# A Practical Algorithm for Chromaticity Correction in Linear Collider Final Focus Systems

P. Krejcik Stanford Linear Accelerator Center Stanford University, Stanford, California 94305

## Abstract

The details of a tuning algorithm are described for minimizing the chromatic error in a final focus system, using the two families of sextupoles incorporated in a chromatic correction section. The chromatic errors are characterized by the beam waist location of off-energy particles with respect to the interaction point location. It is shown that linear combinations of changes in strength to the two sextupole families can move the waist position of off-energy beams independently in the X and Y planes. Measurements at the SLC have shown that the off-energy waist position can be measured as a function of the sextupole multiknob to minimize chromaticity.

#### I. INTRODUCTION

In colliders the luminosity is related to the inverse square of the beam sizes at the interaction point (IP). Chromatic errors in the beam line will result in a degradation of spot size for beams with a finite energy spread. In linear colliders the beam is far from monochromatic with a typical energy spread of 0.5%. Chromatic contributions to spot size are thus very important in determining the useable luminosity.

This paper does not address so much the design of a final focus beam line nor the calculation of the contributions of all significant aberrations to spot size. Rather, it addresses the more immediate issue of how to tune the sextupoles in a given final focus beam line based on the available beam observation tools at hand. The analysis made here is of the final focus beam line of the SLC [1], using the normal conducting final triplet. The technique can be generalized to other final focus layouts, such as with the newly installed superconducting arrangement at the SLC [2].

Quantifying the chromatic effects in terms of readily observable beam parameters requires some reappraisal in a linear collider. In circular colliders the routine measurement of tunes at energies offset from the central energy reveal the change in phase advance around the ring resulting from chromatic errors. In single pass beam lines the phase advance is not easily measured, only the final beam size at the IP is measured with any degree of precision. Verifying that chromatic correction sextupoles are optimally set using only beam size data [3] requires much interpretation of the measurements and can take considerable time. The algorithm described in this paper exploits the change in the beam waist position for off energy beams as a result of chromatic errors. The chromatic correction sextupoles are scanned in a prescribed ratio to each other so that the waist position of the off energy beam changes in either the x or y plane. This type of sextupole scan has the twofold purpose of quantifying the amount of chromatic error present and allows the optimum sextupole setting to be easily implemented.

Figure 1. Beam size in x (top) and y (lower) and dispersion (dashed) in the SLC FF beam line



Work supported by the Department of Energy, Contract DE-AC03-76SF00515

# II. OPTICS OF THE FINAL FOCUS

#### Linear optics

The layout of the SLC final focus beam line is shown in fig. 1. Its design function has been covered in detail elsewhere [1]. Of interest are the two demagnifying transformers separated by a Chromatic Correction Section (CCS). In order to achieve the desired small beta functions of a few tens of millimeters at the IP the final quadrupole triplet is very strong. The strong bending of rays inside the quadrupole leads to a pronounced variation in beam waist position with energy, fig. 2, resulting in large chromatic errors.



Figure 2. Simple view of the effect of chromatic errors on achievable spot size at the final focus

### Chromaticity correction scheme

A string of bend dipoles generate the necessary dispersion inside the CCS where two families of sextupoles S1 and S2 are incorporated to control the chromaticity. The sextupoles act on all particles displaced from the axis of the sextupoles. The design aims at producing a correlation between this offset and energy deviation via the dispersion function but the action of the sextupoles on the beam can also arise from misalignments of the sextupole and this effect should be distinguished when evaluating the tuning algorithm.

The design strength of the sextupoles is set in order to minimize the path length integral of an off-energy ray. This is not a concept that can be easily realized in terms of tuning parameters when dealing with beam measurements. Instead, use is made of the concept of the waist motion with respect to energy deviations in the beam. Minimizing the waist motion with respect to energy can be shown to be equivalent to minimizing the path length integral [6].

### **III. TUNING IN THE FINAL FOCUS**

### First order optical tuning

Optical tuning of the beam line is a necessary precursor to chromatic correction to match to the variety of beam conditions at the entrance to the final focus beam line. The goal of the tuning is to produce the smallest beam spot compatible with the constraints of low background generation for the detector. The logical procedure in which orbits, dispersion, skew and beta matching is done so as to empirically arrive at a minimum spot size has been dealt with elsewhere [4]. Of principle interest here is the routine use of IP beam size measurements based on beam-beam deflection scans (or even wire scans for single beams) combined with systematic scans of the focal length of the final quadrupoles in what is collectively termed a *waist scan*. These waist scans reveal both the minimum spot size of the beam and the distance from the final lens at which the minimum occurs.

## Tuning to minimize chromaticity

If the energy spread in the beam were to be increased then in the conceptual drawing in fig. 2 it is seen that particles of different energies cross the axis at different distances from the lens. However, all that can be observed in practice is the increase in beam size at the IP. This phenomena alone is a difficult criteria to tune the sextupoles by as it is not immediately apparent how much of the beam size is due to chromaticity, or dispersion, skew, beta mismatch etc..

If instead the centroid energy of the beam is shifted by  $\Delta E$ then the waist moves by some amount  $\Delta L$  in a nonchromatically corrected configuration. The amount  $\Delta L$  that the waist moves with energy can be determined from a waist scan using the final quadrupoles. On the other hand the focal length of the off-energy beam can be moved with the sextupoles. The two sextupole families are scanned in a prescribed ratio to each other and one observes the off energy waist move, in much the same way as the final quadrupoles change the focal length in a conventional waist scan. A single off-energy sextupole waist scan is more expedient than conventional waist scans performed at several energies and furthermore allows the sextupoles to be directly dialled to their correct values at the completion of the scan.

