A New Method of Correcting the Trajectory in Linacs*

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ABSTRACT

This paper describes a new method of reducing the transverse emittance dilution in linear colliders due to both transverse wakefields and dispersive errors. The technique is a generalization of the Dispersion-Free[1,2] correction algorithm; the dilutions are corrected locally by varying the beam trajectory. This technique complements BNS damping[3] which primarily corrects the dilutions resulting from coherent betatron oscillations. Finally, the results of simulations are presented demonstrating the viability of the technique.

I. INTRODUCTION

In a linear collider the magnets, accelerating structures, and the Beam Position Monitors (BPMs) are all typically misaligned relative to the ideal centerline. Thus, the beam trajectory is offset in both the magnets and the accelerating structures. This can lead to transverse wakefields and dispersive errors.

Transverse wakefields result from the electromagnetic interaction between the particle bunch and the acceleration structures. When a point charge travels off-axis in a structure, it leaves behind a transverse wakefield that will deflect subsequent particles. These deflections cause a particle's trajectory to be a function of it's longitudinal position within the bunch and thereby cause a dilution of the (projected) transverse emittance. Likewise, dispersive errors arise when the beam travels off-axis in the quadrupoles. If the beam is offset in a quadrupole magnet, it will be deflected. Since particles with different energies are deflected differently, the trajectory will be a function of the energy deviation and the projected transverse emittance will be diluted.

Both of these dilutions depend upon the transverse alignment of the accelerator relative to the beam size. To achieve the necessary luminosity in future linear colliders the beam sizes are very small, and, if uncorrected the wakefields and the dispersive errors would impose extremely tight transverse alignment tolerances on the collider. Thus, correction of these dilutions is crucial for future linear collider designs.

II. THEORY

We can gain some insight into this problem by using two two-particle models: one to determine the wakefield effects and one for the dispersive effects. First, consider two macro-particles, each with charge $N/2$, located at $z = \pm\sigma_z$, where σ_z is the rms bunch length. To determine the effect of the wakefields, we examine the difference between the trajectories of these two macro-particles $\Delta x_w = x(\sigma_z) - x(-\sigma_z)$ where the second particle is assumed to have a "correlated" energy deviation of $\overline{\delta}$; this is an energy deviation that is correlated with z and is used by the BNS damping technique[3]. Next, to find the effect of the dispersive errors, we consider the difference Δx_d between the trajectory of the on-energy head particle and another particle, also located at $z = \sigma_z$, with an "uncorrelated" energy deviation of ξ. The uncorrelated energy spread is typically $\sim 1\%$ when injecting into the linac and it decays as $1/\gamma$ as the beam is accelerated.

Assuming that the wakefields and dispersive errors are small, we can solve for Δx_w and Δx_d perturbatively. The first order solutions are[4]

$$\Delta x_w(s) = \int_0^s ds' R_{12}(s,s')\Big[\overline{\delta}\Big(G + Kx_q\Big) - \Big(\overline{\delta}K + \frac{Nr_0}{2\gamma_0}W_\perp(2\sigma_z)\Big)x + \frac{Nr_0}{2\gamma_0}W_\perp(2\sigma_z)x_a\Big] \quad , \tag{1}$$

$$\Delta x_d(s) = \int_0^s ds' R_{12}(s,s')\Big[\xi\Big(G + Kx_q\Big) - \xi K x\Big] \quad , \tag{2}$$

where G, K, and $W_\perp$ are the normalized strengths of the correctors, quadrupoles, and the transverse wakefield. In addition, x is the trajectory of the on-energy head particle, x_q and x_a are the misalignments of the quadrupoles and the accelerator sections respectively, and $R_{12}(s,s')$ relates a deflection at s' to a position as s.

The first term (enclosed in parentheses) in both equations will be small. This occurs because the beam trajectory is corrected; the dipole correctors G are adjusted to cancel the quadrupole deflections Kx_q. Since the correction is performed locally, keeping the trajectory offsets small throughout the machine, the cancellation is independent of slow variations in $\overline{\delta}$ and ξ. Thus, the primary sources of emittance dilution are the last two terms of Eq. (1) and the last term of Eq. (2).

BNS Damping

Looking at Eqs. (1) and (2), one sees that there are two free parameters that can be varied to correct the dilutions: the correlated energy spread $\overline{\delta}$ and the trajectory $x(s)$. The BNS damping technique[3] does the former; it reduces the wakefield contribution to the second term of Eq. (1) with dispersive errors due to the correlated energy deviation $\overline{\delta}$. Specifically, $\overline{\delta}$ is adjusted so that $\overline{\delta}K$ cancels the wakefield $Nr_0W_\perp/2\gamma$.

