

# B Factory Optics and Beam-Beam Interaction for Millimeter $\beta^*$ and Locally Shortened Bunches \*

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## Abstract

To achieve the enormous luminosity required for B factories it is necessary to increase the factor  $I\xi_v/\beta_v^*$ . We have investigated the possibility of decreasing  $\beta_v^*$ , using locally shortened bunches. The lattice and optics were designed to accommodate the CESR tunnel. The beam-beam interaction was simulated for the following conditions: finite bunch length, longitudinal beam-beam kicks, and crossing angle collision geometry. Estimations and simulations show that using a method of local bunch shortening, it is possible to design the "after  $10^{34}$ " generation of  $e^+e^-$  colliders with luminosities  $\sim 10^{35}$ .

## Introduction

The next step in the development of extra high ( $\sim 10^{35}$ ) luminosity for  $e^+e^-$  colliders can be made by decreasing both  $\beta_y^*$  and  $\sigma_L^*$  to the order of  $1\text{ mm}$ . This step seems to be expensive and technically difficult but not unrealistic. In this paper, we have tried to explore the practical possibilities of a theoretical design of local bunch compression [1] that permits us to have a normal bunch size,  $\sigma_L$ , outside of the IR.

The basic idea of local bunch compression is first to deliver a powerful kick to the particles, producing a horizontal angle  $\Delta x'(s)$  that depends on  $s$ , the longitudinal coordinate of the particle; and then to send the particles into a bending magnet, so that those with different  $s$  will move along different trajectories and will be focussed longitudinally. One needs for this a set of deflecting RF cavities and a rather large horizontal size  $\Delta x$  of all focussing elements:  $\Delta x \sim 10\sigma_L (R/\ell)$ , where  $\ell/R = \phi$  is the rotation angle in the magnetic field. The further transverse focussing into the IP also requires a set of large and powerful elements, not only because of a small  $\beta_y^*$ , but also because of a large horizontal emittance,  $\epsilon_x^*$ . In this design there is a chain of transformations between the entrance to and the exit from the interaction area:

$$\epsilon_L \rightarrow \epsilon_x^* \rightarrow \epsilon_L; \epsilon_x \rightarrow \epsilon_L^* \rightarrow \epsilon_x; \epsilon_L^* \ll \epsilon_L. \quad (1)$$

It is convenient and probably useful to design this system in such a way that bunches will have a disk-like shape

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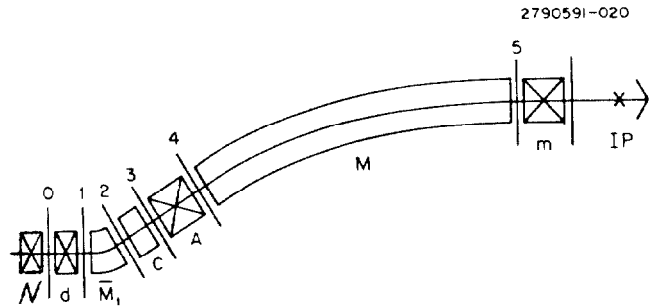


Figure 1: Lattice for Bunch Shortening

at the IP:  $\sigma_L^* \approx \sigma_x^*$  (and  $\sigma_L^* = \beta_y^*$ ). Such "disks" are almost insensitive to the crossing and crabbing angles. Besides that, if the lattice is designed properly in this system, the horizontal oscillations of the particles are almost insensitive to the beam-beam perturbations. The  $x$ -oscillations are perturbed only by longitudinal beam-beam kicks, which are relatively weak. Horizontal beam-beam kicks produce only longitudinal perturbations.

Our rough design and preliminary simulations (without taking into account the errors of the lattice) show that such a project is not unfeasible.

## Lattice for Bunch Shortening

Fig. 1 shows the lattice needed for local bunch shortening. Particles first pass the thin defocussing lens,  $d$ , which eliminates the dependence of  $\sigma_L^*$  on  $\sigma_x$ ,

$$\frac{1}{f} \approx \frac{Rb}{3L} \left( \frac{\ell}{R} \right)^2 \left( 1 + \frac{1}{2bR} \right). \quad (2)$$

where  $\ell/R = \phi$  is the angle of the bending magnet mentioned above,  $b$  is the full strength of the deflecting RF cavities  $C$  (fig. 1)

$$\Delta x' = bs, \quad b = \frac{2\pi U}{\lambda_{RF} \epsilon} = \frac{1}{L \sin \phi}, \quad (3)$$

and  $l$  is the strength of the triplet  $A$  (fig. 1) of quadrupole lenses  $FDF$  with phase shifts  $\pi/4, 3\pi/2, \pi/4$ , with  $2 \times 2$

matrices

$$A_x = \begin{pmatrix} 0 & L \\ -1/L & 0 \end{pmatrix}, \quad A_y = \begin{pmatrix} 0 & -Le^{-3\pi/2} \\ e^{3\pi/2}/L & 0 \end{pmatrix} \quad (4)$$

