VLASOV-MAXWELL SIMULATIONS OF NONLINEAR PLASMA DYNAMICS IN THE PLASMA WAKEFIELD ACCELERATOR

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ABSTRACT

Simulations of the plasma wakefield accelerator are carried out by following the time evolution of the plasma distribution function in one dimension via the Vlasov-Maxwell equations. It is found that highly nonlinear wakes are obtainable in the vicinity of the driving beam, where the thermal velocity spread of the plasma is reduced. In this region, wake amplitudes can significantly exceed the expectations of relativistic warm plasma models and agree closely with cold fluid theory.

I. INTRODUCTION

Novel plasma-based acceleration devices are being actively researched due to their ability to support acceleration gradients in excess of 10 GeV/m. The plasma wakefield accelerator (PWFA) [1] is one such device, wherein a moderate-energy electron beam drives a plasma wave which, in turn, accelerates a high-energy electron bunch. This process has been demonstrated experimentally [2]. Of interest in this scheme are limits on the obtainable accelerating gradients and transformer ratios in the plasma wakefield.

The transformer ratio is defined as $R \equiv E_+/E_-$, where E_- is the peak decelerating field experienced by the driving electron beam and E_+ is the peak accelerating field in the wake. Theoretical results indicate that high transformer ratios may be obtained by operating in the nonlinear regime [3].

The properties of large-amplitude waves in cold plasmas have been studied by a number of researchers. For nonrelativistic plasma waves, it was found that the peak electric field is limited by the nonrelativistic wave-breaking field: [4]

$$E_{max} = mv_{ph}\omega_p/e \quad < \quad E_{wb} \equiv mc\omega_p/e \qquad (1)$$

where m is the electron mass, v_{ph} is the phase velocity of the plasma wave, e, assumed positive, is the elementary charge, c is the speed of light in vacuum, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, and n_0 is the equilibrium plasma density.

Katsouleas and Mori [5] performed a calculation using the relativistic Vlasov-Poisson equations and a "waterbag" distribution function for the plasma electrons to obtain the maximum wave amplitude for a warm plasma. An alternative approach was taken by Rosenzweig [6], who calculated the saturation amplitude of the plasma wave via a three-fluid model for the thermal plasma.

In this paper we present simulations of plasma waves driven by an non-evolving electron beam with velocity v_b. These are carried out by following the time evolution of the plasma distribution function in one spatial dimension via the Vlasov-Maxwell equations. Direct simulation via the Vlasov equation is an ideal way of examining thermal and trapping effects because the artificially high temperatures associated with typical particle simulations are avoided. The simulation code is discussed in Ref. 7.

II. 1D NONLINEAR VLASOV FORMULATION

To simulate plasma wave generation we use the Vlasov-Maxwell equations under a coordinate transformation from (z,p_z,t) to $(\zeta=ct-z,p_z,\tau=t)$:

$$\frac{\partial f}{\partial \tau} + \left(c - \frac{p_z}{\gamma m}\right)\frac{\partial f}{\partial \zeta} + eE_z\frac{\partial f}{\partial p_z} = 0, \qquad (2)$$

and

$$\frac{\partial E_z}{\partial \zeta} = 4\pi e(n + n_b - n_0) \quad . \qquad (3)$$

where $f(\zeta,p_z,\tau)$ is the distribution function, $\gamma = (1 + p_z^2/m^2c^2)^{1/2}$, n_b is the driving beam density, E_z is the electric field and $n(z,\tau)$ is determined from f. For this system, Maxwell's equations reduce simply to Gauss's law.

Initially f is uniform in z and Gaussian in p_z, varying as $\exp(-p_z^2/2mT_0)$, where T_0 is the equilibrium plasma thermal energy. We represent the driving beam as a fixed charge shape that propagates with velocity v_b but that otherwise does not evolve.

For comparison to our simulation results, we consider a cold plasma model similar to that of Refs. 3, 8 and 9. For a non-evolving electron beam with $v_b \simeq c$, steady-state may be assumed and derivatives with respect to τ may be neglected. It is convenient to work in terms of the normalized scalar potential, $\phi = e\Phi/mc^2$ and the normalized bulk fluid velocity, $\beta_z = \bar{v}_z/c$. Following Ref. 8, one finds a self-consistent nonlinear equation describing $\phi(\zeta)$:

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$$\frac{d^2\phi}{d\zeta^2} = \frac{k_p^2}{2}[2n_b/n_0 + (1+\phi)^{-2} - 1] , \quad (4)$$

where n_0 is the ambient plasma electron density and $k_p = \omega_p/c$.

