# Beam transport to the SXLS ring 

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## Abstract

Design of a transport line with highly restrictive space and beam size requirements and with flexibility in the orientation of the injector is presented. An analytic method, based on symbolic algebra is used to find optimal regions of the design parameters. The fine tuning of the parameters were then performed with a standard matrix program. Special interest was paid to the question of dispersion vector matching large angle bend with very small bending radius using non-iron bending magnets with very small gap area.

## 1. Introduction

The SXLS ring [1] is a superconducting compact synchrotron/storage ring of 8.5 m circumference, runing at 696 MeV and providing $\lambda_{\text {critical }}=10 \mathrm{~A}$ for x -ray lithography. It is a two superperiod machine consisting of two superconducting combined function bending magnets, four warm quadrupoles and four warm sextupoles. Injector is a 200 Mev LINAC [2]. Injection takes places at one of the straight sections immediatelly before the ring quadrupole (see Fig. 1). It has to fit into the small space left by the beam line, the cryostat and other elements of the densely packed compact ring. On the other hand, the presence of iron close to the superconducting ring dipoles has to be avoided.

The transport line consists of a triplett, followed by an achromatic bend ( $29^{\circ}$ ) with horizontal/vertical waist in the middle to allow matching of $1 \%$ momentum spread from the LINAC to the $0.1 \%$ momentum spread in the ring. This is followed by a quintuplett, which basically serves to match the phase space ellipses into the ring, and finally a total of $151^{\circ}$ bend. This last bend consists of two combined function bending magnets with small radius of curvature ( $\mathrm{B} \geq 2 \mathrm{~T}$ ) and a septum. There is also a quadrupole between the two bending magnets, separated by drift spaces from them. This last $151^{\circ}$ bend will bring the beam parallell with the stored heam as well as match the dispersion and its derivative into the ring. The high field bending magnets and quadrupole will be pulsed non-iron magncts. The design, as shown on Fig. 1, is such, that by reversing the direction of a $29^{\circ}$ achromatic bend in the transport line, the LINAC can be lined up either with the long axis of the ring or with the closest beam line, thus making future installation flexible.

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## 2. Geometry considerations

The septum magnet is in one of the straight sections immediatelly before the ring quadrupole. From the values of the machine parameters at that point of the ring, the matching conditions can be obtained.

It is desirable to limit the presence of iron in the proximity of the superconducting coils of the $180^{\circ}$ ring dipole. The only element, where iron is needed (to confine the flux) is the pulsing septum. The maximum achievable bending field in the septum is $1.1-1.3 \mathrm{~T}$, and this dictates a minimum bend of $9^{\circ}$ to clear the vacuum pipe.

To establish the geometry of the injection into the ring let us imagine to proceed in the reverse direction, from the septum towards the linac, bending the beam to the right, away from the ring. After the septum (in the reverse direction), the minimum bending field, in order to clear elements in the ring as well as the cryostat itself, is $2.5-2.0 \mathrm{~T}$, the lower field corresponding to the higher operating field in the septum. The larger the bending field is, the more compact the final injection can be made and the easier it is to clear all ring elements as well as the lithography beam lines. From purely geometrical considerations, to keep as far as possible from the lithography lines, $B=3 T$ would be preferable to $2 T$. Calculations in the following sections will be shown for both, $B$ $=2$ and 3 T .

The minimum total bend (including the septum) is $122^{\circ}$, dictated by the closest lithography beamline. Instead, we chose this bend to be $151^{\circ}$ for the following two reasons. Since the SXLS machine is a prototype, it will be more flexible if the design allows future installations at the customer's site with two different linac orientation. This can be achicved if the magnitude of the total final bend $\left(\theta_{\mathrm{tox}}\right)$, and the momentum spread matching achromatic bend ( $\theta_{\text {acht }}$ ), are chosen such that: $\theta_{\text {wot }} \pm$
$\theta_{\mathrm{achr}}=180^{\circ}$ or $122^{\circ}$. These conditions yield $\theta_{\text {ot }}=151^{\circ}$ and $\theta_{\text {achr }}=29^{\circ}$. The second reason is that, as it will be discussed later, the larger the total bend is, the easier is to match the dispersion vector into the ring.

