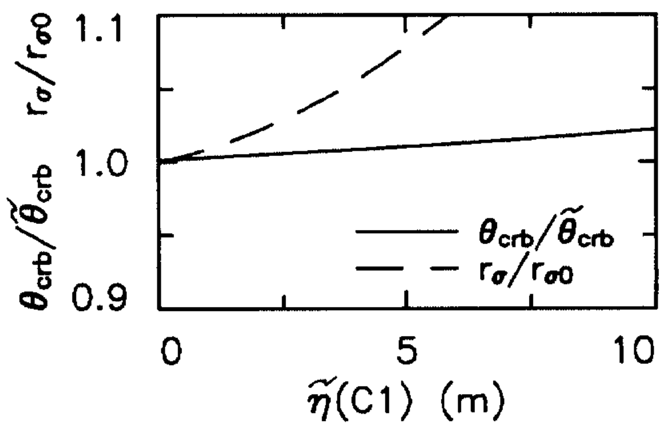

Figure 4: Crab angle and beam aspect ratio as a function of the dispersion at crab cavity C1.

changed in unison. With Figure 4 one sees that it takes a rather large dispersion to effect the crab angle by more than a few percent. This is to be expected since the kick at the crab cavities is, to first order, independent of the dispersion.

One can consider variations in the other parameters[4]. The General conclusion is that with the horizontal and longitudinal tunes far from a coupling resonance then θ_{crb} and r_σ are not 'overly' sensitive to errors in the crab compensation.

Higher Order Effects.

Higher order effects produce a distortion from a Gaussian profile. A systematic analysis is beyond the scope of this paper. These effects, however, are inevitably most significant near the extremities of the beam where the beam density is minimal. As an example of a higher order effect, consider the effect of finite crab cavity RF wavelength. A particle with displacement z will feel a kick in the crab cavity

$$\Delta x' \propto \sin(2\pi z/\lambda_{RF}) \approx \left(\frac{2\pi}{\lambda_{RF}}\right)\left(z - \frac{2\pi^2 z^3}{3\lambda_{RF}^2}\right) . \quad (9)$$

For $\lambda_{RF} = 0.6$ m (500 MHz RF) a particle with a z of 0.023 m ($2.3\sigma_z$) will feel a kick which deviates 1% from nonlinearity. Considering that only 1% of the particles have $z > 2.3\sigma_z$ the finite RF wavelength does not have a large effect on the overall beam shape. This is not to imply however that higher order effects are necessarily negligible since they induce synchrobetatron resonances for large amplitude particles.

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