

Numerical Simulation of a Short RFQ Resonator Using the MAFIA Codes¹

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Abstract

The electrical characteristics of a short ($2\beta\lambda=0.4\text{m}$) resonator with large modulation ($m=4$) have been studied using the three dimensional codes, MAFIA. The complete resonator, including the modulated electrodes and a complex support structure, has been simulated using $\sim 350,000$ mesh points. Important characteristics studied include the resonant frequency, electric and magnetic fields distributions, quality factor and stored energy. The results of the numerical simulations are compared with the measurements of an actual resonator and analytical approximations.

I. INTRODUCTION

A prototype of a Superconducting RFQ (SRFQ) resonator has been designed and built at SUNY, Stony Brook [1].

The short length of the resonator is chosen to facilitate superconducting operation. This design offers many advantages for the acceleration and focussing of low β (0.01 to 0.05) heavy ion beams [2].

The SRFQ resonator has the four rod structure [3]. The short length of the SRFQ makes it possible to simulate the whole structure for a computer, including the modulation of the electrodes and the fringe field regions. We used the MAFIA codes (version 2.04) [4] to compute in detail the electrical characteristics of the resonator.

In this paper we compare the MAFIA numerical simulations with the measurements of the SRFQ resonator as well as the results of an equivalent lumped circuit analysis [5].

II. GEOMETRY DEFINITION IN MAFIA

The definition of the resonator's geometry and the mesh in MAFIA is very important for obtaining accurate results.

The most complicated objects in the SRFQ are the electrodes. Also, maximum detail in the fields is required near the electrodes. Thus we place the electrodes in the x-z and y-z planes of the simulation reference frame. The beam is along the z axis. With this particular orientation we are able to define the

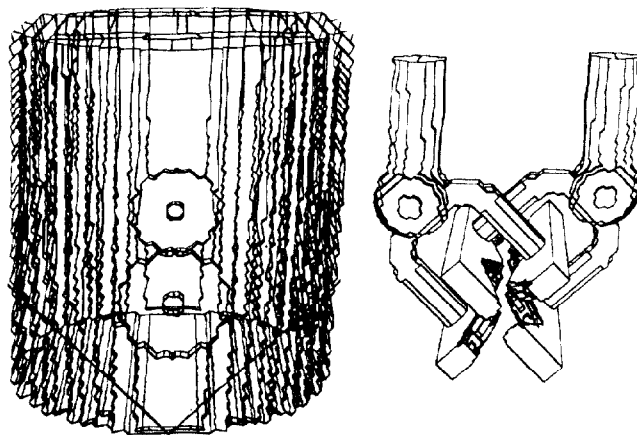


Figure 1: SRFQ outer tank (left), inner electrodes and support structure (right) as simulated by the MAFIA M3 code

modulation as well as the transverse geometry of the electrode tips. The electrodes are built in slices which are one mesh step thick and are modulated along the z direction. This modulation is defined differently for each slice to account for the transverse profile. The electrode geometry in the input file format for the mesh generator code M3 has been created by a program which computes the height and modulation of each slice.

The outer tank and the support tubes appear as inclined cylinders in the reference frame chosen. The code M3 is unable to define inclined cylinders. Therefore, these geometries have been defined by overlapping a few bricks with appropriate aspect ratios. The other components, such as the spheres, the connecting tubes and the beam ports are easy to simulate with the standard shapes available in M3. The result of this geometry simulation is shown in figure 1.

The M3 always makes structure boundaries shift to the closest mesh planes if the M3 input file doesn't define them on the mesh planes. The location of the mesh in the input must be defined very carefully in order to prevent (often unpredictable) distortion of the resonator. For the same reason, the proper position for a change in step size is on the boundaries of the structure. A high mesh density is needed in high field regions and where a high resolution boundary definition is called for. In this particular application a small mesh size was used in the beam region and around the tips of the electrodes. In this simulation the ratio between the largest and the smallest step sizes is 5.4. A smaller value is desirable, but the solution is still acceptable with this ratio.

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It is also advisable to retain the symmetry of the structure in the choice of mesh densities. Otherwise similar objects may distort into different shapes. Our experience shows that detailed drawings of the structure projections on all reference planes are necessary. These drawings should show the surfaces of the structure as well as the mesh lines. This procedure is quite laborious, thus we have developed a computer program which produces the necessary drawings. The use of the auto meshing routine in the M3 is not recommended for optimum placement of mesh planes in complicated structures such as this SRFQ.