## 3. Dispersion matching bend

The final $151^{\circ}$ bend has to provide dispersion vector matching into the ring besides bending the injected bcam parallell with the stored beam. As a consequence, the bend after the septum (still in reverse direction) is broken into two bending magnets with a horizontally focusing quadrupole between them. The strength ( $\mathbf{k}_{\mathrm{q}}$ ) and Iocation (i.e. the $\theta_{3}, \theta_{4}$ split of the total $\theta=\theta_{\text {tot }}$ $\theta_{\text {sequam }}=142^{\circ}$ bend) of the quadrupole is chosen to match the $\eta_{T}=\eta_{T}{ }^{\prime}=0$ in the transport line with $\eta_{R}=-1.2, \eta_{\mathrm{R}}{ }^{\prime}$ $=-.6$ in the ring. One would like to make the bend as compact as possible (no or very short drift spaces and short quadrupole) in order to clear the closest lithography beam line. It turns out, however, that it is impossible to match the dispersion vector without drift space with either $\mathrm{B}=3 \mathrm{~T}$ or with $\mathrm{B}=2 \mathrm{~T}$.

Consider a horizontal composite bend (vertical can be treated similarly), consisting of two bending magnets with bending ficlds $B_{3}$ and $B_{4}$, bending angles $\theta_{3}$ and $\theta_{4}$ $\left(\theta_{3}+\theta_{4}=\Theta\right)$ and ficld indices of $n_{3}$ and $n_{4}$ with a horizontally focusing quadrupole between them. The quadrupole is separated from the dipoles by $\mathrm{L}_{3}$ and $\mathrm{L}_{4}$ drift spaces. The length and the strength of the quadrupole is $l_{q}$ and $\mathrm{k}_{\mathrm{q}}\left(=\mathrm{B}^{\prime} / \mathrm{B} \rho\right)$. Calculating the standard second order transport matrix of the bend as:

$$
M=M^{B_{2}} M^{L_{1}} M^{Q} M^{L_{2}} M^{B_{1}}
$$

the resulting $\eta$ and $\eta$ ' will be:
$\eta=\eta_{a}+q_{12} b_{11} \eta_{b}^{\prime}+q_{21}\left(L_{1} \boldsymbol{\eta}_{b}^{\prime}+\boldsymbol{\eta}_{b}\right)\left(b_{11} L_{2}+b_{12}\right)+$

$$
\begin{equation*}
\mathrm{q}_{11}\left[\mathrm{~b}_{11} \eta_{\mathrm{b}}+\left(\mathrm{b}_{11}\left(\mathrm{~L}_{1}+\mathrm{L}_{2}\right)+\mathrm{b}_{12}\right) \eta_{b}^{\prime}\right] \tag{1a}
\end{equation*}
$$

$$
\begin{align*}
\eta^{\prime}= & \eta_{\mathrm{a}}+\mathrm{q}_{12} \mathrm{~b}_{21} \eta_{\mathrm{b}}^{\prime}+\mathrm{q}_{21}\left(\mathrm{~L}_{1} \eta_{\mathrm{b}}^{\prime}+\eta_{\mathrm{b}}\right)\left(\mathrm{b}_{21} \mathrm{~L}_{2}+\mathrm{b}_{22}\right)+ \\
& \mathrm{q}_{11}\left[\mathrm{~b}_{21} \eta_{\mathrm{b}}+\left(\mathrm{b}_{21}\left(\mathrm{~L}_{1}+\mathrm{L}_{2}\right)+\mathrm{b}_{22}\right) \eta_{\mathrm{b}}^{\prime}\right] . \tag{1b}
\end{align*}
$$

In eqs.(1) $\mathrm{q}_{\mathrm{ij}}$ are the quadrupole matrix elements and $\mathrm{a}_{\mathrm{ij}}$, $\eta_{a}$ and $\eta_{a}^{\prime}$ are the matrix elements representing the $B_{1}$ dipole. The b index refers to the $\mathrm{B}_{4}$ dipole.

