Modelling of the Transverse Mode Suppressor for Dielectric Wake-Field Accelerator

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I. INTRODUCTION

Minimizing transverse wake-field effects in high gradient accelerating structures is of critical importance for future high energy linear colliders, including those using wake-field accelerators. Dielectric-loaded slow wave structures have been proposed as wake-field accelerators and have been under intensive study in the last few years [1]. The principle of the dielectric wake-field accelerator is very simple. It utilizes the strong longitudinal wake field left behind a short and intense driving beam to accelerate a less intense witness beam. Unfortunately, a strong longitudinal wake field can also mean a strong transverse deflection wake field. In order to minimize the deflection of the accelerated beam due to the transverse wake forces produced by a misaligned driving beam, a transverse mode damping device was proposed by E. Chojnacki [2]. This device uses principles similar to that used in the slotted cavity structure [3]. The configuration of the device is shown in Fig. 1. The principles are as follows. The deflection modes are non-axisymmetric hybrids containing both axial electric and magnetic fields. These hybrid modes are comprised of all six cylindrical field components and will require *both* azimuthal surface currents and axial surface currents. If the conductors are segmented to allow only axial surface currents, the deflection modes will not be confined and will radiate beyond the outer wall, establishing a surface wave, or trapped mode, within the outer uniformly conducting boundary. If this outer region is filled with rf absorbing material, the deflection modes will be highly attenuated. A description of the device with its experimental demonstration is given in reference [2].

In this paper, we provide an analytical description of this dielectric-lined waveguide with an axially slotted conductor and give solutions for all the wake fields of any order. We show the characteristics of the device and compare the calculations with experimental results.

II. ANALYTIC SOLUTIONS

A. General Solutions

Considering the device shown in Fig. 1, we denote the volume inside the vacuum hole ($0 < r < b$) as Region 0, inside the non-dispersive dielectric material ($b < r < a$) as Region 1, and the region containing the absorber from the axial wire boundary to the conducting wall ($a < r < d$) as Region 2. All of the fields and properties of the materials are labeled accordingly. Assume that a point charge e passes through the vacuum hole at $r = r_0$ and $\theta = 0$ with speed $v = \beta c$. In Regions 1 and 2, the wake fields produced by the motion of a charged particle are given by

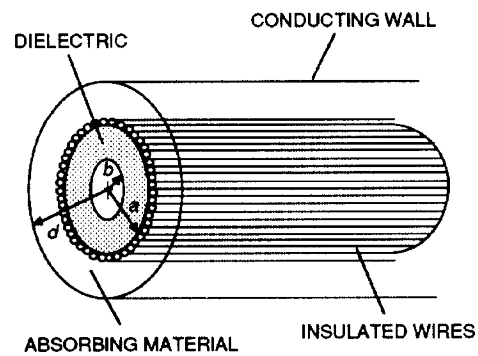

Figure 1: Schematic diagram of a dielectric wake-field device with transverse mode suppression.

Maxwell's equations which can be manipulated to yield the wave equations

$$\nabla^2\mathbf{E} - \frac{\mu\epsilon}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{4\pi\mu\sigma}{c}\frac{\partial\mathbf{E}}{\partial t} = 0 \quad (1)$$

$$\nabla^2\mathbf{B} - \frac{\mu\epsilon}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{4\pi\mu\sigma}{c}\frac{\partial\mathbf{B}}{\partial t} = 0 \ , \quad (2)$$

where μ, ϵ and σ are the permeability, dielectric constant and conductivity of the material, respectively. The Fourier transforms of the fields are

$$\mathbf{E}(r,\theta,z,t) = \sum_{m=-\infty}^{\infty} e^{im\theta}\int_{-\infty}^{\infty} d\omega\, e^{i\omega(z-vt)/v}\,\mathbf{E}_m(r,\omega) \quad (3)$$

$$\mathbf{H}(r,\theta,z,t) = \sum_{m=-\infty}^{\infty} e^{im\theta}\int_{-\infty}^{\infty} d\omega\, e^{i\omega(z-vt)/v}\,\mathbf{H}_m(r,\omega) \quad (4)$$

