THREE-DIMENSIONAL SPACE CHARGE AND IMAGE CHARGE EFFECTS IN RADIO-FREQUENCY-QUADRUPOLE ACCELERATORS*

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Abstract

Image charges, combined with an appropriate space-charge treatment, are one method of handling simulation of the self-fields of a charged-particle beam in a beamline partially or totally enclosed by conducting surfaces. If current density is high and if surrounding conducting surfaces are not symmetric about the beam as in an off-center beam in a cylindrical beam pipe, or if the beam comes close to a conducting surface as it does in skimming the vanes of a radio-frequency quadrupole (RFQ), then image forces can be important. A new version of PARMTEQ with 3-D space charge and an approximate image charge treatment has been written and used to simulate several high-current RFQs. This paper explains the calculational method and gives simulation results.

I. INTRODUCTION

The percentage of input beam transmitted through high-current, high-brightness RFQs is not always calculated well by the standard version of PARMTEQ. Often the code prediction is higher than experimental results. There are physics effects that could reduce transmission that have not yet been included in the code. An experimental version of PARMTEQ has been written that includes two of these physics effects, image charges and 3-D space charge. It was thought that these particular effects might be important for high-current, high-brightness RFQs because of high charge density in the beam. This paper discusses the methods that were used to include 3-D space charge and image charges in PARMTEQ, and gives some results. Only the space charge treatment is different; accelerating and focusing fields are unchanged.

II. CALCULATIONAL METHOD

A. 3-D Space Charge

The 3-D space charge is similar to that used in a modified version of PARMILA that was written to accommodate the 3-D geometry of funnel design work. It is a point-to-point non-relativistic treatment in which space-charge forces on each particle are calculated by summing repulsive forces from all other particles. Singularities in the space-charge calculation are avoided by representing particles with charge clouds rather than points. Particles in as many as five leading and following bunches can be included. With no image charges, two or three neighbor bunches are all that is necessary; more than three do not change the result. The effect of neighbor bunches is reduced by image charges because the total charge of a bunch, integrated over all particles of the bunch itself and all resulting images, is zero. It was found that a single neighbor bunch (leading and following) is sufficient when both space charge and image charges are included.

B. Image Charges

The effect of conducting boundaries is added by the image charges, which combine with bunch charges to give the total non-relativistic electric field due to the beam itself. Every particle in the beam has a primary image in each vane. There are also an infinite number of higher-order reflections but they are ignored in the code for reasons explained later. An electric potential U is generated around the vane because of this image. A particle at a certain position, the "test point," will be affected by this potential which is caused by a particle, the "point charge," at another position. The approximate image-charge calculation uses the Green's function at a test point for U caused by the image of a point charge exterior to an infinitely long conducting cylinder:

$$U\,(a,r,z,\Phi) = \frac{-q}{2\pi^2\varepsilon_o} \sum_{m=0}^{\infty} 2_m \cos(m\Phi) \times \int_0^{\infty} \frac{I_m(kR)\,K_m(ka)\,K_m(kr)}{K_m(kR)} \cos(kz)\,dk \qquad (1)$$

where R is the cylinder radius;
  a and r are the radii of the test point and the point charge respectively, measured from the cylinder's axis;
  Φ and z are the angle measured at the cylinder's axis, and axial distance, between the test point and the point charge;
  I_m and K_m are modified Bessel functions;
  $2_0 = 1$, $2_{m>0} = 2$.

In the numerical sum and integration, the Bessel function order m goes from zero to 30 and the integral over the wave number k goes from k=0 to k=100 in gradually increasing steps starting with Δk=0.1. These values were found to produce sufficient accuracy in the calculation.

The cylindrical approximation to the vane-tip shape is reasonably good over a single cell but in many RFQs the vane-tip radius changes gradually along the length of the vane. This change is approximated in the code by using several cylindrical vane-tip radii R during the calculation and

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changing these radii in steps. In running a problem with the code, a four-dimensional grid for $U\ (a,r,z,\Phi)$ is set up for each vane-tip radius R. A new grid is read into the image-charge subroutine when R changes significantly.

The electric force on a test particle at radius a from the vane-tip axis due to the image of a charge at r is calculated by differentiating and interpolating the potential grid. The code sums over primary images (those due to first reflection) in all four vane tips but it does not take into account reflections of reflections. These higher-order reflections are much farther from the actual beam charges and they tend to cancel out because they have alternating signs and a decreasing amount of charge. Image forces on particles are strong close to a vane but decrease rapidly as distance increases; so secondary and higher-order reflections in and from other vanes will have much less effect than the primary images.

