Degradation of Brightness by Resonant Particle Effects *

Y.-J. Chen, G. J. Caporaso, A. G. Cole, A. C. Paul, and W. C. Turner
Lawrence Livermore National Laboratory
University of California, L-626
Livermore, California 94550

ABSTRACT

A resonance between the periodic focusing force and the envelope oscillations or the particle orbits can lead to beam halo formation and a dramatic decrease in brightness. We have developed a theoretical model for this parametric transport instability. We have also investigated the instability on the Experimental Test Accelerator II (ETA-II)[1]. The instability can be excited by altering the excitation of just one solenoid.

I. INTRODUCTION

Most accelerator configurations consist of some periodic structures such as a periodic focusing or acceleration system. The Experimental Test Accelerator-II (ETA-II)[1] is the induction linac designed to drive a 140 GHz microwave FEL. Reference 2 showed that when the envelope oscillations are in resonance with the periodic magnetic field of the first two 10-cell blocks of ETA-II, 20-25% of particles walk away from the bulk of the beam and form a halo which seriously degrades the beam brightness. The present paper aims to provide some theoretical understanding of the resonant particle effects and the cures. A theoretical model is given in Sec. II. We have investigated this parametric transport instability on ETA-II by detuning a single focusing solenoid deliberately and observing the brightness degradation. Comparisons between the simulation results and experimental data are presented in Sec. III.

II. THEORETICAL MODEL

A. *Envelope Oscillations*

In this paper, we only consider a solenoid transport system. In general, we find the matched tune for the beam by solving the envelope equation:

$$R'' + \frac{(\gamma\beta)'}{\gamma\beta}R' + \left(\frac{k_c^2}{4} - k_s^2\right)R - \frac{\mathcal{E}^2}{\gamma^2\beta^2R^3} = 0 \quad , \qquad (1)$$

where $R(z)$ is the r.m.s. beam radius and $\gamma\beta$ is the parallel beam momentum normalized by mc. Here $k_c = eB/\gamma\beta mc^2$ is the cyclotron wavenumber, $k_s^2 = I/I_o\gamma^3\beta^3R^2$ is the tune shift due to the defocusing space charge force, I is the beam current, $I_o = mc^3/e \approx 17$ kA, and $\mathcal{E}^2 = \epsilon_n^2 + P_\theta^2/c^2$ is the effective normalized emittance. P_θ and ϵ_n are the canonical angular momentum and normalized emittance of the beam. According to Eq. (1), the matched magnetic field $B_o(z)$ for a preferable slowly varying envelope $R_o(z)$ is given by

$$|B_o(z)| = \frac{2\gamma\beta mc^2}{e}\left[k_s^2 + \frac{\mathcal{E}^2}{\gamma^2\beta^2R^4} - \frac{R''}{R} - \frac{(\gamma\beta)'}{\gamma\beta}\frac{R'}{R}\right]_o^{\frac{1}{2}} \quad . \qquad (2)$$

The subscript "o" represents the matched envelope. Note that the magnetic field B_o can be both positive and negative. Hence, we can use either the continuous or the alternating solenoid field to transport the beam. Furthermore, for a smoothly varying beam envelope, the matched magnetic field B_o should also be smoothly varying except near the acceleration gaps. Let us assume that the actual magnetic field used in the machine differs from the matched magnetic field by $\delta B(z)$, and δB is much less than B_o. Then the envelope will differ from the matched value by

$$R(z) = R_o(z) + \delta R(z) \quad . \qquad (3)$$

Linearizing Eq. (1) around the matched envelope yields the following envelope oscillation equation:

$$X'' + K_e^2 X = -\left(\frac{k_c^2 R}{2}\right)_o \frac{\delta B}{B_o} + \left(\frac{2\mathcal{E}^2}{\gamma^2\beta^2R^3}\right)_o \frac{\delta\mathcal{E}}{\mathcal{E}_o} \quad , \qquad (4)$$