In the smooth approximation, where K and $W_\perp$ are smooth functions of s, one can solve for a $\overline{\delta}$ such that this term is always zero. Unfortunately, this local cancellation

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is not possible† in the alternating-gradient focusing structures used in high-energy machines. While the wakefield $W_\perp$ has a constant sign, an alternating-gradient focusing structure usually contains a periodic array of discrete focusing magnets with both positive and negative K values. Since the energy spread $\overline{\delta}$ cannot be changed rapidly with s, at best one can adjust $\overline{\delta}$ to cancel the integral of this term over a cell of the focusing structure. Furthermore, since this cancellation depends upon the position x in the quadrupoles and the accelerator sections, exact cancellation is only possible if $x(s)$ is correlated from point-to-point. This is the case for a coherent betatron oscillation, but it is not true if the particle is steered to or deflected by random errors as is the case for a corrected trajectory. Thus, while the BNS technique can cancel the wakefield effects due to a coherent betatron oscillation, it may reduce, but cannot cancel, the effects of wakefields due to a corrected trajectory.

<u>Dispersion-Free Correction</u>

The Dispersion-Free[1,2] (DF) correction technique uses the other approach to correct the dispersive emittance dilution. Here, the trajectory $x(s)$ is varied so that over any short region of the accelerator the integral in Eq. (2) is small. The technique "measures" the dispersive errors by measuring the difference of two trajectories while changing the beam energy, or equivalently, while changing the magnet strengths. The equation for this difference orbit is identical to Eq. (2) except that ξ is replaced by the effective energy change; this is typically around 10%. Thus, the difference orbit will accurately reflect the emittance dilution except for measurement errors and effects of the non-linearity of the dispersive error; a complete analysis of all the errors is given in Ref. [1]. By correcting the difference orbit, in concert with the actual trajectory, the DF correction technique can reduce the dispersive emittance dilution to negligible values.

<u>Wake-Free Correction</u>

Given the performance of the DF algorithm, we have attempted to extend it to also correct wakefields. The goal is to find a new trajectory along which both the wakefield and the dispersive effects cancel. The wakefields are caused by trajectory offsets in the accelerator sections which are due to both misalignments of the accelerator sections and a non-zero trajectory. If we ignore the accelerator misalignments, the effective offset in a section is just the average of the position in the two adjacent quadrupoles. By varying the quadrupole strengths in a specified manner, one can measure a difference orbit where the orbit in the quadrupoles will *mimic* the effects of the wakefields due to the trajectory. From the second term in Eq. (1), we find that, to mimic the wakefield effect, the quadrupole strengths must vary as

$$\frac{\delta K}{K} \propto \frac{L_{\mathrm{acc}}(s)}{\gamma(s) K L_{\mathrm{quad}}(s)} \sqrt{\frac{\beta_{\mathrm{acc}}}{\beta_{\mathrm{quad}}}} \cos \Delta\phi \quad , \qquad (3)$$

where β_{acc} and β_{quad} are the beta functions at the middle of the accelerator sections and the adjacent quadrupoles and $\Delta\phi$ is the betatron phase advance between the two. In addition, L_{acc} and L_{quad} are the lengths of the accelerator sections and the quadrupoles. Finally, note that because the correction is local, this condition can fluctuate slowly with s.

Condition (3) specifies that the quadrupole strength variation $\delta K/K$ has opposite signs at focusing (QFs) and defocusing quadrupoles (QDs). In contrast, when creating the difference orbit to measure the dispersive error, $\delta K/K$ has the same sign at both the QFs and the QDs. To correct both the wakefields and the dispersive errors one minimizes both of these difference orbits along with the actual trajectory. Alternately, one can use an equivalent procedure where one minimizes a difference orbit created by varying only the QFs and a difference orbit created by varying only the QD magnets.

To summarize, the correction algorithm is: (1) measure a difference orbit $\Delta x_{QF}(s)$ created by varying the QFs and the associated dipole correctors, (2) measure the difference orbit $\Delta x_{QD}(s)$ created by varying the QDs and the associated dipole correctors, (3) measure the actual trajectory $x(s)$, and finally, (4) one minimizes all three of these orbits. When developing the DF algorithm, it was found that a weighted least-squares is the best minimization procedure. Thus, in this variation, one minimizes the sum:

$$\sum_{j\in\{BPM\}} \frac{(\Delta x_{QF})_j^2}{2\sigma_{\mathrm{prec}}^2} + \frac{(\Delta x_{QD})_j^2}{2\sigma_{\mathrm{prec}}^2} + \frac{x_j^2}{\sigma_{\mathrm{BPM}}^2 + \sigma_{\mathrm{prec}}^2} \quad , \qquad (4)$$

where each term is weighted by the accuracy of the respective measurement: σ_{BPM} is the estimated rms of the BPM misalignments and σ_{prec} is the rms precision (reading-to-reading jitter) of the BPM measurements. Although it does not correct the wakefields due to the accelerator section misalignments, we refer to this technique as Wake-Free (WF) correction because the resulting trajectory does not in itself cause wakefield or dispersive dilutions.

