

RF Transfer in the Coupled-Cavity Free-Electron Laser Two-Beam Accelerator

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Abstract

A significant technical problem associated with the Coupled-Cavity Free-Electron Laser Two-Beam Accelerator is the transfer of RF energy from the drive accelerator to the high-gradient accelerator. Several concepts have been advanced to solve this problem. This paper examines one possible solution in which the drive and high-gradient cavities are directly coupled to one another by means of holes in the cavity walls or coupled indirectly through a third intermediate transfer cavity. Energy cascades through the cavities on a beat frequency time scale which must be made small compared to the cavity skin time but large compared to the FEL pulse length. The transfer is complicated by the fact that each of the cavities in the system can support many resonant modes near the chosen frequency of operation. A generalized set of coupled-cavity equations has been developed to model the energy transfer between the various modes in each of the cavities. For a two cavity case transfer efficiencies in excess of 95% can be achieved.

1 Introduction

A standing wave free-electron laser (FEL) two-beam accelerator (TBA) has recently been proposed[1] in which the FEL interaction takes place in a series of drive cavities rather than in an open ended waveguide. This configuration mitigates a number of microwave extraction problems associated with previous designs. RF extraction is based on an extension of the "beat-coupling" method proposed by Henke[2] for a relativistic klystron TBA design.

Due to the short RF wavelengths under consideration (10 - 20 mm) a fundamental mode cavity allows neither adequate electron beam clearance nor a long enough interaction length to extract significant RF energy from the electron beam by the FEL mechanism. To circumvent these problems a coupled-cavity method is proposed here in which the drive (FEL) structure is taken to be an oversized cavity assumed to be shock excited in a single resonant cavity mode. Power is coupled out of the drive cavity through a series of holes into a second oversized cavity,

which in turn is similarly coupled to the high gradient accelerator structure. Oversized refers to a cavity in which the density of modes is high in the vicinity of the chosen resonant frequency, i.e. the cavity mode does not exist in isolation, irrespective of mode degeneracies.

2 Formalism

The equation describing the time evolution of an arbitrary cavity mode is based on that developed by Bethe[3] and is given by

$$\ddot{p}_n + \frac{\omega_n}{Q_n} \dot{p}_n + \omega_n^2 p_n = \sum_{m=1}^{N_m} [C_{nm} \ddot{p}_m + \omega_n^2 \mathcal{D}_{nm} p_m],$$

where

$$C_{nm} = (-1)^{\delta_{nm}} \sum_{j=1}^{N_h} \alpha_{E_j} (\mathbf{F}_n \cdot \mathbf{F}_m)_{\mathbf{r}_j},$$

and

$$\mathcal{D}_{nm} = -\frac{(-1)^{\delta_{nm}}}{k_n^2} \sum_{j=1}^{N_h} \alpha_{M_j} [(\nabla \times \mathbf{F}_n) \cdot (\nabla \times \mathbf{F}_m)]_{\mathbf{r}_j},$$

where p_n is the mode amplitude of the n^{th} mode, dots refer to time derivatives, ω_n is the resonant frequency of the n^{th} mode, Q_n is the unperturbed quality factor of the n^{th} cavity mode, N_m is the total number of modes in all cavities, C_{nm} and $\mathcal{D}_{nm}$ are inter-mode coupling constants, N_h is the number of holes common to the m^{th} and n^{th} modes, δ_{mn} is the Kronecker- δ ($= 1$ if $m = n$, $= 0$ otherwise), k_n is the mode wavenumber $= \omega_n c$, c is the speed of light, $\alpha_{E_j} = -2\pi r_{oj}^3/3$ and $\alpha_{M_j} = 4\pi r_{oj}^3/3$ respectively are the electric and magnetic polarizabilities of the j^{th} hole, r_{oj} is the radius of the j^{th} hole, $\mathbf{F}_n$ is the vector electric potential of the n^{th} cavity mode, and $\mathbf{r}_j$ is the position of the j^{th} hole common to the two modes.

Above, the C_{nm} and $\mathcal{D}_{nm}$ terms respectively represent cavity coupling from the electric and magnetic polarizability of the holes. The magnitude of each of these terms also depends on the field components of the two modes evaluated at the position of each of the holes. The holes cause

each mode to interact with itself, perturbing the resonant frequency of each cavity mode.