where $X = \sqrt{\gamma\beta}\,\delta R$, and

$$\begin{aligned} K_e^2 = {} & \frac{k_c^2}{4} + k_s^2 + \frac{1}{4}\left[\frac{(\gamma\beta)'}{\gamma\beta}\right]^2 - \frac{1}{2}\frac{(\gamma\beta)''}{\gamma\beta} + \frac{3\mathcal{E}^2}{\gamma^2\beta^2R^4} \\ & + \left[\frac{\partial(\gamma\beta)'}{\partial R} - \frac{(\gamma\beta)'}{\gamma\beta}\frac{\partial(\gamma\beta)}{\partial R}\right]\frac{R'}{\gamma\beta} \\ & - \left(\frac{k_c^2}{2} - 3k_s^2 - \frac{2\mathcal{E}^2}{\gamma^2\beta^2R^4}\right)\frac{R}{\gamma\beta}\frac{\partial(\gamma\beta)}{\partial R} \quad . \end{aligned} \qquad (5)$$

There are two driving forces for the envelope oscillations [Eq. (4)]. We only consider the oscillations excited by the perturbation in the magnetic field here. We can express the magnetic driving term by a Fourier expansion, and rewrite Eq. (4) as

$$X'' + K_e^2 X = \sum_{k_b} F_{k_b} e^{ik_b z} \quad . \qquad (6)$$

Note that K_e^2 is a slowly varying function. Let $X = \sum X_{k_b} e^{ik_b z}$. For $K_e \neq k_b$, the envelope oscillation does not resonate with the periodic magnetic force, and

$$X_{k_b} \approx \frac{F_{k_b}}{K_e^2 - k_b^2} \quad . \qquad (7)$$

For $K_e = k_b$, the orbits of the bulk of the beam resonate with the periodic focusing force. Then the parametric instability occurs, and

$$X_{k_b} \approx \frac{F_{k_b}}{2ik_b} z \quad . \qquad (8)$$

* Work performed jointly under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract W-7405-ENG-48, for the Strategic Defense Initiative Organization and the U.S. Army Strategic Defense Command in support of SDIO/SDC MIPR No. W43-GBL-0-5007

B. *Phase Mixing and Halo Formation*

A single particle orbit is given by

$$\eta'' + \left[\frac{(\gamma\beta)'}{\gamma\beta} - ik_c\right]\eta' - k_s^2(|\eta|)\eta = 0 \quad , \qquad (9)$$

where $\eta = x + iy$. Let $\eta = \eta_o + \delta\eta$. To obtain the consistent perturbation in k_s, we need to solve either the Vlasov or the continuity equation. However, for simplicity, we assume that both the perturbed and the unperturbed space charge force is linear in radius. Linearizing Eq. (9) around the unperturbed orbit yields

$$\zeta'' + K_s^2\zeta = -\sqrt{\gamma\beta}\;\eta_o e^{-i\int\frac{k_{co}}{2}dz}\left(k_{co}k_o\frac{\delta B}{B_o} + 2k_{so}^2\frac{\delta R}{R_o}\right) , \qquad (10)$$

where

$$\zeta = \sqrt{\gamma\beta}\;\delta\eta\; e^{i\int k_{co}dz/2} \quad , \qquad (11)$$

$$K_s^2 = \frac{k_{co}^2}{4} - k_{so}^2 + \frac{1}{4}\left[\frac{(\gamma\beta)'}{\gamma\beta}\right]^2 - \frac{1}{2}\frac{(\gamma\beta)''}{\gamma\beta} + \frac{ik_{co}}{2}\frac{B_o'}{B_o}$$
$$+ k_{co}k_o\eta_o\left[\frac{\partial B_o/\partial\eta}{B_o} - \frac{\partial(\gamma\beta)/\partial\eta}{\gamma\beta}\right] + \frac{\partial}{\partial\eta}\left[\frac{(\gamma\beta)'}{\gamma\beta}\right] , \qquad (12)$$

