Low Momentum Compaction Lattice Study for the SSC Low Energy Booster

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Abstract

To avoid emittance growth from transition crossing it is desirable that the Low Energy Booster have a transition energy well above its extraction energy or even imaginary. A number of lattices have been designed with this feature. Examples will be given along with the general principles underlying their design. One of the lattices has been tentatively chosen as reference design.

I. Introduction

The SSC injector system consists of a linac and a chain of three booster synchrotrons. The first of these, the Low Energy Booster (LEB), will accelerate protons from 1.22 GeV/c to 12.5 GeV/c with a repetition rate of 10 Hz. The magnetic field and gradient are 1.3 T and 14.9 T/m, respectively, and the circumference is to be in the 500 m to 600 m range. The ring will contain proton bunches spaced at 5 m intervals, each containing 10^{10} protons for the nominal luminosity of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. However, the machine must be capable of higher intensities for accelerating test beams.

In order to maximize performance and reliablility, the lattice should have the following properties:

- Transition energy γ_t either imaginary or well above the extraction energy;
- Low or zero dispersion function η in the straight sections;
- Adequate straight sections for rf, injection, extraction, etc.;
- Provision for acceleration of polarized beams.

In addition, the lattice should satisfy conventional requirements such as linear behaviour up to amplitudes of 4σ (for a test-beam emittance four times the nominal emittance of 0.6π mm-mr), moderate peak values of the orbit functions, and an area in tune space large enough to encompass the space-charge tune spread.

These design goals are not all obviously compatible. This paper contains a brief account of recent LEB lattice studies.

II. Obtaining high γ_t

Three methods have been explored for raising γ_t: these might be labeled the high-tune, harmonic, and modular methods.

The high-tune approach is to design a simple FODO-cell ring with sufficiently large cell number and phase advance to raise the tune to the value desired for γ_t , since these quantities are nearly equal in such lattices. This method leads to higher chromaticity and, at fixed circumference, to small peak dispersion values, both of which drive up sextupole strengths, possibly compromising single particle stability (cf. Talman [1]).

The harmonic approach involves enhancing a higher-order Fourier component of the momentum compaction α. The expression for α is given in [2]:

$$\alpha = \frac{\nu^3}{R}\sum_k \frac{|\alpha_k^2|}{\nu^2 - k^2},$$

where α_k^2 is nonzero only for $k = 0$ and k equal to multiples of the periodicity P of the quantity $\beta^{3/2}/\rho$. If ν is made close to, but less than, P the momentum compaction will be reduced and can even be made negative. This can be done in a FODO lattice by superposing a P^{th} harmonic perturbation in the gradient or bending distribution [3]. Two problems with this approach are that the η oscillations can be very large and that it is not obvious how to include dispersion-free straight sections.

The modular approach is to design simple modules or supercells that give negative η in some or all of the bending magnets. Sets of these modules are combined into an arc. This approach is convenient for conceptually separating the design of the supercells from that of the straight sections, each with desired characteristics. A drawback is that it sometimes leads to lattices with numerous magnet types.

III. Obtaining low η in the straight sections

We have constructed lattices with zero η in the straight sections in two ways. One way is to make the horizontal matrix of each arc be unity, leading to an integer tune across the arc. As a result the dispersion function will be identically zero in the straight sections.

The second way is to make the arc out of modules with unconstrained phase advance, bordered by dispersion suppressors. The suppressors should be designed so as not to raise the momentum compaction by an unacceptable amount.

IV. Polarized Beams

A polarized beam will tend to be depolarized as it crosses the energies corresponding to depolarizing resonances. These are of two kinds: intrinsic resonances, induced by betatron oscillations, occur at

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$$\gamma G = kP \pm \nu_y$$

with the relativistic energy factor γ, the magnetic moment anomaly $G = (g-2)/2$ (=1.7928 for protons), any integer k, the lattice periodicity P and the vertical betatron tune ν_y. A high periodicity will limit the number of intrinsic resonances to be crossed. Imperfection resonances are due to closed-orbit excursions and occur at integer values of the "spin tune," which measures the number of spin revolutions per turn.

A method used successfully to overcome intrinsic resonances is the tune jump. A set of very fast pulsed quadrupoles jumps the tune across the resonant value in approximately 1.5 μs, followed by a slow decay of the quadrupoles in about 3 ms [4]. Problems arise if the tune jump required is too large or the resonances follow each other too closely.

Imperfection resonances can be corrected by programmed orbit correctors, allowing different corrector settings as each resonance is crossed. A more elegant method has been developed in the form of a partial Siberian snake [5]. For LEB energies this would be a 4 Tm solenoid, rotating the spin by 10°...18°.

V. Examples

In this section we present four examples, each illustrating one or more of the design methods discussed above.