The number of mesh points required for the geometry shown in figure 1 is 347,733 ($81 \times 81 \times 53$). The average mesh density is ~ 1 mesh point/cm³, the highest is 11.8/cm³ and the lowest is 0.5/cm³.

III. COMPUTATION AND RESULTS

Following the mesh generation we use the eigenvalue solver E31 to compute the electromagnetic fields. The E31 requires considerable memory space. Total running time depends on the availability of on-line memory. The E31 requires frequent access to large arrays, thus a lot of virtual memory storage results in excessive I/O activity. To run the E31 in fast mode with $\sim 350,000$ mesh points we need about 70 Megabytes of core memory. Since last publication [5] several improvements have been made to increase the precision of the solution. We have described the technique by which we generate the transverse profile of the electrodes. The application of this technique requires a higher mesh density in beam region and in electrode tip area. However, rounding the electrode tips leads to a better simulation of the structure. As a result, we observe an increase in the computed resonant frequency from 54.7 MHz to 56.5 MHz. The CPU time of the $\sim 350,000$ mesh point problem was about 1 hour on a CRAY 2.

The accuracy of the solution is also dependent on E31 input parameters. In this simulation the measure of the accuracy was $\nabla \times (\nabla \times \vec{E}) = 9.4 \times 10^{-11}$, $\nabla \cdot \vec{D} = 5.5 \times 10^{-14}$ and $\nabla \cdot \vec{B} = 9.1 \times 10^{-14}$ (MKS units). This precision has been obtained by using 10 resonant modes and optimizing the highest mode frequency in the computation. Other techniques [6] have also been used to improve the solution.

Table 1 lists the main electrical characteristics computed by MAFIA. The values from bead pulling measurement and those from approximate expressions derived from a lumped circuit model [5] are also given.

The experimental value of the capacitance is derived from an axial bead-pull measurement in a given cell of the SRFQ, using the following expression:

$$C_{total} = \frac{\pi}{2} a^3 \epsilon_0 k^2 A_{10}^2 (\Delta f/f)_{peak}^{-1}$$

where a is the radius of the metallic bead, $(\Delta f/f)_{peak}$ is the measured fractional peak frequency deviation, $k = 2\pi/\beta\lambda$ and A_{10} is taken as the theoretical two term potential value. Units are MKS. A better agreement should be obtained once we get A_{10} from the complete analysis of the bead-pull data.

Table 1. MAFIA Results vs Measurements and Approximate Expressions

Characteristic	MAFIA	Approx.[5]	Measure.
f (MHz)	56.493	63.4	57.372
$Q^{(1)}$	10400	8450	7200
C_{total} (pF)	41	45	53
$U^{(2)}$ (J)	3.6	3.9	4.7
Γ (Ω)	20.2	17.2	14.1
$E_s^2/U^{(3)}$ ($[MV/m]^2/J$)	72	62	40
$E_s^2/U^{(4)}$ ($[MV/m]^2/J$)	1.1	1.0	1.1
E_s/E_s	0.12	0.13	0.17
B_s^2/U (G^2/J)	7.4×10^1	3.3×10^1	3×10^1
$\Delta V/V$ (%)			
Ends	4.0	3.8	—
Centre	0.04	3.8	—

Notes:

- (1) For room temperature copper.
- (2) At a designed inter-vane voltage of $V=0.419MV$ [2].
- (3) At the middle of a SRFQ cell.
- (4) Includes transit time factor and fringe field effect.

IV. DISCUSSION

As we see in Table 1, the agreement between the MAFIA and the experiment in frequency is reasonable, considering the complexity of the structure. As mentioned above, rounding the sharp edges over the tips of the electrodes has increased the frequency by 1.8 MHz. Sharp corners lead to an anomalously high energy density which lowers the frequency. The present simulation still contains some sharp corners which do not exist in the real resonator. We estimate that by rounding the remaining electrode edges the MAFIA frequency will go up by 0.87 MHz to 57.37 MHz, in remarkable agreement to the measured value. This estimate is obtained by scaling the frequency change of 1.8 MHz by the ratio of the length of the tips and the electric energy density there to the length and energy density of the remaining sharp edges.

We can not explain the higher Q value and geometric factor Γ in MAFIA relative to the measurement. We note that similar discrepancies occur frequently between simulations and measurement. This may be the result of oxidation of the copper surface.

The electric unbalance $\Delta V/V$ [5] calculated from the MAFIA field distribution shows a difference between the ends of the electrodes and the center. This difference is due to transmission line effects along the electrodes. The approximate calculation does not include this effect.