and $k_o \approx k_{co}/2 \pm (k_{co}^2/4 - k_{so}^2)^{1/2}$ is the betatron wavenumber for the matched beam. Solving Eq. (10) by using Eqs. (4) - (8) and (11) gives the pertubed orbit. Generally, $K_s \neq K_e + k_o - k_{co}/2$, a given Fourier component in the perturbed force can only resonate with the envelope oscillations or the particle orbits. The perturbed orbit for this single resonance case is given by

$$\frac{\delta\eta}{\eta_o} = \sum_{\substack{k_b\neq K_e \\ k_b\neq K_s - k_o + \frac{k_{co}}{2}}} \frac{k_{co}^2}{K_s^2 - (k_b + k_o - \frac{k_{co}}{2})^2} \times \left[-\frac{k_o}{k_{co}} + \frac{1}{\sqrt{\gamma\beta}}\frac{k_{so}^2}{K_e^2 - k_b^2}\right]\frac{\delta B_{k_b}}{B_o}\, e^{ik_bz}$$
$$+ \frac{k_{co}^2}{K_s^2 - (K_e + k_o - \frac{k_{co}}{2})^2}\left[\frac{k_o}{k_{co}} + \frac{i}{2\sqrt{\gamma\beta}}\frac{k_{so}^2 z}{K_e}\right]\frac{\delta B_{K_e}}{B_o}\, e^{iK_ez}$$
$$- \frac{ik_{co}^2 z}{2K_s}\left[-\frac{k_o}{k_{co}} + \frac{1}{\sqrt{\gamma\beta}}\frac{k_{so}^2}{K_e^2 - (K_s - k_o + \frac{k_{co}}{2})^2}\right] \times \frac{\delta B_{K_s - k_o + \frac{k_{co}}{2}}}{B_o}\, e^{i(K_s - k_o + \frac{k_{co}}{2})z} \qquad (13)$$

However, when $K_s = K_e + k_o - k_{co}/2$, the periodic perturbation can excite parametric instabilities simultaneously in both the envelope oscillations and the particle orbits. When the double resonance occurs, the orbit is given by

$$\frac{\delta\eta}{\eta_o} = \sum_{k_b\neq K_e} \frac{k_{co}^2}{K_s^2 - (k_b + k_o - k_{co}/2)^2} \times \left[-\frac{k_o}{k_{co}} + \frac{1}{\sqrt{\gamma\beta}}\frac{k_{so}^2}{K_e^2 - k_b^2}\right]\frac{\delta B_{k_b}}{B_o}\, e^{ik_bz}$$
$$- \frac{k_{co}^2 z}{2K_s}\left[i\frac{k_o}{k_{co}} - \frac{1}{2\sqrt{\gamma\beta}}\frac{k_{so}^2 z}{K_e}\right]\frac{\delta B_{K_e}}{B_o}\, e^{iK_ez} \qquad (14)$$

The first term in Eqs. (13) and (14) corresponds to the nonresonant orbit. The other terms in these equations describes the unstable orbits when the orbit or the envelope oscillation resonates with the B field. Equation (12) shows that K_s is a function of radius due to the space charge potential depression, and the radial dependence of the solenoidal field and the accelerating field. Hence, $\delta\eta/\eta_o$ varies in radius implicitly through $\gamma\beta$ and K_s regardless whether the particle orbit resonates with the perturbation or not. This leads to $(\delta\eta/\eta_o)'$ also varying in radius. Therefore, phase mixing occurs whenever there is an oscillatory magnetic perturbation or envelope oscillations.

