Heating of the LSS Wiggler Beam Tube Due to Induced Surface Current

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I. INTRODUCTION

During 1990, The Boeing Company was designing a high-power, free-electron laser, referred to as the Laser Subsystem (LSS). Among the considerations determining the wiggler beam tube cooling requirements, was the heating caused by induced surface currents flowing on the inside of the beam tube.

The expected heating is calculated here. Additionally, the effect of pulse shape and length on the heating is investigated.

Two methods are used to evaluate the power dissipated in the LSS wiggler beam tube. The first is an expansion of methods used in reference 1. In this method, the beam charge is assumed to be uniformly distributed in a cylinder of length l and radius r_0. The beam center is taken as the z axis. The moving beam induces currents in the beam tube wall, resulting in the heating of the wall. This heating is given by

$$P(\zeta) = 2\pi\sigma \int_{b}^{\inf} E_z^2(r,\zeta) r dr \tag{1}$$

Here ζ = z-ct, z = longitudinal distance, t=time, r=distance from beam tube center, E_z=longitudinal component of the electric field, b=beam tube radius and σ = conductivity. It is assumed that the radial part of the E-field is unimportant.

Following reference 1, E_z and the charge distribution are Fourier transformed.

$$E_z(r,\zeta) = \int_{-\inf}^{\inf} \tilde{E}_z(r,k) e^{ik\zeta}\, dk \tag{2}$$

$$\rho(r,\zeta) = \int_{-\inf}^{\inf} \tilde{\rho}_z(r,k) e^{ik\zeta}\, dk \tag{3}$$

ρ=charge distribution and the wiggle indicates the Fourier transform. These are inserted into Maxwell's equations, and a new set of equations results. These are solved for E_z in three regions $r<r_0$, $r_0<r<b$ and $r>b$. The solutions are Hankel and Bessel functions, but only their asymptotic forms are needed. Boundary conditions are applied and E_z in the metal wall is determined.

Inserting this result for E_z and equation (2) into (1), simplifying, performing the integration over r, and averaging over the pulse length, one obtains

$$P = \frac{8\pi\sigma q^2 \beta^4}{b(RL)^2} \sum \sqrt{2Rk_n} \left\{ \frac{\sin[k_n(L+l)/2] - \sin[k_n(L-l)/2]}{k_n l} \right\}^2 \tag{4}$$

$$R = 4\pi\beta\sigma/c \quad k_n = \pi n/L \quad \beta = v/c \tag{5}$$

Here P=average power dissipated per unit length, q=charge per bunch and L=separation between bunches.

The second method is largely based on eq 1.55 from reference 2. This is the energy loss of beam through an arbitrary, transverse-charge multiple moment. It is assumed that all particles travel at c. For a cylindrically symmetric beam, only the zeroth moment contributes and it reduces to

$$P = 2\pi c^2 f \int_{-\inf}^{\inf} d\omega \left|\tilde{\rho}(\omega)\right|^2 R_e Z_0''(\omega)/D \tag{6}$$

$$\tilde{\rho}(\omega) = \frac{1}{2\pi c} \int_{-\inf}^{\inf} e^{-i\omega z/c} \rho(\rho) dz \tag{7}$$

Here $\rho(z)$ is the longitudinal charge distribution and fL=c. $Z_0(\omega)/D$ is the zeroth order longitudinal impedance of the beam pipe per unit length. In general, Z is a difficult quantity to evaluate. However, for a smooth, cylindrical beam pipe, this can be easily evaluated by combining equations 1-48, 1-36, 1-14, 1-7 and 1-9 of Ref. 2. The result is:

$$\frac{Z_0''}{D} = \frac{2}{cb} \frac{1}{\sqrt{\frac{2\pi\sigma}{c|k|}} + i\left[\sqrt{\frac{2\pi\sigma}{c|k|}}\, s_k - \frac{kb}{2}\right]} \tag{8}$$

s_k=sign of k, k=ω/c

The only approximation used in deriving the above result is that wavelengths of about 1km or greater are ignored. The predominant wavelength will be concentrated around distances which describe the beam-pulse structure. These are about a cm and shorter, so the neglected part of the impedance should be unimportant.

II. SOFTWARE

A short program was written to evaluate the terms of the expression for P of method 1. It was found that 40,000 terms allow evaluation of the beam power to a few percent.