III. SIMULATIONS IN THE LINEAR REGIME

Simulations in the linear regime showed excellent agreement with theory for $T_0 \le 5$ keV. Here, we considered $n_0 = 2 \times 10^{14}$ cm^{-3}, such that $\lambda_p = 2\pi c/\omega_p = 0.236$ cm and $E_{wb} = 1.36$ GeV/m. The driving beam, defined in the region $(c-v_b)\tau < \zeta < (c-v_b)\tau + L_b$, was of the form

$$n_b = n_{b0}\sin\{\pi[\zeta - (c-v_b)\tau]/L_b\} \quad (5)$$

with $n_{b0}/n_0 = 0.1$, $L_b = 0.24$ cm $\approx \lambda_p$ and $\gamma_b = 100$. For these runs, we modelled a region of phase space bounded by $0 \le \zeta \le 1.024$ cm and $-10.2 \le p_z/mc \le 30.7$ with simulation parameters $\Delta\zeta = 10^{-3}$ cm and $\Delta p_z/mc = 4 \times 10^{-2}$. After 1.6 cm of propagation, a near-steady state was established.

We diagnosed the plasma temperature as

$$T = P/n \quad (6)$$

with P given by [10]

$$P = \int dp_z (p_z - \bar{p}_z)(v_z - \bar{v}_z) f, \quad (7)$$

where $\bar{p}_z$ and $\bar{v}_z$ are the average momentum and velocity as given by moments $f(z,p_z,\tau)$.

IV. SIMULATIONS IN THE NONLINEAR REGIME

A series of simulations were performed with the beam profile given in Eq. (5) with $n_{b0}/n_0 = 0.1, 0.2, 0.3, 0.4$ and 0.5, $L_b = 0.24$ cm $\approx \lambda_p$, $\gamma_b = 100$ and $T_0 = 19$ keV. Peak electric fields and corresponding theoretical values are plotted versus n_{b0}/n_0 in Fig. 1. Simulation parameters were as in the linear-regime runs.

Excellent agreement with the cold plasma equations was observed up to the first accelerating E_z peak. In this region the temperature, given by Eqs. (6-7), drops by an order of magnitude. Results did not vary when the initial plasma temperature was decreased to $T_0 = 5$ keV. The amplitude of the E_z peak was also found to be independent of γ_b for $\gamma_b \ge 10$ and was not reduced by the presence of trapped particles.

The close agreement between the simulation and the cold plasma equations in the region near the beam is an unexpected result. It is caused by the reduction in the thermal velocity spread of the plasma electrons which is associated with the bulk acceleration of the plasma in this

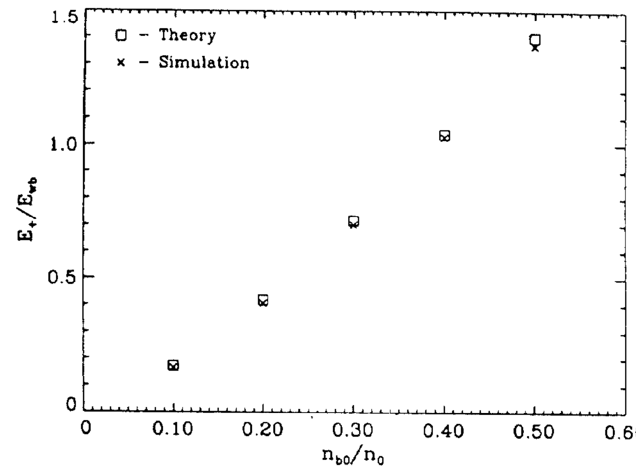

Fig. 1. **Peak electric fields and corresponding theoretical values plotted versus beam density.**

region in the direction opposite the phase velocity of the wave. From Eqs. (6-7),

$$T \sim \int dp_z\, f\Delta\gamma\left(\frac{1}{\gamma^2} - \frac{1}{\bar{\gamma}^2}\right) \sim \frac{1}{\bar{\gamma}^3}, \quad (8)$$

where $\Delta\gamma = \gamma - \bar{\gamma}$ and we have assumed that $n(\zeta)$ is constant over the accelerating region and that $\gamma >> 1$ and $\Delta\gamma << \gamma$ for regions in which $f > 0$.

V. NONLINEAR PWFA

For a beam pulse with $n_b = n_p/2$ and $L_b > \lambda_p$, theory suggests that transformer ratios $R >> 2$ may be obtained. We consider such a beam pulse defined over the region: $(c-v_b)\tau < \zeta < (c-v_b)\tau + L_b$. The nonlinear cold plasma equations have been solved for this case [4].