A. SCDR Lattice

The first example, demonstrating the high-tune approach, is the lattice designed by Y. Y. Lee for the SSC Site-Specific Conceptual Design Report [6]. This ring is made up of 54 FODO cells. Missing magnets give the ring six superperiods, one of which is shown in Figure 1. The

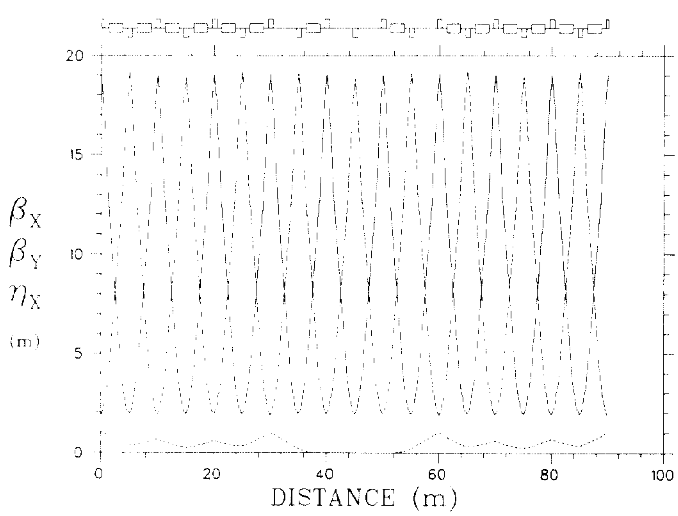

Figure 1: Lattice functions of the high-tune lattice.

number of cells and the phase advance per cell, 112°, together conspire to raise γ_t to 14.5, somewhat above the extraction energy, while the tune is 16.8. The missing-magnet half cells make straight section space for injection, extraction and rf, and their distribution serves to give nearly zero dispersion in the center empty cell of the superperiod. This design shows a very straightforward way to make a non-transition crossing LEB. The main reasons to explore other possibilities were that still higher γ_t values are desirable, and these could be made with this approach only by making the ring too crowded or too large in circumference.

B. The $-I$ Straight Section Lattice

This lattice is a simple example of the modular approach. The ring is made up entirely of modules, each consisting of two bending cells and two empty cells, the latter each having 90° phase advance. The two empty cells constitute a straight section with a $-I$ matrix, which transforms the momentum vector from $(0, \eta')$ at one end to $(0, -\eta')$ at the other, forcing η to have negative values in the bending cells (moreover, they are invisible to the β-functions of the bending cells). Hence α is negative and γ_t is $i7.4$. Figure 2 shows the half-superperiod of a three-

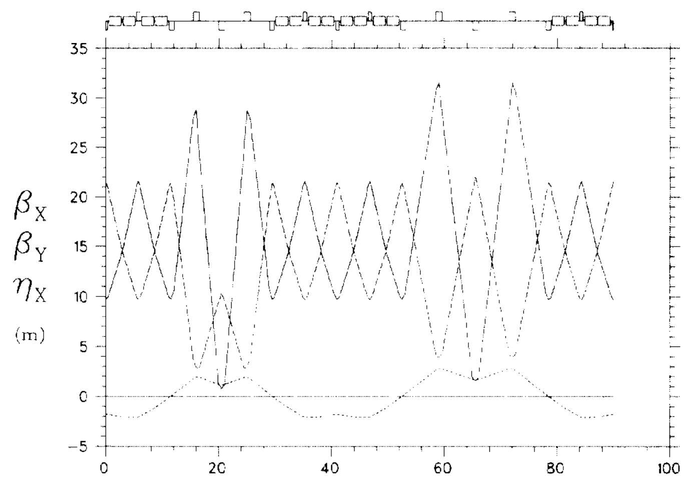

Figure 2: Lattice functions of the $-I$ straight-section lattice.

superperiod ring containing two module types that differ in the length of their empty cells. The short ones serve the purpose of reversing η' only, while the long ones are also used for injection, extraction and rf. Lattices of this type do not have zero η in the straight sections. Similar lattices have been devised by D. Trbojevic [7].