The program for method 2 accepts input specifying the beam-pipe characteristics and the charge distribution. One possible way of specifying the charge distribution is by reading it in from a file. A different option instructs the software to generate a gaussian which can be symmetrically cut off at any distance from the centroid. An additional option allows the first gaussian to be continued by another gaussian of different width. The two gaussians are matched in slope and vertical height at the cut in the first gaussian.

A fast Fourier transform (FFT) of up to 65536 points is performed on the charge distribution. The number of points and the spatial interval used for the transform can be specified as input. Once the transform is done, a Simpson's rule integration is used to obtain the power dissipation.

III. RESULTS

The following input was used in all calculation, unless otherwise noted:

5.76mm = beam tube radius
3.43×10^7/(ohm.m) = conductivity of aluminum
10nc = microbunch charge
27.088 MHz = micropulse rate

Figure 1 shows the PARMELA-predicted[3] charge distribution at the entrance to the LSS wiggler. Using method 2, the calculated heating will be 90 watts/m. The numerical integration is a possible source of error. To check how serious this might be, the integration is done using different Fourier spatial intervals (S), and different numbers of points (N). Some results are shown in Table 1. If N is kept at 65536, the calculation should be accurate to better than 1%.

TABLE 1 Effect of number of points (N) and Fourier interval (S), on calculated results.

S(m)	N	Heating (w/m)
.1	4096	90.23
1	4096	90.84
10	4096	95.13
.1	65536	90.23
1	65536	90.87
10	65536	90.10

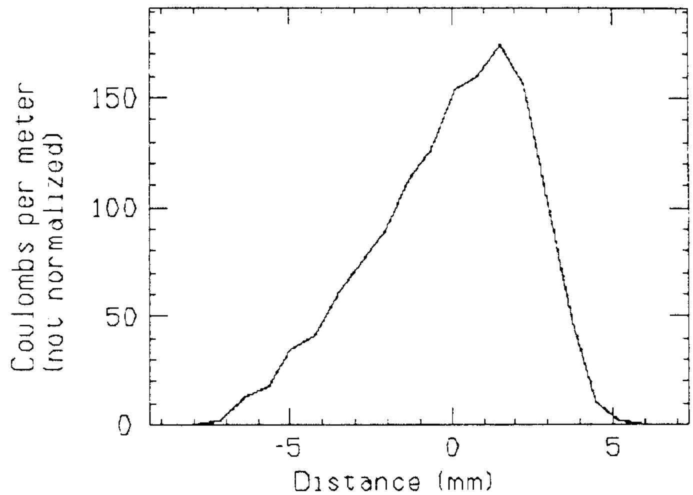

Figure 1. LSS wiggler longitudinal charge density.

Consider the imaginary term in equation (8). Letting

$$\frac{kb}{2} = \sqrt{\frac{2\pi\sigma}{c|k|}}$$

one gets $k = 6.74 \times 10^4$/m for beam-pipe parameters used here. The r.m.s. width of the charge distribution used above is a = 2.43mm. Taking ka = 1 gives $k = 4.12 \times 10^2$mm. Thus if one uses a gaussian charge distribution of r.m.s. width around 2.43mm, the kb term in the impedance can be ignored. When this is done, equation (6) can be integrated to give (in MKS):

$$P = 1.27 \times 10^8 \frac{q^2 f}{b\sigma^{1/2}\, a^{3/2}}$$

This gives 85.9w/m at a 27.088Mhz pulse rate. The program of method 2 gives the same result for the same gaussian distribution. This verifies that the program is working correctly. It also indicates that -3/2 scaling of the bunch length can be used to estimate power dissipation for LSS-type beam pulses.

To investigate the effect of pulse shape, figure 2 was produced. The lowest point at 85.3 watts is generated using a gaussian with the same r.m.s. width (a = 2.43mm) as the PARMELA prediction. The gaussian FWHM = 5.71mm. The remaining points are generated by using a wide central gaussian (a = 30mm). It is cut off and a second gaussian is appended to give the indicated rise and fall length. The FWHM is kept at 5.71mm. All calculations use a spatial interval of 5cm and 65536 points. Because of the short, spatial interval, low-frequency components are underestimated and all results are about 1% low.