There is one equation for each of the N_m modes. The system of N_m equations together with the initial condition constitutes an eigenvalue problem. A computer code was written which accepted parameters describing a system of coupled rectangular cavities: the number of cavities, the modes in each of the cavities, and the number and positions of each of the holes joining successive pairs of cavities. The linear algebra problem was then set up and solved using standard routines.

3 General Results

When operated in an open ended waveguide the FEL excites a low order TE waveguide mode such as the TE-0,1. Closing the ends of the waveguide to form a cavity would then produce a high order axial cavity mode: TE-0,1, m with $m \gg 1$. In a TBA the power in this mode must then be efficiently transferred to the high gradient accelerator. It is not immediately evident whether such a high order mode can be effectively used without losing power to a variety of other modes nearly resonant with it in the oversized cavity.

Very encouraging results have been obtained with respect to the mode competition problem. It is a problem as expected but not as severe as anticipated. Several factors contribute to this result:

A) Position of Holes: Placing holes on the centerline of a wall generates a geometrical symmetry and a consequent selection rule. In this case odd modes in one of the indices cannot couple as there is a null in the field at the position of the hole. The concept can be extended to more elaborate arrays of holes.

B) Matching Condition: If the wavelength in a particular direction is the same in two coupled cavities, then regardless of where the holes are placed the two modes will couple in phase at each of the holes. Thus a long length of holes will constructively interfere only with modes of the proper wavelength and destructively interfere with all others leading to selective mode coupling.

C) Cavity Bandwidth: The Q of the cavity determines its linewidth about the resonant frequency. The unperturbed cavity Q 's of the oversized cavities under consideration are typically between 10^4 and 10^5 . Adding holes to the cavity loads the cavity, reducing its Q and increasing the number of nearly resonant modes which can be excited. Thus the smaller the coupling the higher the loaded Q and the fewer modes which are excited.

D) Degeneracy Splitting: The presence of the holes shifts the resonant frequency of all the modes. This includes the degenerate TE and TM modes. If the splitting is large enough the coupling to the degenerate TM mode can be reduced for the reasons explained in C).

These results generate some simple rules of thumb which

have been verified by numerical calculation: 1) Long (many wavelengths) holes arrays are mode selective, 2) It is desirable to keep the coupling small and the loaded Q 's high. Rule 2) leads to very long beat times. This is restricted only by ohmic losses: one cannot make the transfer so long that all the power is lost in the walls before arriving at the final cavity. The trade-off is not simple as by reducing the transfer time more power may be lost through coupling to undesirable modes. Thus 2) becomes $\tau_{transfer} \leq \tau_{ohmic} = Q/\omega$. It is also known from [1] that the FEL pulse length, τ_p , must satisfy $\tau_p \ll \tau_{transfer}$.

For a given cavity configuration there is a constellation of neighboring modes (usually with indices differing by ± 1 or ± 2 from the principle mode of interest) which are of concern. By application of the above rules the number of modes and the power coupled to them can be substantially reduced.

4 Numerical Example

Figure 1 shows the geometry, hole configuration, and dimensions of a pair of coupled cavities. Cavity I is resonant in the TE-0,1,16 mode while cavity II is resonant in the TE-0,1,40. The unperturbed resonant frequency of both cavities is 11.7 GHz and the unperturbed cavity Q 's of the resonant modes are 63137 for cavity I and 41231 for cavity II. The cavity I mode constellation used in the calculation consists of the TE-0,1,17, TE-0,1,16, TE-0,1,15, TE-1,1,16, TE-1,1,15, TM-1,1,16, and TM-1,1,15 while that for cavity II is TE-0,1,41, TE-0,1,40, TE-0,1,39, TE-1,1,40, TE-1,1,39, TM-1,1,40, and TM-1,1,39. The coupling constants were changed by varying the hole radius.