Simulations of the $n_b = n_p/2$ pulse were performed with: $n_p = 2 \times 10^{14}$ cm^{-3}, $T_0 = 19$ keV and $\gamma_b = 100$. Here, we used $L_b/\lambda_p = 0.83$, 1.63 and 2.79 such that the cold plasma equations give R = 2, 3 and 4 respectively.

Simulation results are summarized in Table I. As an example, the beam density, electric field and perturbed plasma density, $n_1 = n - n_0$, are plotted in Fig. 2 for the $L_b/\lambda_p = 2.79$ case. Note that in our plotting convention $E_z > 0$ is accelerating.

Agreement with the cold plasma model up to the first peak is made clear when Fig. 2 is compared to Fig. 1 of Ref. 3. In fact, the $L_b/\lambda_p = 2.79$ case shows a peak electric field which, while down by 6% from the cold plasma result, significantly exceeds the expectations of Refs. 5 and 6 for a $T_0 = 19$ keV plasma. This surprising result is caused by the reduction in the thermal velocity spread in the plasma in accordance with Eq. (8) as can be observed in Fig. 3, which shows plots of T and $\bar{\gamma}$ for this case.

L_b/λ_p	R	R [theory]	E_+/E_{wb}	E_+/E_{wb} [theory]
0.83	2.09	2.09	1.81	1.83
1.63	2.90	3.00	2.71	2.82
2.79	3.78	4.00	3.63	3.86

Table 1. Simulation results for the nonlinear PWFA runs. Theoretical results are from numerical solutions to Eq. (4).

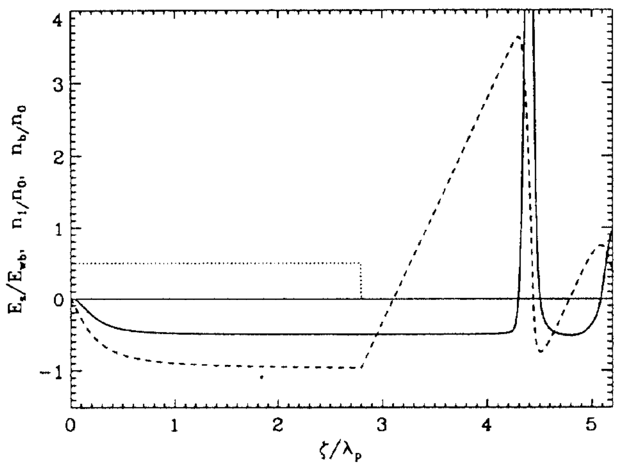

Fig. 2. Simulation result showing beam density (dotted), electric field (dashed) and perturbed plasma density (solid) plotted versus ζ at fixed time for the $L_b/\lambda_p = 2.79$ case.

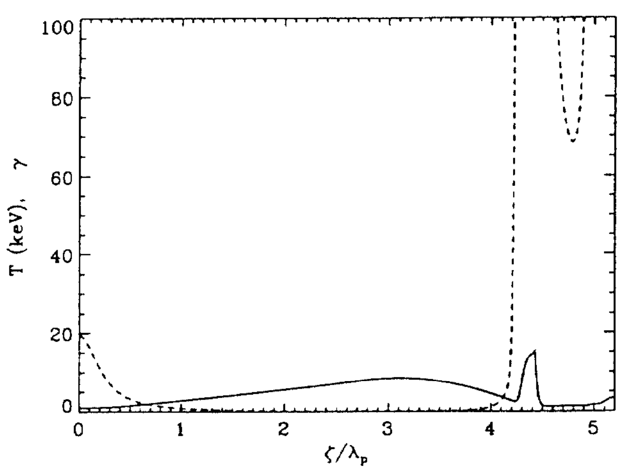

Fig. 3. Plots of temperature T (dashed) and average gamma γ (solid) for the $L_b/\lambda_p = 2.79$ case show a reduction in T as expected from Eq. (8).

VI. CONCLUSIONS

Simulations of the plasma wakefield accelerator were carried out by following the time evolution of the plasma distribution function in one dimension via the Vlasov-Maxwell equations. Results were in close agreement with numerical solutions of the nonlinear relativistic cold plasma equations in the vicinity of the driving beam, where the thermal velocity spread of the plasma is reduced by the mechanism of Eq. (8). This reduction in the plasma thermal velocity allowed the generation of wake amplitudes that exceeded of the predictions of relativistic warm plasma models as given in Refs. 5 and 6.

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