III. SIMULATIONS

In Table 1, the performance of the DF and WF techniques is compared against a standard correction algorithm, the 1-to-1 method. The 1-to-1 algorithm adjusts the trajectory to zero the BPM measurements; typically, in this technique one only uses the BPMs and correctors located near the focusing quadrupoles. The correction was simulated in a preliminary design of the Next Linear Collider[5] (NLC) where the vertical beam size is tiny, roughly 2 μm.

† It was assumed here that $\overline{\delta}$ is due to an energy deviation. It is also possible to vary the focusing strength with RF quadrupoles.

Table 1. Correction in the NLC.

Method	ϵ_y	Trajectory rms
1-to-1	$22.9 \pm 21.3\,\epsilon_{y0}$	$72 \pm 3\,\mu$m
DF	$9.3 \pm 7.3\,\epsilon_{y0}$	$55 \pm 5\,\mu$m
WF	$1.09 \pm 0.05\,\epsilon_{y0}$	$44 \pm 3\,\mu$m

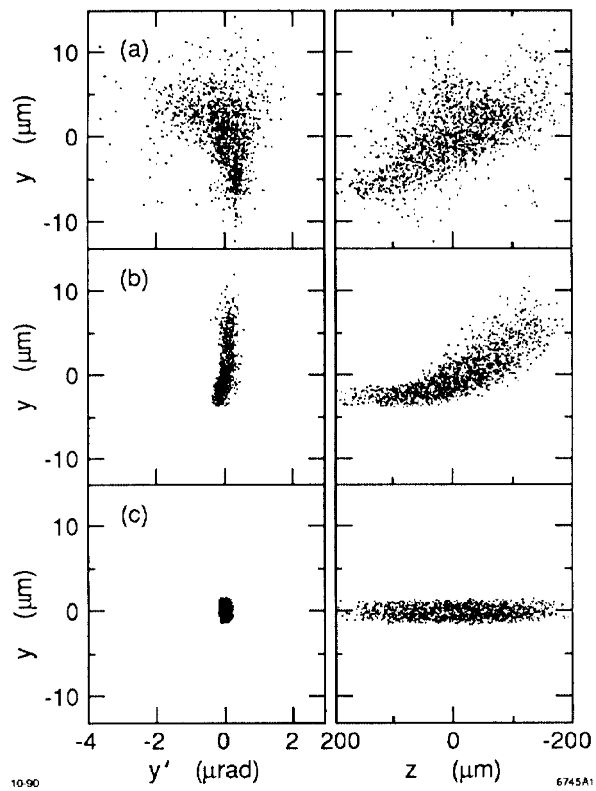

Fig. 1 The beam distribution after (a) 1-to-1, (b) DF, and (c) WF correction. The left-hand plots are the y-y' phase space while the right-hand plots are the beam in y-z space.

The results in Table 1 are the average of correcting 20 sets of random error distributions and ϵ_{y0} is the initial undiluted vertical emittance. The error distributions have $70\,\mu$m rms vertical quadrupole and BPM misalignments, and $2\,\mu$m rms BPM precision errors; the accelerator sections were aligned to the ideal machine centerline. In addition, the optimal BNS energy spread has been added to the beam in all three cases. Finally, the initial conditions (y_0, y'_0) were optimized[6] after 1-to-1 correction to further reduce the dilution. While this procedure reduces the dilution by a factor of two, from $50\,\epsilon_{y0}$ to $23\,\epsilon_{y0}$, when using the 1-to-1 algorithm, it yields little improvement when using DF or WF correction. The WF technique performs extremely well; it virtually eliminates all of the emittance dilution and it does a better job correcting the actual trajectory than the other two methods.

Figure 1 shows plots of the beam distribution after (a) 1-to-1, (b) DF, and (c) WF correction for one of the 20 cases in Table 1. The scatter-plots on the left are the projections of the beam distributions in the y-y' phase space while the right-hand plots are projections onto the y-z plane. One can immediately see that the beam emittance is seriously diluted after 1-to-1 correction. Next, after DF correction, the dispersive errors are corrected, but the distribution displays the tails characteristic of transverse wakefields; these arise from the random trajectory. Finally, after WF correction, one can see that the dilution due to both the dispersive errors and the wakefields is negligible.

IV. DISCUSSION

To conclude, we note that the WF technique reduces the emittance dilution due to misaligned quadrupoles and a non-zero trajectory extremely well. Since the technique is very similar to the DF method, we know that it is a robust algorithm and it is not sensitive to effects such as jitter and calibration errors. Furthermore, the technique will effectively decouple the emittance dilution from the transverse alignment of the quadrupoles and the BPMs. Finally, variations on the WF technique may be able to directly measure and correct the wakefield dilutions. Here, one would measure a difference orbit while varying the bunch length, the current, or the beam energy to measure the wakefield effects. Thus, the WF technique, and in general, the use of the trajectory to *locally* cancel emittance dilutions, will likely prove indispensable in future linear colliders.

V. REFERENCES

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