Figure 2 shows the fraction of the total power coupled into the various possible channels: P_Ω = the ohmic power loss, P_{II} = the power coupled to the resonant mode of cavity II (TE-0,1,40), and P_s = spurious power residing in other modes. Note that the beat time, the time it takes the power to transfer from the resonant mode of cavity I to the resonant mode of cavity II, scales approximately as τ_o^{-3} . Ohmic losses dominate for small hole radii in which the transfer times are long (> 100 ns). At the other extreme of large hole radii power is rapidly scattered into a large number of modes and spurious mode losses dominate the transfer efficiency. Figure 2 shows a high efficiency point at $\tau_o = 2.8$ mm despite the presence of strong mode competition effects. This is the result of a number of spurious modes serendipitously coupling to the resonant cavity II mode at the proper time and proper phase. For hole radii slightly larger or smaller than 2.8 mm the power coupled to the TE-0,1,40 mode decreases rapidly making this a poor operational design point. However, for hole radii between 1.5 and 2.5 mm (beat times between 80 and 15 ns) the ohmic and spurious mode losses are low yielding transfer efficiencies as high as 96.6% (losses are almost equally split between ohmic and spurious modes).

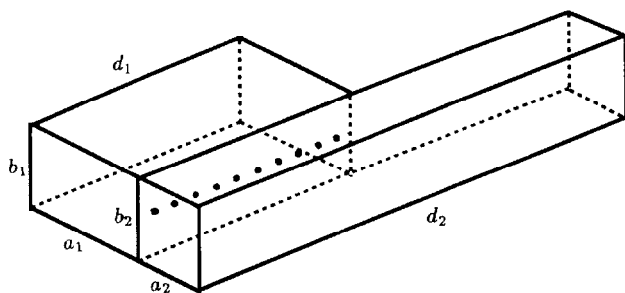
Similar results have been obtained for other two and three cavity configurations. However, for three coupled cavities there are more modes and thus more loss channels, resulting in slightly lower peak transfer efficiencies (between 80 and 90%).

5 Conclusions

The above results show that it is quite feasible to use oversized coupled cavities to transfer energy from the drive accelerator to the high-gradient accelerator in a TBA. A number of factors substantially reduce the severity of mode competition effects, primarily hole geometry and cavity bandwidth. Considerations based on these and other results yield a necessary condition for high transfer efficiency between cavities: $\tau_p \ll \tau_{\text{transfer}} < \tau_{\text{ohmic}}$. Despite mode competition effects transfer efficiencies for a model two cavity system have been found to be greater than 95%. Preliminary results for three cavity systems show transfer efficiencies in the range of 80 to 90%.

References

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- [2] H. Henke, "An Energy Recuperation Scheme for a Two-Beam Accelerator," CERN Internal Report CERN-LEP-RF/8-55 (unpublished, 1988).
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$a_1 = 80.00 \text{ mm}$	$a_2 = 32.00 \text{ mm}$
$b_1 = 40.00 \text{ mm}$	$b_2 = 40.00 \text{ mm}$
$d_1 = 216.39 \text{ mm}$	$d_2 = 540.96 \text{ mm}$

Figure 1. Coupled cavity geometry used for numerical results.

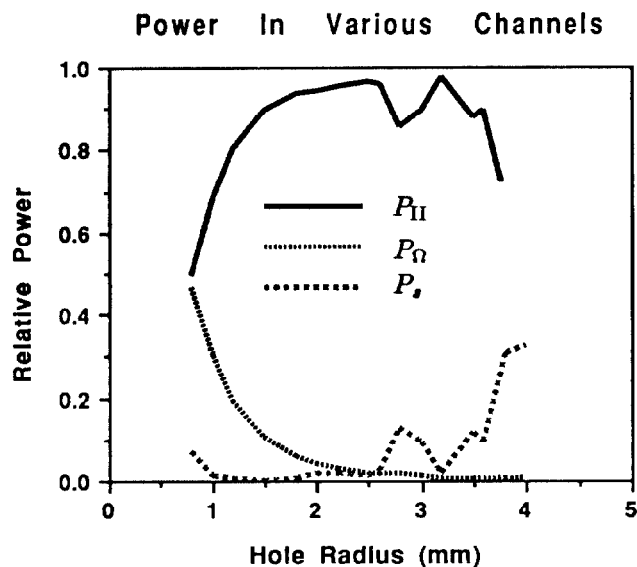


Figure 2. Distribution of power at time of maximum power transfer to the resonant mode of cavity II.

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