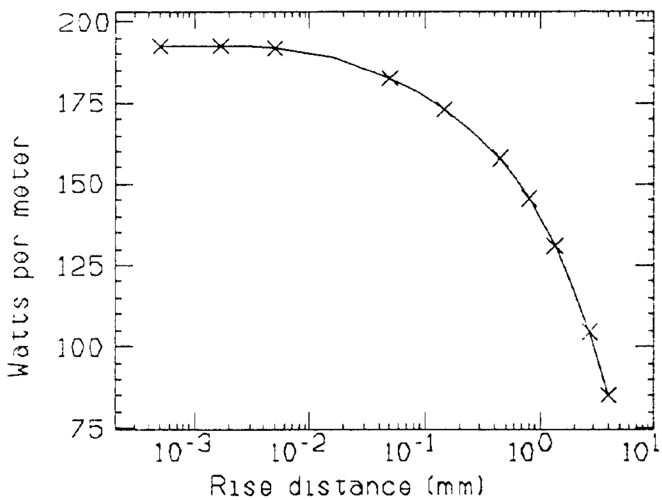

Figure 2. Dissipated power vs rise distance.

The interesting features of this graph are that the power dissipation increases until a rise distance of about .01mm is reached. For shorter distance, the dissipation does not increase. This can be understood in the following way. It is shown in reference 2 that in a beam bunch, charges separated less than about

$$d=x^{1/3}b \qquad x=1/(b\sigma\mu_0 c)$$

the leading charge repels the trailing charge, while the trailing charge has no effect on the leading charge. Thus, the trailing charge loses energy. If the trailing charge is further behind than d, it is attracted to the leading charge, and thus gains energy. A charge well within a long, uniform cylinder of charge will be about equally repelled and attracted by leading charges, and thus, will gain or lose little energy. This is approximately the situation inside gaussian distribution, whose width is much greater than d. For a finite cylinder of charge with zero-rise and fall distances at the two ends, charges within a few times d of the ends will be strongly decelerated. These charges will lose orders of magnitude more energy than the charges in the cylinder interior.

For the situation considered here, d = .014mm. If the rise distance is long compared to this, all charges will lose small amounts of energy. As the rise distance is reduced, more charges will have large, decelerating forces and power dissipation increases. If rise distance is short compared to .014mm, then no more increase in power loss should be expected.

Figure 3 shows the heating as a function of pulse length for the cylindrical pulse shape of method 1. Also shown, are calculations done with method 2, for a pulse-rise distance of .0005mm. The solid curve is $Aa^{-3/2}$, where A was adjusted so the curve falls near the data for both methods. This indicates that the $a^{-3/2}$ dependence arrived at using the gaussian pulse shape is approximately valid for a rectangular charge distribution. Since the rectangular shape is drastically different from the gaussian distribution, it seems reasonable to use a $-3/2$ scaling to estimate beam-tube heating for most pulse shapes.

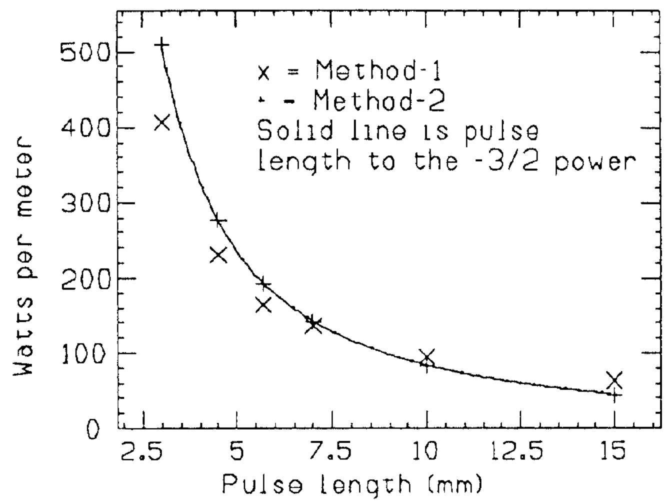

Figure 3. Dissipated power vs pulse length for a cylindrical charge bunch.

IV. CONCLUSIONS

The expected heating in the LSS wiggler beam tube will be 90 watts per meter. If the pulse rise and fall times are shortened, the heating will increase, but this increase will be limited to a factor of three. If the pulse length is shortened, the increase in heating can be estimated using a -3/2 scaling. Thus, a 10% shortening of the pulse length will result in a 15% increase in beam-tube heating.

It would be conceivable to have much greater heating than 200 watts per meter if the beam pulse becomes chopped up with many, rapidly rising and falling edges.

V. REFERENCES

(1) P.L. Morton, V.K. Neil, A.M. Sessler, *App. Phys.*, 37, p. 3875, 1966.
(2) A.W. Chao "SLAC-PUB-2946", June, 1982.
(3) B. McVay, Los Alamos National Laboratory, Private Communication.