Here  $L = \exp(3\pi/2) \sqrt{\frac{BK}{\partial B/\partial R}}$ , and  $\bar{R}$  in (2) is the radius of the magnet (with an inverse field) next to lens  $d$ . This magnet eliminates the dependence of  $\sigma_L^*$  on  $\sigma_\varepsilon$ ; its (inverse) angle  $\bar{\alpha}$  and  $4 \times 4$  matrix  $\bar{M}$  are

$$\bar{\alpha} = \frac{\bar{\ell}}{\bar{R}} = \frac{R(\phi - \sin \phi)}{L \sin \phi} \ll 1, \quad (5)$$

$$\bar{M} \approx \begin{pmatrix} 1 & \bar{R}\bar{\alpha} & 0 & 0 \\ -\bar{\alpha}/\bar{R} & 1 & 0 & -\bar{\alpha} \\ \bar{\alpha} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

The main magnet,  $M$ , whose matrix is

$$M = \begin{pmatrix} \cos \phi & R \sin \phi & 0 & R(1 - \cos \phi) \\ -\sin \phi/R & \cos \phi & 0 & \sin \phi \\ -\sin \phi & -R(1 - \cos \phi) & 1 & -R(\phi - \sin \phi) \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

transforms  $x$  deviations (which now depend on  $s$  after a particle has passed through the triplet  $A$ ) into the deviations of  $s$ ,

$$\Delta s = - \int_0^z dz x(z)/R. \quad (8)$$

Finally, lenses  $m$  form the dispersion function  $\eta^*$  at the IP (taking into account a given focussing system between  $m$  and the IP), and lenses  $N$  form the dependence of  $\sigma_L^*$  on  $\sigma_x^*$ . (All sizes  $\sigma_x, \sigma_x^*, \sigma_s, \sigma_\varepsilon$  are given at the entrance of the lattice.)

Multiplying all matrices, including the  $4 \times 4$  matrix of the RF cavity,

$$C \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ b & 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

we get the matrix for the transition from the entrance of the lattice to the IP in the horizontal plane:

$$S = \begin{pmatrix} 0 & 0 & 0 & \eta^* \\ 0 & -r/\eta^* & -1/\eta^* & 0 \\ 0 & -r & 0 & 0 \\ 1/r & 0 & 0 & 1 \end{pmatrix} \quad (10)$$

$$S^{-1} = \begin{pmatrix} -r/\eta^* & 0 & 0 & r \\ 0 & 0 & -1/r & 0 \\ 0 & -\eta^* & 1 & 0 \\ 1/\eta^* & 0 & 0 & 0 \end{pmatrix}. \quad (11)$$

The remarkable feature of the  $S^{-1}$  matrix is that it cancels the influence of the horizontal beam-beam kicks  $(\delta x)^*$  on the  $x$ -movement ( $S_{21}^{-1} = S_{22}^{-1} = 0$ ). The kick  $(\delta x)^*$  itself depends on  $y^*$  and on the deviation  $\Delta p/p$  at the entrance

of the lattice, because, according to (10),  $S_{11} = S_{12} = S_{13} = 0$ ,  $S_{14} = \eta^*$ , and the  $x^*$ -coordinates at the IP depend only on  $\Delta p/p$ . Therefore, kick  $(\delta s)_{entrance}$  depends only on  $(\Delta p/p)_{entrance}$  and  $y^*$ . The  $x$  coordinates link with the longitudinal coordinates through relatively small longitudinal kicks  $(\Delta p/p)^*$  ( $S_{14}^{-1} = r \neq 0$ ). Four deflecting cavities with  $\lambda_{RF} = 0.6 m$  can give  $b \approx 1.5 \times 10^{-2} m^{-1}$ . With  $L \approx 100$  condition (3) gives  $\sin \phi \approx 0.65$ . When  $R = 40 m$ ,  $\ell = 26 m$ . When  $\epsilon_x = 10^{-7}$  and  $\epsilon_y = 0.25 \times 10^{-9}$ ,  $\sigma_x < 1 cm$  and  $\sigma_y < 1.5 mm$  over the entire lattice. If  $\sigma_L = 0.5 cm$  and  $\sigma_\varepsilon = 5.5 \times 10^{-4}$ , then  $\epsilon_x^* = 2.75 \times 10^{-6}$ ;  $\sigma_x^* = 1 mm$ , and  $\sigma_y^* = 5 \times 10^{-7} m$ .