It is necessary to take vane modulations into account. This is done by stepping along the cell using (for these problems) six steps per cell. The vane surface is assumed to be parallel to the RFQ axis within each step with a vane radial distance from the RFQ axis that is the average of the actual vane radial distance over a step. The vane modulation advances 180° per cell so that a step represents a 30° advance. Because of vane modulation, the vane surface is not necessarily parallel to the axis as it would be in an unmodulated cylinder. However, during passage through two cells (a total of 360° of vane modulation) the angle at a particular radial distance averages almost to zero, so to a first approximation the effect of the non-zero surface angle is canceled.

Image-charge forces for each particle are summed over all the particles in the bunch and over corresponding particles in neighboring bunches. This is done for each of the four vanes. Finally the x', y' and energy of the particle is adjusted. This is done once per cell at the same time as the space-charge calculation.

III. RESULTS

Beam transmission percentages are shown in Table 1 for several PARMTEQ runs of 1000 particles each. Some actual RFQs studied were the ATS (Los Alamos Accelerator Test Stand), Chalk River RFQ1, and CERN RFQ2. Some unbuilt designs studied were two early versions of the SSC (Superconducting Super Collider) design (used only as an example; the final SSC design is considerably different), and the ATW (Accelerator Transmutation of Waste) design. In Table 1, input beams were matched and aligned unless otherwise noted.

Certain trends were seen in the results. Some are expected and consistent with effects that have been observed in PARMILA, or that might be expected from image charges. Trends that show up in the results are as follows:

1. The main effect of images is that beam transmission usually decreases. Particles close to the vanes are strongly attracted by image charges and are deflected out of the beam. There is a correlation between beam loss and beam charge density close to the vanes; as charge density increases, so does percentage of beam loss. The CERN RFQ did not show the transmission reduction effect, but this machine has a larger aperture than the other RFQs and the beam does not spend as much time close to the vanes.

Table 1.
Beam transmission percentages for various RFQs
with different space-charge and image-charge treatments

RFQ Identity and Input Current	Transmission
ATS RFQ, 100 mA	
2-D	89.8%
3-D, no images	92.5%
3-D, with images	88.8%
Experimental results [1]	~85%
Chalk River RFQ1, 90 mA	
2-D	83.1%
3-D, no images	86.1%
3-D, with images	74.0%
Experimental results [2]	~74%
CERN RFQ2, 220 mA	
2-D	91.0%
3-D, no images	85.1%
3-D, with images	85.2%
Experimental results [3,4]	~90%
SSC RFQ (early version 4-92), 30 mA	
2-D	90.1%
3-D, no images	93.0%
3-D, with images	90.7%
SSC RFQ (early version 5-92), 50 mA	
3-D, no images, misaligned beam	81.4%
3-D, with images, misaligned beam	75.6%
SSC RFQ (early version 5-92), 70 mA	
3-D, no images, misaligned beam	73.8%
3-D, with images, misaligned beam	56.9%
3-D, with images, aligned beam	59.9%
ATW RFQ, 140 mA	
2-D	90.6%
3-D, no images	89.7%
3-D, with images	74.8%

2. Transverse emittances and Courant-Snyder parameters α, β, and γ are not much affected by image charges. A high-current, high-brightness RFQ acts as a filter because the beam

fills the aperture for much of the RFQ's length. Consequently although image charges may reduce transmission and increase emittance in individual particles, output emittance change is small because lost particles are preferentially high-emittance ones.

3. The 3-D runs without image charges show a tendency toward slightly more transverse and slightly less longitudinal emittance growth than do the 2-D runs. The same tendency has been observed in PARMILA.

4. Bunch length and longitudinal emittance are usually slightly reduced by image charges. Particles at the ends of the bunch see less space-charge repulsion from the middle of the bunch because this repulsion is partially cancelled by the oppositely-charged images produced by the bunch center.

5. Misalignment or mismatch of the input beam, even if within tolerance, may cause some beam loss in the standard PARMTEQ code. Image charges tend to amplify this beam loss, probably because particles in the beam spend more time close to the vanes if the input beam is misaligned or mismatched than in a perfectly aligned and matched beam.

From the results of these few runs it is not obvious how beam loss changes with beam parameters or RFQ configuration, except that for a particular RFQ the percentage of beam loss increases with current or emittance. More study is required to understand the main factors responsible for differences in beam loss from one RFQ design to another.

High-current, high-brightness RFQs that might show a measurable image-charge effect are few, and beam measurements can be complicated and difficult. Where experimental transmission is less than the prediction of the standard PARMTEQ code, some of the discrepancy may be explained by image-charge effects such as those presented here in results from the modified code.

IV. CONCLUSIONS

The 3-D space-charge treatment made little difference in the PARMTEQ results. Image charges, on the other hand, caused significant beam loss in some cases. This effect should be taken into account in designing high-brightness, high-current RFQs. To this end, more theoretical, computational and experimental work is necessary to quantify the factors affecting image-charge beam loss.

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VI. REFERENCES

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