Since the resonance condition is a function of radius, only the particles at certain radius can resonate with the periodic field. These particles can walk away from the bulk of the beam with an amplitude proportional to z for the single resonance case and to z^2 for the double resonance case. Eventually, a halo will be formed. Generally, $(\gamma\beta)' \approx 0$ and $B_o' \approx 0$. The resonance condition for the envelope oscillations and for the particle orbits is $k_b = K_e$ and $k_b \approx 2K_s$, respectively. Then, double resonance occurs when $k_b \approx 0.63k_{co}$ and $k_{so} \approx 0.39k_{co}$. Therefore, the double resonance can happen only if the beam is strongly space charge dominated. These double resonance conditions can be easily satisfied for the ETA-II beam since the perturbation in a single solenoid field contains many Fourier components. If the parametric instability is excited over a large distance, the quality of the beam will be damaged beyond repair.

III. EXPERIMENTS

We should choose a magnetic field B_o such that K_e does not equal an integer times the wavenumber of the periodic system. Furthermore, allowing K_e to vary along the z axis can prevent the parametric instability from growing over an extended distance. The result of particle simulations for the old ETA-II experiment reported in Ref. 2 is presented in Figs. 1 (a)-(c). As shown in Fig. 1(b), with the magnetic field of the old tune, the bulk of the beam could resonate with the periodic magnetic structure over the first 20 cells. Large envelope oscillations were excited. Hence, a halo was formed. Figures 1 (d)-(f) present the simulation results for the current ETA-II tune. The new tune is chosen so that the new beam envelope (Fig. 1(e)) is roughly the same as the old envelope (Fig. 1(b)) without the large parametric instability excitation. Comparing the envelope oscillations in Figs. 1(b) and (e) shows that the new K_e varies relatively fast in z. Therefore, it is harder for the parametric instability to ruin the beam quality.

The magnetic field of a single solenoid consists of many long wavelength Fourier components. Therefore, the instability can be excited by altering the excitation of a single solenoid. We have investigated the transport parametric instability experimentally on the newly modified version of the ETA-II. The instability was excited at the exit of the injector deliberately by detuning one focusing solenoid. The stronger space charge forces at that location allowed