Due to the finite conductivity of the material in Region 2, one can assume that the fields E and H are proportional to $e^{-\omega_i(z-vt)/v}$, where ω_i is the damping factor. The wave equations applied to the Fourier transforms then become

$$\nabla^2[\mathbf{E},\mathbf{H}] + k_0^2[\mathbf{E},\mathbf{H}] = 0 \ , \quad (5)$$

with

$$k_0^2 = \mu\epsilon\frac{\omega^2}{c^2}\left(1 + i\frac{4\pi\sigma}{\omega\epsilon}\right) = \mu'\epsilon\frac{\omega^2}{c^2} = \mu\epsilon'\frac{\omega^2}{c^2} \quad (6)$$

$$\mu' = \mu + i\frac{4\pi\mu\sigma}{\epsilon\omega} \quad (7)$$

$$\epsilon' = \epsilon + i\frac{4\pi\sigma}{\omega} \ . \quad (8)$$

It is easily seen that if one uses a complex magnetic permeability μ' or complex dielectric constant ϵ', one can solve this wake field problem using known methods [1], modified in that one must use complex Bessel functions and that the resonant frequencies will be complex. Then the damping term appears naturally.

The unique effect of the axial wire boundary at $r = a$ in Fig. 1 is that the electromagnetic fields will have $E_z = 0$ for $r \geq a$ and E_θ simply continuous at $r = a$. The general solutions for the longitudinal fields in the regions 0, 1, and 2 can be written

$$E^{(0)}_{zm} = \alpha_m I_m(kr) - \eta_m K_m(kr) \quad (9)$$
$$E^{(1)}_{zm} = A_m P_m(r,a) \quad (10)$$
$$E^{(2)}_{zm} = 0 \quad (11)$$
$$B^{(0)}_{zm} = \beta_m I_m(kr) \quad (12)$$
$$B^{(1)}_{zm} = D_m N_m(s_1 r) + E_m J_m(s_1 r) \quad (13)$$
$$B^{(2)}_{zm} = F_m Q_m(r,d) \quad , \quad (14)$$

where

$$k = \frac{\omega}{v}\sqrt{1-\beta^2} \quad (15)$$
$$s_1 = \frac{\omega}{v}\sqrt{\epsilon_1\beta^2 - 1} \quad (16)$$
$$s_2 = \frac{\omega}{v}\sqrt{\mu'\epsilon_2\beta^2 - 1} \quad (17)$$

and

$$\alpha_m = \alpha'_m I_m(kr_0) \quad (18)$$
$$\beta_m = \beta'_m I_m(kr_0) \quad (19)$$
$$\eta_m = \frac{ie\omega(1-\beta^2)}{\pi v^2} I_m(kr_0) \quad (20)$$
$$P_m(r,a) = J_m(s_1 a) N_m(s_1 r) - N_m(s_1 a) J_m(s_1 r) \quad (21)$$
$$Q_m(r,d) = J'_m(s_2 d) N_m(s_2 r) - N'_m(s_2 d) J_m(s_2 r) \quad (22)$$

The transverse electric fields E_r and E_θ can be computed from the longitudinal components E_z and B_z. Using the boundary conditions of continuity of E_z, E_θ, D_r and H_z, we get

$$\alpha_m = \frac{a_1 a_2 z K_m(kb) - a_3\left[K'_m(kb)/kv + a_4 K_m(kb)\right]}{a_1 a_2 z I_m(kb) - a_3\left[I'_m(kb)/kv + a_4 I_m(kb)\right]}\eta_m \quad , \quad (23)$$

where

$$r_m(r,a) = J'_m(s_1 a) N_m(s_1 r) - N'_m(s_1 a) J_m(s_1 r) \quad (24)$$
$$r'_m(b,a) = J'_m(s_1 a) N'_m(s_1 b) - N'_m(s_1 a) J'_m(s_1 b) \quad (25)$$