C. Lattice with Low-α Modules and Dispersion Suppressors

Here a more complex example of the modular method is given. The ring has three superperiods, each with a zero-η straight section and an arc. Figure 3 shows half of one reflection-symmetric superperiod; the left end is at the center of the straight section, the right end at the center of the arc. The half arc shown has a dispersion suppressor on the left, and one low-α module on the right. The module,

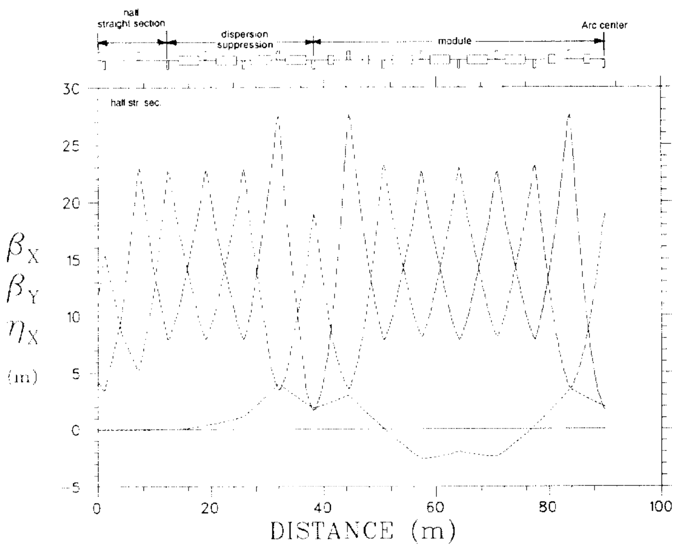

Figure 3: Lattice functions of the low-α lattice.

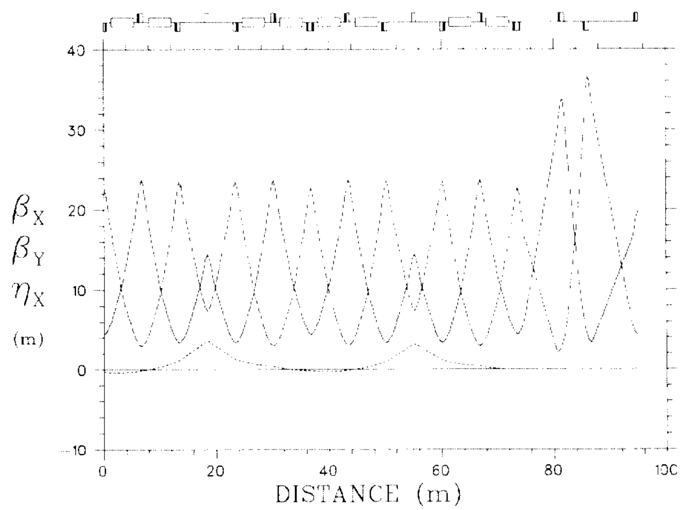

Figure 4: Lattice functions of the 4^{th} lattice example.

also symmetric, has four cells. The short cell at each end has about 90° phase advance and short dipoles, the two longer cells in the center each have 60° phase advance and long dipoles. This combination causes η to be negative on the average. The module has a phase advance $5/6 \times 2\pi$ in each plane. The sextupoles in one arc module are thus at a phase interval from the corresponding ones in the other, causing cancellation of the third-order resonances.

The dispersion suppressor has the same focusing structure as a half module, but the lengths of the dipoles in the two cells are different from the corresponding ones in the module. Unlike more standard suppressors in this one the bending is *increased* rather than decreased about a point 90° away from its end, to prevent η from becoming negative. Because the suppressor has the half-module structure, it has the same β-functions as the module.

The half straight section has four quadrupoles, which are adjusted to produce a waist at the center and give the ring the desired tune, 8.8. The γ_t value of this ring is 21.3.

D. *Lattice with reduced number of spin-resonance crossings*

The last example (Fig. 4) shows a threefold symmetric lattice designed using the harmonic method. The arcs by themselves are a FODO lattice with a superiodicity P of 12, introduced by the missing-magnet cells and enhanced by a modulation of the quadrupole strength. They are similar to structures proposed by Senichev *et al.* for the Moscow Kaon Factory [8]. The total horizontal phase advance is $9 \times 2\pi$, and thus, due to its reflection symmetry, each arc is a second-order pseudo-achromat in the horizontal plane, which suppresses *eta* in the straight sections. The insertion points for the straight sections are chosen to minimize $\hat{\eta}$ in the arcs. The straight sections are tuned to give the desired fractional tune in the horizontal plane. The same principles were used before in the design of some of the TRIUMF KAON Factory lattices [9].

In the vertical plane the arcs are used to achieve the desired fractional tune, while the straight sections are unit sections with a phase advance of 2π each. Thus they are transparent to both the particle orbit and the spin; the apparent symmetry w.r.t. first-order intrinsic depolarizing resonances is restored to be 12-fold, and only four such resonances are in the acceleration range. These are fairly evenly spaced and calculations show they can be crossed with reasonable tune-jumps of less than 0.2 in magnitude. The straight sections allow for inclusion of a partial snake.

The nominal tune of the machine is 11.60; γ_t was chosen to be about 22. The lattice can be tuned to any working point within the rectangle in tune space defined by $10.9 \leq \nu \leq 11.9$ in either plane.

This lattice has been tentatively adopted as reference design for the LEB.

VI. References

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