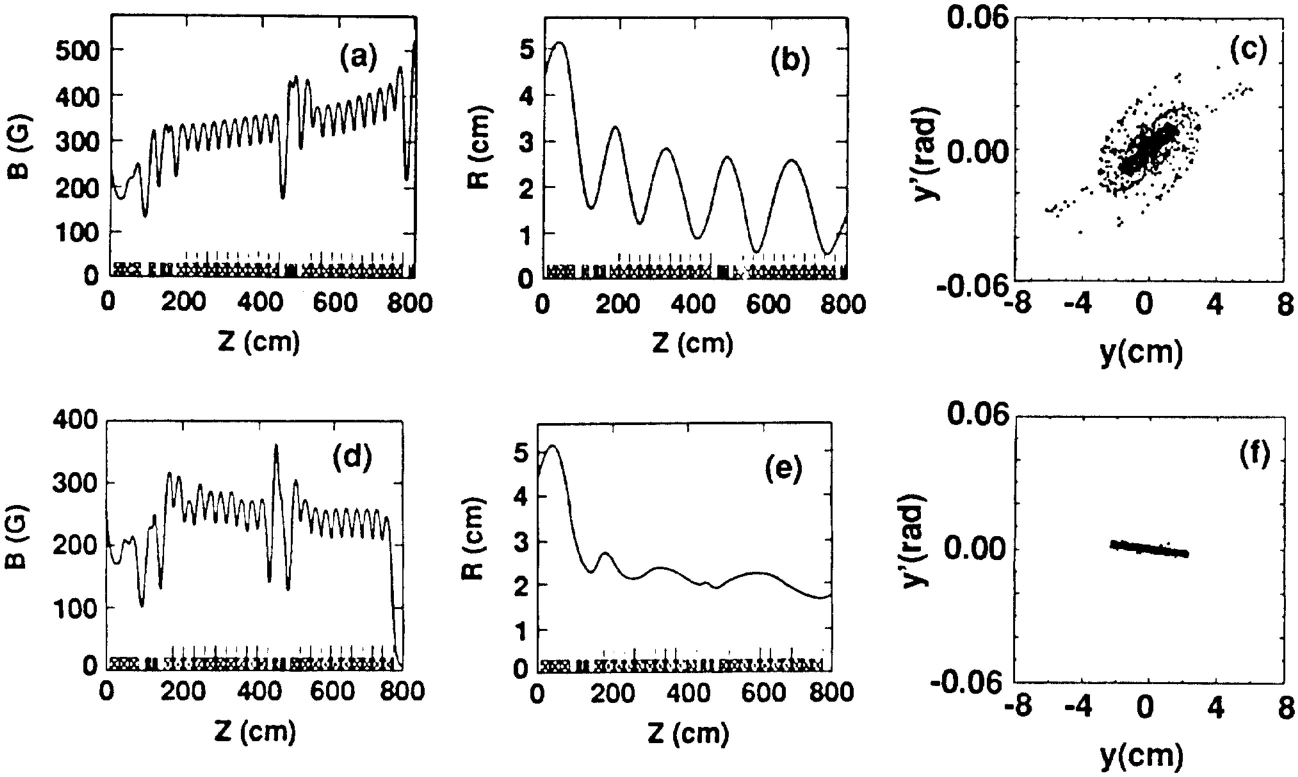

Fig. 1 Reduction of resonant degradation of brightness with choice of focusing tune. The old tune, the beam envelope and the beam phase space at the end of the 20 cells is given in (a), (b) and (c), respectively. The plots for the new tune are presented in (d), (e) and (f).

us to observe a large brightness degradation. The beam brightness was measured by using a pepper-pot emittance diagnostic[3]. Since the beam does not stay in resonance with the periodic magnetic structure over a long distance with the current ETA-II tune, a relatively large perturbation in the solenoid current excitation must be used. The nominal excitation on the first solenoid after the injector is 60.81 A. We varied the current from the nominal current to 90A. The brightness was reduced up to a factor of 3.67. The experimental data are summarized in Table I. Here, I_{C1}, I_{B4} and I_{T3} are the current excitation of the solenoid, the beam current at the pepper-pot mask, and the transmitted beam current after the mask, respectively. The intrinsic beam brightness[3] is defined as

$$J = \frac{2I}{(\pi\gamma\beta a\theta)^2} \quad , \qquad (15)$$

where a is the beam radius, and θ is the intrinsic angular spread of the beam. Both the experimental and simulated brightness data are given in Table I.

IV. CONCLUSIONS

We have studied a parametric instability of envelope oscillations in a periodic solenoid focusing system. A theoretical model was developed to understand this parametric instability and its phase mixing. We have investigated the instability on ETA-II. A magnetic tune has been chosen for the ETA-II focusing system so that the envelope oscillations can not resonate with the periodicity of the focusing system over a significant distance. Therefore, it is difficult to excite this transport parametric instability and to degrade the beam quality.

Table I.
Brightness Degradation by Mismatch

I_{C1} (Amp)	I_{B4} (Amp)	I_{T3} (Amp)	J_{exp} (A/(m-rad)^2)	J_{sim} (A/(m-rad)^2)
60.81	1108	2.39	9.15×10^8	4.5×10^9
70	933	2.05	6.02×10^8	1.9×10^9
80	718	1.70	3.64×10^8	1.0×10^9
90	533	1.41	2.49×10^8	6.8×10^8

References

1. W. E. Nexsèn, et al., "The ETA-II Induction Linac as a High Average Power FEL Driver", *Proc. of the 11th Int. FEL Conf.*, Naples, FL., (August 28-Sept. 1, 1989).

2. Y.-J. Chen, et al., "Measurement and Simulation of Whole Beam Brightness on the ETA-II Linear Induction Accelerator", in *Proc. 1990 Linac Conference*, Albuquerque, NM., Sept. 10-14, 1990.

3. A. C. Paul, et al., "ETA-II Beam Brightness Measurement", this conference.