High Accelerating-Gradient Accelerator based on Magnetic Field Decay Mechanism

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Abstract

We propose a new concept of accelerating device, where the magnetic field energy is efficiently stored in a specially prepared material by a continuous electrical current. If the current drops abruptly to zero, the magnetic field stored inside the material decays, thereby generating accelerating-electric field. A high-current electron beam with relatively low energy propagates through a hole along the axis of a cylindrical field-storage device, sustaining azimuthal magnetic field in the field-storage material. A theory of the magnetic field diffusion is developed, in order to estimate the accelerating gradient whenever the sustaining electric current drops to zero. According to a theoretical calculation, a high-accelerating electric field occurs at the axis, which can be a substantial fraction of mega volt/cm for optimized values of the magnetic permeability and conductivity of the field-storage material.

I. INTRODUCTION

In recent years, there has been strong progress in high current electron beam accelerators. The most advanced device for intense electron beam acceleration is the induction linear accelerator[1], where each module of many local accelerators applies its local electric field to a cluster of traveling electrons. In this way, several research institutes have already produced very high-energy high-current electron beams. The physical-acceleration mechanism underlying the induction linear accelerator is the time varying magnetic field. A change of the magnetic field is an excellent means for charged particle acceleration. Typical energy and current of these electron beams are more than 10 MeV and more than 10 kA. The accelerating gradient of the induction linear accelerator is typically 0.5 mega volt/m. Since the accelerating gradient is proportional to the change of the magnetic field flux, most of the present induction linear accelerators require cores made of a ferromagnetic material. These cores are very bulky and heavy. In this regard, we propose a new concept of accelerating device, which requires less core material and provides more accelerating gradient. This new concept also makes use of the accelerating-electric field initiated by an abrupt change of the magnetic field.

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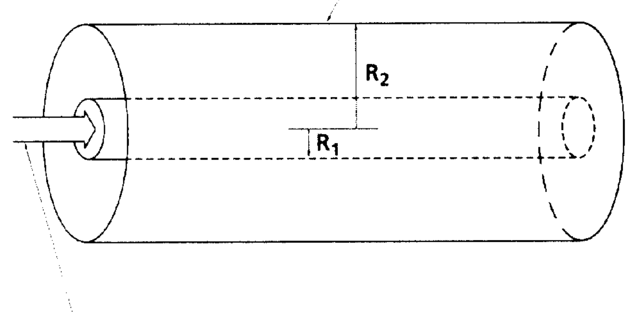

Figure 1. Schematic presentation of an accelerator module.

II. THEORY OF MAGNETIC FIELD DECAY

As shown in Fig. 1, we assume that an electron beam with current I propagates through a hole with radius R_1 in a magnetic field-energy storage with radius R_2. Here we assume that the electron beam radius is less than the hole radius. The material inside the field-energy storage device has a high permeability μ and a relatively-low conductivity σ. During the time, when the beam current is a constant value I, a high concentration of the magnetic field is stored in the material. The beam current I drops abruptly to zero at time t = 0 and the magnetic field stored in the material starts to decay, thereby generating a strong induced-electric field. This electric field accelerates a trailing electron current if there is any. The Ampere's law in the Maxwell equation is written by

$$\nabla \times B = \frac{\mu}{c}\frac{\partial}{\partial t}E + \frac{4\pi\mu}{c}J_T, \qquad (1)$$

where **B** is the magnetic field, **E** is the electric field and J_T represents both the steady-state beam current and the induced current.

In the steady-state case when $t < 0$, the azimuthal magnetic field is approximately given by

$$B_\theta^b(r) = \begin{cases} 2I\mu/cr, & R_1 < r < R_2, \\ 0, & \text{otherwise}, \end{cases} \qquad (2)$$

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where the magnetic field outside the storage material is neglected simply because of a large value of the permeability of the material. The superscript b in Eq. (2) represents "before" t = 0. The azimuthal magnetic field after t = 0 is obtained from the Maxwell equation

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{B_\theta^a}{\mu}\right)\right] - \frac{4\pi\sigma}{c^2}\frac{\partial B_\theta^a}{\partial t} - \frac{\epsilon}{c^2}\frac{\partial^2 B_\theta^a}{\partial t^2} = 0, \tag{3}$$

where ϵ is the dielectric constant and use has been made of the induced current density $J_{in} = \sigma E_z(r,t)$. Most of the azimuthal magnetic field is concentrated inside the field-storage material. We therefore neglect other regions. Assuming that the most dominant radial wavevector k is order of $1/R_1$ in magnitude, we recognize that the third term in Eq. (3) is negligible, provided

$$4\pi\sigma\sqrt{\mu/\epsilon}\,R_1 >> c, \tag{4}$$

which can be easily satisfied in practical cases.