$$z = \frac{r_m(b,a)}{r'_m(b,a)} - \frac{r_m(b,b)}{r'_m(b,a)}\frac{s_1}{s_2}\frac{r_m(a,a)}{y r'_m(b,a)} Q'_m(a,d) \quad (26)$$
$$y = \frac{Q_m(a,d)}{\mu} + \frac{s_1}{s_2}\frac{r_m(a,b)}{r'_m(b,a)} Q'_m(a,d) \quad , \quad (27)$$

and

$$a_1 = \frac{m s_1}{bv}\left(\frac{1}{k^2} + \frac{1}{s_1^2}\right) \quad (28)$$

$$a_2 = \frac{m}{b}\left(\frac{1}{k^2} + \frac{\epsilon_1}{s_1^2}\right) \quad (29)$$
$$a_3 = 1 + \frac{s_1 z}{k}\frac{I'_m(kb)}{I_m(kb)} \quad (30)$$
$$a_4 = \frac{\epsilon_1}{s_1 v}\frac{P'_m(b,a)}{P_m(b,a)} \quad . \quad (31)$$

The general longitudinal electric wakefield can then be written as

$$E_z^{(0)}(r,\theta,z,t) = \sum_{m=-\infty}^{\infty} e^{im\theta}\int_{-\infty}^{\infty} d\omega\, e^{i\omega(z-vt)/v}\alpha_m I_m(kr) \quad (32)$$

where the Fourier amplitudes α_m are all calculable.

B. Monopole Fields (m = 0)

When $m = 0$, under the ultra relativistic limit, $\beta \to 1$ and hence $kr \ll 1$, it is straightforward to show that the longitudinal wakefield at coordinate (r, z_0), where z_0 is the longitudinal distance behind the charged particle, reduces to

$$E^{(0)}_{z0} = \frac{4e}{\epsilon_1 b}\sum_\lambda\left[\frac{P_0(b,a)\cos\left(\frac{s_1 z_0}{\sqrt{\mu\epsilon_1 - 1}}\right)}{\frac{d}{ds_1}\left(P'_0(b,a) + s_1 b P_0(b,a)/2\epsilon\right)}\right]_{s_1=s_\lambda} , \quad (33)$$

where s_λ satisfies the condition

$$P'_0(b,a) + \frac{s_\lambda b}{2\epsilon}P_0(b,a) = 0 \quad . \quad (34)$$

This agrees with the field in reference [1] and is an expected although important result. It shows that the axisymmetric modes $m = 0$ are not affected by the segmented wire and absorber in Region 2. In this way, the accelerating wake field ($m = 0$) is unchanged from that of the usual dielectric-lined waveguide with uniform metallic boundary, as experimental data showed in reference [2].

C. Multipole Fields (m ≠ 0)

Again, we are primarily interested in the case of wakefields generated by a charged particle moving close to the speed of light, i.e., $\beta \to 1$ and $kr \ll 1$. The longitudinal wakefield is then

$$E_z^{(0)} = 8e\sum_m e^{im\theta}\left(\frac{rr_0}{b^2}\right)^m \sum_\lambda\left[\frac{\exp\left(\frac{is_1(z-vt)}{\sqrt{\epsilon_1 - 1}}\right)}{\frac{d}{ds_1}C(s_1)}\right]_{s_1=s_\lambda} \quad (35)$$

where

$$C(s_1) = \frac{s_1 b^2}{m+1} + \frac{b}{z}\left(1 + z\epsilon_1\frac{P'_m(b,a)}{P_m(b,a)}\right) - \frac{m\epsilon_1 + m}{s_1} \quad (36)$$

and s_λ satisfies the condition

$$C(s_\lambda) = 0 \quad . \quad (37)$$

Equation (35) is the central equation of this paper. If either ϵ_2 or μ_2 is a complex number, the roots of $C(s_1)$ will be complex roots and the resonant frequencies ω_λ will be complex as given by eq. (16). As we stated earlier, the imaginary part of ω is the damping factor of the wake fields, so the wake fields are then damped naturally. This result is drastically different from the $m = 0$ case. The $m \neq 0$ modes are affected by the material in Region 2 and can be damped if the material is lossy.

