# Beam Loading in a High Current Accelerating Gap<sup>\*</sup>

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#### Abstract

Energy exchange between a high-current beam and a source at an accelerating gap is treated with a simple transmission line theory. There exists a matching condition for which the beam energy gain is equal to the source voltage. The total energy gain in a multigap system is expressed in terms of individual source voltages and the beam current.

#### I. INTRODUCTION

Recently, multigap high-current accelerators[1,2] have attracted considerable attention because of their potential applications in diverse areas. Unlike a low current beam in the conventional accelerators, a high-current beam accompanies a substantial amount of field energy. Thus, as a high-current beam passes through an accelerating gap, the energy transfer takes place not only from the source to the beam but also from the beam to the source. The latter is easily ignored in the low-current accelerator systems.

In this work, the energy exchange between a beam and a source at an accelerating gap is treated with a simple transmission line theory. The beam energy gained as it passes through the accelerating gap is expressed in terms of the source voltage, the beam current, and the characteristic impedance of the transmission line. There exists a matching condition at which the accelerating voltage is equal to the source voltage. The analysis is extended to a case where the accelerating gap is shunted with a resistor. The beam energy gained in a multigap accelerator system is expressed in terms of relevant parameters.

#### II. TRANSMISSION LINE MODEL

The interaction between a beam and an accelerating gap may be described with a discontinuity in a transmission line in which the beam terminates the end of the transmission line as shown in Fig. 1. As a pulse produced by a pulsed power source arrives the discontinuity, continuities are required of the voltage and current from the transmission line to the beam (Kirchhoff's voltage and current laws). We consider a case when a pulse of constant amplitude,  $V_S$ , from the source and a beam current of constant



Figure 1: Schematic representation of an accelerating gap and a high-current beam.

amplitude,  $I_B$ , are arriving at the gap simultaneously. The voltage and current of the pulse are related by  $V_S = I_S Z_0$  in the transmission line, where  $Z_0$  is the characteristic impedance of the transmission line. The boundary condition that the sum of currents at the discontinuity equals zero necessitates a reflected pulse  $I_-$  such that

$$I_S + I_- = I_B, \tag{1}$$

where the voltage of the reflected pulse is given by  $V_{-} = -I_{-}Z_{0}$ . The beam experience a accelerating voltage,  $V_{B}$ , which is the sum of voltages of the incident and reflected pulses appearing across the gap given by

$$V_S + V_- = V_B. \tag{2}$$

Eliminating  $I_{-}$  and  $V_{-}$  from Eqs. (1) and (2), one finds

$$V_B = (2I_S - I_B)Z_0. (3)$$

It is apparent from Eq. (3) that the voltage across the beam,  $V_B$ , which is the accelerating voltage, is not always equal to the source voltage  $V_S = I_S Z_0$ . The matching condition for which the accelerating voltage is equal to the source voltage,  $V_B = V_S$ , is only when

$$I_{Sm} = I_B \quad \text{or} \quad V_{Sm} = I_B Z_0, \tag{4}$$

i.e., the source voltage is equal to the beam current times the characteristic impedance. Under this condition, the full energy transfer takes place from the source to the beam. This result is illustrated in Fig. 2. It is interesting to note that when  $V_S = 0$ , Eq. (3) reduces to  $V_B = -I_B Z_0$ 

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Figure 2: Accelerating voltage vs source voltage.

i.e., when the beam passes through a gap which is not powered, the beam energy is lost by the amount of the beam current times the characteristic impedance of the transmission line. Also note that when the source voltage is one-half of the matched value,  $V_S = I_B Z_0/2$ , the net beam energy gain is zero.

In many practical accelerator systems, a shunt resistor is employed in the accelerating gap[1] as schematically shown in Fig. 3. It is straightforward to generalize Eq. (3) to the case that includes a shunt resistance R as

$$V_B = (2I_S - I_B)Z_{\text{eff}},\tag{5}$$

where the effective impedance is  $Z_{\text{eff}} = RZ_0/(R + Z_0)$ . The matching condition for which the accelerating voltage equals to the source voltage,  $V_B = V_S$ , is found to be

$$V_{Sm}/Z_0 = RI_B/(R - Z_0).$$
 (6)

The results are illustrated in Fig. 4. In this case the pulse energy from the source is completely absorbed by both the beam and the shunt resistor. It is obvious from Eq. (6) that the shunt resistance R must be greater than the characteristic impedance.

## III. MULTIGAP ACCELERATOR

In multigap accelerator, the total accelerating voltage in the system is the algebraic sum of individual accelerating voltages given by Eq. (5). Assuming identical accelerating gaps with shunt resistors and constant current through the accelerating gaps, the total beam energy in terms of individual source voltages is found by summing Eq. (5) with different  $I_{si} = V_{si}/Z_0$  as

$$V_{\rm tot} = \frac{2R}{R + Z_0} \sum_{i=1}^{n} V_{Si} - nI_B \frac{RZ_0}{R + Z_0},$$
 (7)



Figure 3: Schematic representation of an accelerating gap with a shunt resistor and a beam.



Figure 4: Accelerating voltage vs source voltage for a shunted gap.

where n is the total number of accelerating gaps. It should be noted that the total accelerating voltage is not the algebraic sum of individual source voltages. A few examples are noteworthy. If all of the accelerating gaps are not powered, the total accelerating voltage is negative,  $V_{\text{tot}} = -nI_B Z_{\text{eff}}$ . If one half of the gaps are not powered,  $V_S = 0$ , and the other half are powered with voltage,  $V_S = I_B Z_0$ , the net energy gain is zero. Only when every source has matched voltage,  $V_{Sm}$ , the total accelerating voltage is  $nV_{Sm}$ .

It should be noted that Eq. (5) is derived under the assumption of a constant beam current throughout the gaps such as in a relativistic electron beam accelerator. Therefore, Eq. (5) is not applicable to a low energy ion beam accelerator in which the beam current changes from gap to gap. It is also noted that since the present work assumes the beam current is a load to the transmission line, the results of present work are not applicable to a gap in which no beam is present such as that of the *voltage adder*[3].

## IV. Conclusions

The beam current as it passes through an accelerating gap has been treated with a discontinuity in a transmission line. There exists a matching condition for which the accelerating voltage is equal to the source voltage and pulse energy from the source is completely absorbed by the beam and the shunt resistor. The total beam energy gain in a multigap accelerator system is expressed in terms of individual source voltage and the beam current.

### References

- R. J. Briggs, D. L. Birx, G. J. Caporaso, T. J. Fesseden, R. E. Hester, R. Melendez, V. K. Neil, A. C. Paul, K. W. Struve, IEEE Trans. Nucl. Sci. NS-28, 3360 (1981).
- [2] M. G. Mazarkis, D. L. Smith, R. B. Miller, R. S. Clark, D. E. Hasti, D. L. Johnson, J. W. Poukey, K. R. Prestwich, and S. L. Shop, IEEE Trans. Nucl. Sci. NS-32, 3237 (1985).
- [3] T. W. L. Sanford, J. A. Halbleib, J. W. Poukey, G. T. Baldwin, G. A. Carlson, W. A. Styger, G. A. Mastin, T. Sheridan, R. Mock, J. A. Alexander, E. R. Brock, and C. O. Landron, J. Appl. Phys. 67, 1700 (1990).