The azimuthal magnetic field $B_\theta^a(r,t)$ in the range of $R_1 < r < R_2$ is given by

$$B_\theta^a(r,t) = \int_0^\infty dk\, a_k J_1(kr)\exp(-\lambda_k^2 t), \tag{5}$$

where $J_\alpha(x)$ is the Bessel function of order α, k is the radial wavevector, the parameter λ_k is defined by

$$\lambda_k^2 = \frac{k^2c^2}{4\pi\sigma\mu}, \tag{6}$$

and the coefficient a_k must satisfy

$$\int_0^\infty dk\, a_k J_1(kr) = B_\theta^b(r). \tag{7}$$

Substituting Eq. (2) into Eq. (7), multiplying both sides of Eq. (7) by $rJ_1(kr)$ and making use of the orthogonality of the Bessel function, we obtain

$$a_k = \frac{2I\mu}{c}\left[J_0(kR_1) - J_0(kR_2)\right]. \tag{8}$$

Substitution of Eq. (8) into Eq. (5) gives a complete expression of the magnetic field inside the field storage material. We neglect the magnetic field outside the storage material.

The induced-electric field $E_z(r,t)$ is obtained from the relationship

$$\frac{\partial E_z}{\partial r} = \frac{1}{c}\frac{\partial B_\theta^a}{\partial t}, \tag{9}$$

where use has been made of the approximation $\epsilon = 1$. Substituting Eqs. (5) and (8) into Eq. (9), the induced-electric field at the axis is expressed as

$$E_z(t) = \frac{2I\mu}{c^2}\int_0^\infty \frac{dk}{k}\lambda_k^2\exp(-\lambda_k^2 t)\cdot\left[J_0(kR_1) - J_0(kR_2)\right]^2. \tag{10}$$

Carrying out the integration[2] over k space in Eq. (10), the induced-electric field at the axis is finally given by

$$E_z(t) = \frac{I}{2\pi\sigma R_1^2\tau}\left[\exp\left(-\frac{1}{\tau}\right)I_0\left(\frac{1}{\tau}\right) + \exp\left(-\frac{1}{\tau}\frac{R_2^2}{R_1^2}\right)I_0\left(\frac{1}{\tau}\frac{R_2^2}{R_1^2}\right) - 2\exp\left(-\frac{1+R_2^2/R_1^2}{2\tau}\right)I_0\left(\frac{1}{\tau}\frac{R_2}{R_1}\right)\right], \tag{11}$$

where the dimensionless time τ is defined by

$$\tau = \frac{c^2 t}{2\pi\sigma\mu R_1^2}. \tag{12}$$

The contribution of the second and third terms in the right-hand side of Eq. (11) is less than 7 percent if the parameter $R_2/R_1 > 3$ in a practical application. The induced field in Eq. (11) is a monotonically decreasing function of the normalized time τ. We remind the reader that Eq. (11) has been derived under the assumption that the deriving-beam current I drops **abruptly** to zero at t = 0. The falling time of the driving-beam current is probably small. In reality, there is a finite beam-falling time, which will determine values of the normalized time τ. If the driving-beam current I is extracted from a photo-cathode, the rising and falling times of the driving current I can be controlled reasonably well.

III. ACCELERATING GRADIENT OF INDUCED FIELD

The magnetic field in Eq. (2) at $r = R_1$ before the time of t = 0 increases with the parameter μI until the saturation value, which is one of characteristic properties of the field-storage material. In other word, the maximum value of the parameter μI is limited by the magnetic-field saturation value of the material. For example, the saturation value of the magnetic field for some iron alloy is 15 kG, where as the field saturation value in vacuum is undefined. We want to increase the saturation value of the magnetic field in the field-storage material as much as possible. This is one area, which should be investigated aggressively. As an example, we consider the case when the inner and outer radii of the field-storage device are R_1 = 2 cm and R_2 = 6 cm, respectively. Of course, some people may choose a fat field-storage device. The strength of the induced-electric field $E_z(t)$ at the axis is calculated from Eq. (11). Shown in Fig. 2 is the accelerating-gradient field versus the normalized time τ for $\mu\sigma = 4$ siemens/m and $\mu I = 300$ kA, which corresponds to the saturation magnetic field of 30 kG at $r = R_1 = 2$ cm. The normalized time τ is related to the real time t by $\tau = t$ (in unit of nano-second) for $R_1 = 2$ cm and $\mu\sigma = 4$ siemens/m. In a reasonably-practical parameter regime, the accelerating field can be easily more than 10 mega volt/meter, which is one order in magnitude larger than the conventional accelerating field. In addition, the system is very compact and light.