III. NUMERICAL EXAMPLES

We have given analytic expressions for the wake fields; in this section we show some of the numerical results and compare them with experiment. To find the roots of the complex function $C(\omega)$, we use the IMSL routine DZANLY. We examine only the case for $m = 1$. All other higher modes can be calculated in the same manner.

We calculate the wake fields for the parameters used in the experiment : $b = 1.27$ cm, $a = 1.905$ cm, $d = 2.5$ cm and $\epsilon_1 = 2.55$, the rms (assumed Gaussian) bunch length of both the driving and witness beams is $\sigma = 3$ mm. Other parameters like μ_2, ϵ_2 are unknown but are estimated. We used equation (35) and Panofsky-Wenzel theorem [4] to calculate the transverse forces. r_0 is taken as the centroid of the driving beam. In order to compare the calculations with the experimental data, we set the value of $r_0 = 1$ mm.

Figure 2 shows the calculated transverse wake using the above parameters and estimated $\epsilon_{2r} = 20$, $\mu_{2r} = 2$, and $\mu_{2i} = 3$. The transverse wake field damps out after a few cycles, in qualitative agreement with the experimental data as shown in Fig. 3. This indicates that our theoretical treatment closely describes the general properties of the transverse mode damping device. The amplitude of the calculated wake is slightly smaller than that measured, but we think this is because we have ignored all the higher order radial and azimuthal modes. For comparison, the transverse wake in the case of the uniform conducting boundary ($b = d$ and no damping) is also shown in Fig. 2.

IV. CONCLUSION

We have calculated the wake fields in a dielectric waveguide with deflection-mode damping. An interesting property of the structure is that the axisymmetric mode is unaffected by the lined wire boundary and all higher order modes can be damped very quickly (in a few cycles). This has important implications for the dielectric-based wakefield accelerator. By reducing the beam-beam deflection, one may use multiple drive bunches to enhance the axial acceleration wake field without severe accumulated transverse wake-field effects. A large number of bunches also means that the total charge in each bunch can be reduced, thus reducing the head-tail instabilities within the driving beam, although the head-tail instabilities can be controlled to a certain degree by alternating focussing and defocussing magnetic fields applied around the wake-field device [5].

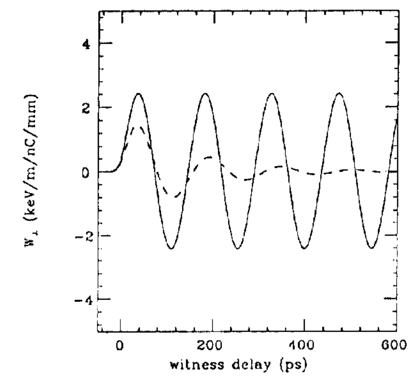

Figure 2: Calculated transverse wake fields. The broken (solid) line refers to damped segmented (undamped uniform conductor) structure.

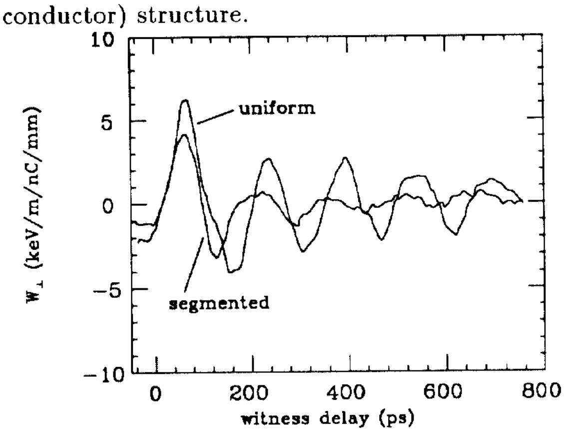

Figure 3: Measured transverse wake fields for both uniform outside conductor and segmented conductor dielectric wake-field devices.

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