### Computer modelling

In order to perform the off-energy waist scans with the sextupoles it is necessary to predict the ratio in which the two sextupole families need to be changed with respect to each other. Changing the sextupole families in a prescribed ratio allows the off energy waist to be moved independantly in the x and y planes. An optics code is used in which the energy of the beam is offset and the off-energy waist position is found in each plane as each of the sextupole families are changed in turn. This immediately gives the matrix coefficients in eq. (1).

$$\begin{bmatrix} \Delta L_{x} \\ \Delta L_{x} \end{bmatrix} = \begin{bmatrix} \left( \frac{\Delta L_{x}}{\Delta S_{x}} \right)_{\Delta S_{\vec{y}}=0} \left( \frac{\Delta L_{x}}{\Delta S_{y}} \right)_{\Delta S_{\vec{x}}=0} \\ \left( \frac{\Delta L_{y}}{\Delta S_{x}} \right)_{\Delta S_{\vec{y}}=0} \left( \frac{\Delta L_{y}}{\Delta S_{y}} \right)_{\Delta S_{\vec{x}}=0} \end{bmatrix} \begin{bmatrix} \Delta S_{x} \\ \Delta S_{x} \end{bmatrix}$$
(1)

A useful feature of this type of analysis is that the same matrix coefficients can also be found by direct measurement with the beam by observing the waist position as as each sextupole family is scanned in turn, while the beam energy is maintained with an energy offset of e.g. 0.2%. A matrix inversion of eq. (1) gives directly the multiknob coefficients required, where the matrix coefficients in eq. (2) now indicate how much each sextupole family should be adjusted in order to move the off-energy waist by a prescribed amount.

$$\begin{bmatrix} \Delta S_{x} \\ \Delta S_{x} \end{bmatrix} = \begin{bmatrix} a_{11}a_{12} \\ a_{21}a_{22} \end{bmatrix} \begin{bmatrix} \Delta L_{x} \\ \Delta L_{x} \end{bmatrix}$$
(2)

### Implementation

The SLC Control Program (SCP) has a generic multiknob software capability for controlling devices, such as magnets, and keeping their strengths in a prescribed ratio. The ratios of the two sextupole families, calculated from the modelling to give orthogonal control of x and y chromaticity, are incorporated in four *chromaticity* multiknobs to allow x and y scans in each beam. The units of the multiknobs are in cm of waist motion per 100 MeV of energy offset.

### IV. RESULTS OF CHROMATICITY SCANS

An example of chromaticity scan is shown in fig.3 for positrons in the south final focus. The beam energy was shifted by 100 MeV and the vertical chromaticity multiknob scanned to find the location of the off-energy waist. The graph shown is the operators output from the scan, where the square of the measured beam size is plotted as a function of the knob value. In this example the waist was found to be at B=-1.565 cm from the IP. The knob is then dialled to this value to achieve the minimum chromatic error.



Figure 3 Measured beam waist position as a function of sextupole multiknob for the SLC beam with an energy offset of 0.2%

After implementing a chromaticity correction using this tool a normal waist scan is made (using the final quadrupoles) of the on-energy beam to check if the waist has moved. This becomes necessary if the orbit errors in the sextupoles are significant so that they add linear focusing terms to the onenergy beam. Changing the sextupoles via the chromaticity knob would in this case change the first order optics and hence the waist position. In order to distinguish between waist motion due to sextupole misalignments and true chromatic waist motion a larger energy offset for the beam is chosen so that chromatic effects become more dominant in the measurements.

### V. CONCLUSION AND FUTURE OUTLOOK

The implementation of orthogonalized sextupole scans in the SLC final focus has been found to be a relatively speedy way of checking and correcting the chromaticity contribution to spot size at the IP. The procedure requires that the beam energy be briefly offset in energy by 0.5% while the sextupole multiknobs are scanned. Augmentations are possible to the technique, where the beam energy is scanned over several data points and the waist location is independently verified by further scanning the quadrupoles in the final triplet [5].

At further reduced magnifications, such as those anticipated for the Next Linear Collider, the chromatic contribution to the final spot size becomes increasingly important. More attention has been given to the optical layout of the chromatic correction section, such as in SLAC's Final Focus Test Beam Project [6], where noninterlaced sextupole families aid in the global control of aberrations. The algorithm described in this paper is applicable to these situations and should help in matching the beam line to the variety of initial phase space conditions that one encounters in practice.

#### VI. ACKNOWLEDGEMENTS

Thanks are due to W. Kozanecki for passing on his experience with commissioning the SLC final focus system and to K. Brown for helpful discussions on chromatic corrections in beam lines. Also to the many people who have worked on the design and tuning of the final focus on whose experience this work was built.

#### **VII. REFERENCES**

- [1] SLAC Linear Collider Design Handbook, SLAC 1984.
- [2] W. Ash et al., "New Final Focus System for the SLC", these proceedings.
- [3] W. Kozanecki, private communication.
- [4] P. Bambade et. al., SLAC Pub. 4776, January 1989.
- [5] N. Toge et. al., "Chromaticity Corrections in the SLC Final Focus System", these proceedings.
- [6] K. Brown, private communication.