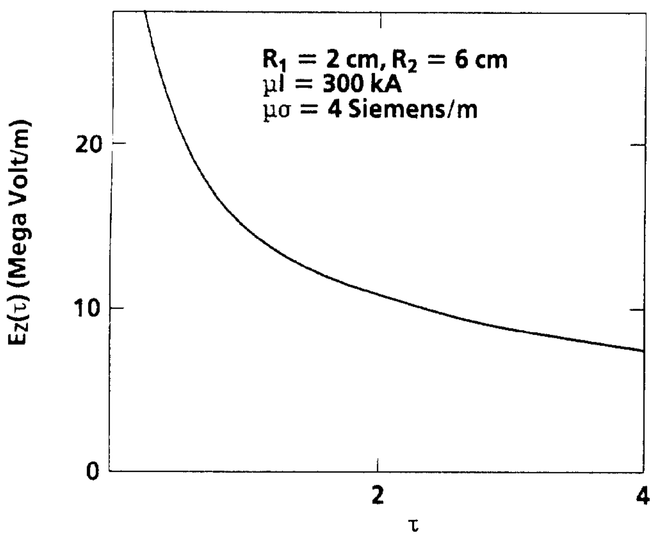

Figure 2. Plot of the induced-electric field $E_z(t)$ at the axis versus the normalized time τ obtained from Eq. (11) for $\mu I = 300$ kA, $\mu\sigma = 4$ siemens/m, $R_1 = 2$ cm and $R_2 = 6$ cm.

The major issues to be solved in this accelerator concept are development of a right material, which satisfies all the necessary requirements. First, the saturation value of the magnetic field should be larger than 15 kG to be practical. Higher the saturation value of the magnetic field in the material is better. Second, the material must withstand a high breakdown field. Third, the material must have a large value of permeability, but has a reasonably small conductivity. As an example, for $\mu I = 300$ kA and $\mu\sigma = 4$ siemens/m in Fig. 2, the driving-beam current I and conductivity σ are $I = 750$ A and $\sigma = 10^{-2}$ siemens/m for the permeability $\mu = 400$. The conductivity of $\sigma = 10^{-2}$ siemens/m is very close to the conductivity of limestone. There is no single element to satisfy these conditions. A composite of several different elements may satisfy these necessary conditions. The dielectric constant or permeability of heterogeneous mixtures can be determined from the functional form[3]

$$v_2 = \frac{\mu_1^{1/3} - \mu^{1/3}}{\mu_1^{1/3} - \mu_2^{1/3}}, \qquad (13)$$

where v_2 is the volume fraction of the dispersed component, μ_2 is the permeability of this component, μ_1 is the permeability of the other component and μ is the desired permeability of the field-storage material. Assuming that pure iron with permeability $\mu_1 = 4000$ is mixed with a material with permeability $\mu_2 = 1$, we find from Eq. (13) that the volume fraction of the material in the iron powder is given by $v_2 = 0.57$ for the desired permeability of $\mu = 400$. The conductivity of the final field-storage material should be also evaluated from properties of the mixture components. Preparation of a right field-storage material is a very painstaking job and the success of this acceleration concept may depend on it.

As mentioned in the beginning of this article, a high-current electron beam with relatively low energy propagates through the hole bored along the field-storage device. Obviously, there should be a physical mechanism to maintain an equilibrium condition of the beam. The axial magnetic field is an excellent means to hold the electron beam. However in this case, the magnetic field may not penetrate very well through the high permeability material. We thus propose to use the ion-focused-regime (IFR) propagation[4] of the electron beam. When a relativistic electron beam propagates through a preionized plasma channel, channel electrons are expelled by the electrostatic force generated by head of the beam, leaving an ion channel behind. This ion channel partially neutralizes the space charge field of the electron beam, thereby permitting a focused beam. The preionized plasma channel is created by photo-ionization by a laser beam preceding the electron beam. The IFR propagation of a relativistic electron beam has been well demonstrated.

IV. CONCLUSIONS

Based on a theoretical calculation, a concept of accelerating device has been proposed. This device makes use of the accelerating-electric field initiated by an abrupt change of the magnetic field stored in a specially prepared material. Theoretical calculation carried out for a high permeability material satisfying Eq. (4) indicates that a high accelerating-electric field occurs at the axis. The accelerating field can be easily a substantial fraction of mega volt/cm. This new accelerating device is very compact and light. The magnetic-field decay mechanism without the restriction in Eq. (4) is currently under investigation by the author and the result will be presented elsewhere.

V. REFERENCES

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