## Interaction Region Optical Design

There are two very challenging problems in designing interaction region optics with  $\beta_y^* \sim 1 mm$  controlling the vertical chromaticity, and providing aperture for a large horizontal emittance. High chromaticity causes a reduction of the range of energies with stable focussing. Indirectly, it causes a reduction in the dynamic aperture through increased sextupole strengths that are required to compensate for the chromaticity. High chromaticity is basic to all millimeter  $\beta_y^*$  optics for B factories; focussing magnets simply cannot be made strong enough or fit close enough to the interaction point. The vertical beta function grows so rapidly with distance from the interaction point ( $\beta_y \approx s^2/\beta_y^*$ ) that the contribution to the vertical chromaticity,  $\Delta \xi_y = \frac{1}{4\pi} \int K \beta_y ds$  becomes very large. Peak  $\beta_y$  of 500  $m$  or more are unavoidable given a 'detector stay clear' cone of half angle 0.3 radians, an interaction point beam pipe radius of  $\approx 2 cm$ , and the limits of magnetic materials.

The problem of finding an adequate aperture for high horizontal emittance is specific to the scheme described in this paper, where the horizontal emittance is locally enlarged in order to shorten the bunch length. An example of an engineered conceptual design of magnetic elements and vacuum chamber that would provide a 1  $mm \beta_y^*$  is given in figure 2. The horizontal stay clear criterion is  $X_{STAYCLEAR} [m] = 10\sigma_H + .005\sqrt{\beta_x [m]}/40$  and is the same as that used in the Cornell B factory design. The vertical stay clear criterion is  $Y_{STAYCLEAR} = X_{STAYCLEAR}$ . The latter was chosen because the large local horizontal emittance dominates the aperture. In this interaction region design, the contribution to vertical chromaticity from each side of the interaction point is  $-62$ . With such a large chromaticity, adequate single beam stability is doubtful. There are a couple of ideas that could be used to reduce the effect of the chromaticity. One is to put sextupole components on the windings of the superconducting quadrupoles. Although there is no conventional  $\eta_y^*$ , the effect of the bunch compression optics would correlate energy with horizontal position and could be used to cancel the chromaticity tuneshift locally. The other idea is to "tune out" the chromaticity using a series of optimized

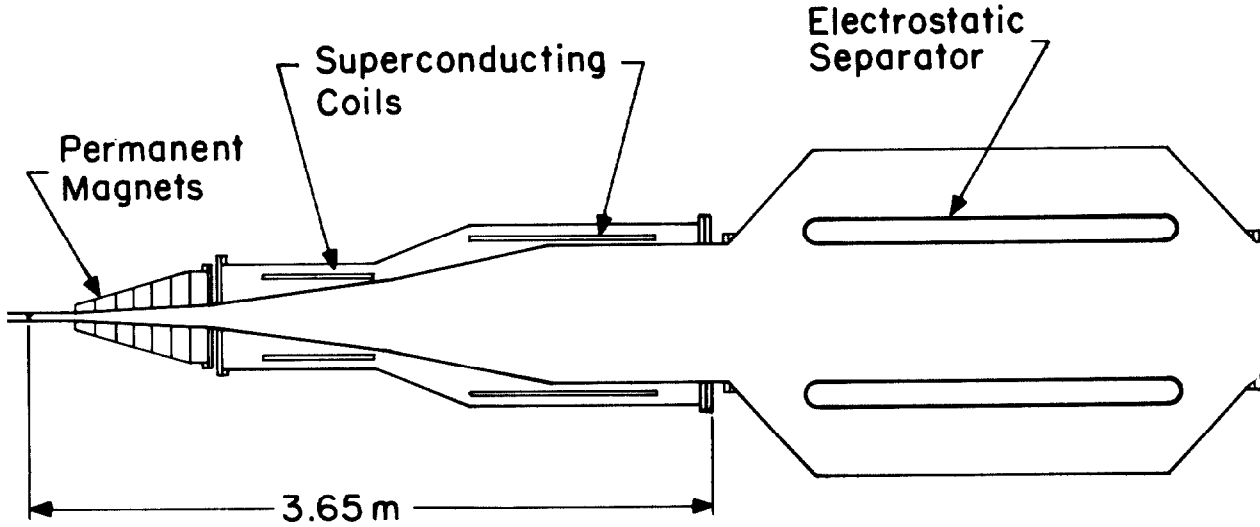


Figure 2: The interaction region quadrupoles and vacuum chamber are designed for  $1 \text{ mm } \beta_y^*$ , a crossing angle of  $\pm 16.6 \text{ milliradians}$ , and a (local) horizontal emittance  $\epsilon_x = 2.75 \times 10^{-6} \text{ m}$ . The electrostatic separator is used to further separate the beams into separate vacuum chambers.

quadrupoles.