The approximate analytical estimate for the total capacitance C_{total} in Table 1 also includes the contributions of the fringe regions and the support structure. A better estimate of the various capacitances has also improved the precision in the calculation of the $\Delta V/V$ as compared to a previous publication [5].

We also note that MAFIA calculates higher peak surface electric field E_s and magnetic field B_s than measured. This can be explained in part by sharp corners which appear in the simulation. There are several reasons for the sharp corners. First, the finite mesh density results in sharp corners at the mesh

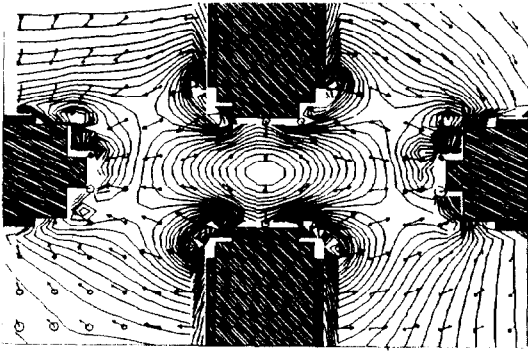


Figure 2: Electric field (arrows) and electric energy density (contour lines) distribution at a cross section of the electrodes in the center of the resonator.

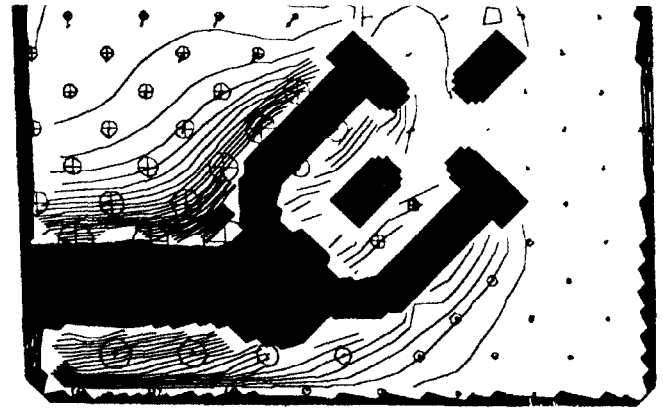


Figure 3: The magnetic energy density (contours) and magnetic field vectors around one of the support tubes.

cube boundaries. The second reason is that the available lattice construction elements (cylinders, spheres, blocks etc) can not match the real structure perfectly, so sharp corners may be created. For example, the support tubes, being inclined in the x-y plane had to be constructed of blocks. This resulted in sharp corners which enhance the peak surface fields. This enhancement can be estimated as approximately $\sqrt{2}$. For example, the value of B_n^2/U which appears under the MAFIA column in Table 1 is too high by approximately a factor of two. Once this correction is done the agreement becomes quite good.

The acceleration field E_a includes the transit time factor and the effect of the fringe fields. Some of the results of the MAFIA calculations are presented in another contribution to these proceedings [1]. This includes the electric energy density distribution on the beam axis and the normalized transit time factor curve. A comparison with an approximate analytic calculation and with the measurement is also shown in [1]. The electric field density has an interesting feature to which we have found no convincing explanation. The magnitude of the fringe field at high energy end of the SRFQ resonator is larger by about 20% than at the low energy end. This feature appears both in the simulation and the experiment.

Fig.2 shows the electric field and the electric energy density in the cross section of the electrodes at the center of the resonator. We expect the field to have a quadrupole symmetry, but we can see that the energy density distribution departs from this symmetry. This fact can be understood by observing the support structure as seen in Fig.1. The two pairs of U shaped tubes surround two of the four electrodes. Therefore the field on the electrodes adjacent to these tubes is changed by this asymmetric geometry. When the polarity of the electrodes is considered we find the changes are in expected direction. This may affect the peak surface electric field but should have a negligible effect on the beam dynamics. Electric unbalance $\Delta V/V$ (see Table 1) is also the result of the asymmetric internal geometry.

Another interesting feature seen in Fig.2 is that the peak energy density appears at a finite distance

off the surface. This is an artifact of the MAFIA algorithm, which works with fields rather than potentials. A field value on a node is averaged with values on adjacent nodes through the solution of Maxwell's equations in a finite-difference equations [7]. Thus the value of the field on a node which is near a metal surface is reduced by the influence of the vanishing field inside the metal.

Fig.3 shows the magnetic energy density contours and magnetic field vectors around one of the support tubes. The location of the peak surface magnetic field in this resonator is next to the joint of the support tube and the tank roof, facing the tank wall.

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