## Program Description and Simulation Results

In the simulation we have used  $10^3$  “particles” per bunch executing  $5 \times 10^3$  turns. Only the symmetric case (equal energy beams) was considered. When treated as the “strong” beam, every bunch was divided into several equally charged slices. We have developed the basic program used in [2] for the flat beam in the following respects:

(a) Strong-strong collisions, in the approximation of unperturbed bunch shapes during a collision. In reality, the relative perturbation of vertical bunch size during one collision is about  $10^{-3}$ .

(b) Longitudinal kicks,  $\delta\epsilon/\epsilon$ . They are essential in this design even without a crossing angle because of the relatively large angles  $\sigma_{x'}^*$ ,  $\sigma_{y'}^*$ . In the presence of the crossing angle  $\theta \ll 1$  (without crabbing), we simply use the angle  $(x' + \theta)$  instead of  $x'$  when calculating the longitudinal kick. Neglecting quadratic terms  $(\delta x')^2$  and  $(\delta y')^2$ , we have

$$\frac{\delta\epsilon}{\epsilon} = \frac{1}{2} [(x' + \theta) \delta x' + y' \delta y']. \quad (12)$$

(c) Crossing angle. In addition to  $\delta\epsilon/\epsilon$ ,  $\theta$  changes the horizontal distance between the “weak” particle position  $x$  and the position  $x_c$  of the center of the strong bunch slice, at the moment of collision with this slice:

$$\Delta = (x - x_c) = x_{ip} + s^{(i)} (x'_{ip} + \theta) - \bar{s}_c \alpha, \quad (13)$$

where  $s^{(i)} = (s_{ip} - \bar{s}_c)/2$ ,  $x_{ip}$ ,  $x'_{ip}$ , and  $s_{ip}$  are coordinates of the particle in the “weak” bunch system,  $\bar{s}_c$  is the  $s$ -coordinate of the slice in the “strong” bunch system, and  $\alpha$

$\nu_h$	$\nu_v = 0.66$	$\nu_v = 0.67$	$\nu_h$	$\nu_v = 0.68$	$\nu_v = 0.69$
.70	$K = 0.74$ $\mathcal{L} = 6.79$ $L = 1.0$	$K = 0.71$ $\mathcal{L} = 6.5$ $L = 0.97$	.66	$K = 0.74$ $\mathcal{L} = 6.96$ $L = 1.0$	$K = 0.59$ $\mathcal{L} = 0.54$ $L = 0.8$
.71	$K = 0.73$ $\mathcal{L} = 6.93$ $L = 1.0$	$K = 0.75$ $\mathcal{L} = 6.98$ $L = 1.0$	.67	$K = 0.79$ $\mathcal{L} = 7.08$ $L = 1.0$	$K = 0.61$ $\mathcal{L} = 5.7$ $L = 0.85$

Table 1: Examples of stable regions.  $\mathcal{L}_0$  is the initial luminosity;  $K = \mathcal{L} (10^{32}) / \mathcal{L}_0 (10^{32})$ .  $L = \mathcal{L} n_B (10^{35})$ ;  $n_B = 150$ ;  $I_B = 20 \text{ mA}$ ,  $I = 3 \text{ A}$ .  $E = 5.3 \text{ GeV}$  (both beams); crossing angle  $\theta = 33 \text{ mradians}$ .

is the crabbing angle;  $\alpha = 0$  in our case. For the short disk-like bunch,  $\sigma_{L'}^* \approx \sigma_x^*$ , we have  $\sigma_{L'}^* \theta \ll \sigma_x^*$ , so the influence of the crossing angle on  $\Delta$  is very small.

(d) Feedback dipole corrections. We have included such corrections in the program because without them some dipole beam-beam instabilities could develop.

The main preliminary result of the simulations is that the tune map of the luminosity,  $\mathcal{L} = \mathcal{L}(\nu_x, \nu_y)$ , in this design is at least as good as the maps of usual colliders.

## References

- [1] Y. Orlov, “Bunch Length Compression Using Crab Cavities”, *Proceedings of the Berkeley B Factory Workshop* (1990).
- [2] S. Krishnagopal and R. Siemann, “Bunch-Length Effects in the Beam-Beam Interaction”, *Phys. Rev. D* **41** (1990), 2312.