Eqs.(1) can be written in a simple format if $L_{1}=L_{2}=0, \rho_{3}=\rho_{4}, n_{3}=n_{4}$ and if the thin lens approximation for the quadrupole is valid $\left(q_{11}=q_{22}=1, q_{12}=0\right.$ and $\left.\mathrm{q}_{21}-\delta--1 / \mathrm{f}_{\mathrm{x}}\right)$ :

$$
\begin{align*}
& \eta=\frac{\rho}{h^{2}}\left\{1-\delta \frac{\rho}{h}\left[1-C\left(h \theta_{3}\right)\right] S\left(\mathrm{~h} \theta_{4}\right)-\mathrm{C}(\mathrm{~h} \boldsymbol{\theta})\right\}  \tag{2a}\\
& \eta^{\prime}=\frac{1}{\mathrm{~h}}\left\{-\delta \frac{1}{\mathrm{~h}}[1-\mathrm{C}(\mathrm{~h} \Theta)] \mathrm{S}\left(\mathrm{~h} \theta_{4}\right)+1-\mathrm{C}(\mathrm{~h} \Theta)\right\} \tag{2b}
\end{align*}
$$

$\eta$ was calculated from cq.(1a) at $B_{3}=B_{4}=3$ and $2 T$, and is plotted on Figs. 2 as a function of $k_{q}$ for $L_{1}=L_{2}=0$, $n_{3}=n_{4}=.5, \Theta=142^{\circ}, l_{4}=.15 \mathrm{~m}$ and different values of $\theta_{4}$.


Figs. 2: $\eta$ as a function of the quadrupole strengths for different values of $\theta_{4}$ at (a) $B=3 \mathrm{~T}$ and (b) $\mathrm{B}=2 \mathrm{~T}$.
One can see, that there is a minimum value of the $\eta\left(\mathrm{k}_{\mathrm{q}}\right)$ function which can be reached with a given set of parameters. The minimum and the location of the minimum ( $\mathrm{k}_{\mathrm{q}}$ ) sligthly depends on the gradient in the bending magnet but strongly depends on the bending field, on the $\theta_{3}, \theta_{4}$ split of the total bending angle and on the quadrupole lengths: It can be shown, that (a) lower bending field or larger field index help in lowering $\eta_{\text {min }}$ and $\mathrm{k}_{\mathrm{q}}$, (b) longer quadrupole results in weeker quadrupole, but increases $\eta_{\text {min }}$, (c) smaller $\theta_{4}$ results in strongly lower $\eta_{\text {min }}$, and slighty weeker quadrupole, and (d) $\eta$ (and $\eta^{\prime}$ ) has a minimum as a function of $\Theta$, and this minimum is reached at $180^{\circ}$.

Let us now consider the conditions of dispersion vector matching. The eqs.(1) nonlinear equations have to be solved for the strength ( $\mathrm{k}_{\mathrm{q}}$ ) and location $\left(\theta_{4}\right)$ of the quadrupole with a given set of the remaining eight parameters; $\Theta, \rho_{3}, \rho_{4}, n_{3}, n_{4}, l_{4}, L_{1}$ and $L_{2}$. First, we will explore the most compact geometry, when $\mathrm{L}_{1}=\mathrm{L}_{2}=0$. The solutions can be seen on Figs.3, where $\mathrm{k}_{\mathrm{q}}$ and $\theta_{4}$ are plotted as a function of the $B$ bending field with $n=.0$ and .7 field indices and $\mathrm{l}_{\mathrm{q}}=.15 \mathrm{~m}$ quadrupole length. One can scc, that already at $B=1.5 \mathrm{~T}$ strong quadrupole ( $\mathrm{k}_{\mathrm{q}}>10$ ) is needed. For higher bending fields, drift spaces have to be allowed. On Figs. $4, \mathrm{k}_{\mathrm{q}}$ and $\theta_{4}$ are plotted as a function of the $\mathrm{L}_{1}$ drift space with $\mathrm{L}_{2}$ as parameter and with $\mathrm{B}=3 \mathrm{~T}$, $\mathrm{I}_{\mathrm{q}}=.15, \Theta=142^{\circ}, \mathrm{n}_{3}=.8, \mathrm{n}_{4}=.6$.


Figs.3: Location $\left(\theta_{4}\right)$ and strengths $\left(k_{q}\right)$ of the matching quad is shown as a function of the $B$ bending field for $\Theta=142^{\circ}, l_{q}=.15 \mathrm{~m}$. and $\mathrm{n}=.0, .7$

In addition to matching the beam into the ring, the beam size have to fit within the very small aperture ( $\approx 1.0-1.5 \mathrm{~cm}^{2}$ ) of the pulsing bending magnets. The beam size, divergence and orientation of the phase space ellipse are given by the matrix clements as:

$$
\mathrm{x}=\sqrt{\Sigma_{\mathrm{x}, 11}}, \quad \mathrm{x}^{\prime}=\sqrt{\Sigma_{\mathrm{x}, 22}}, \quad \alpha=\frac{\mathrm{r}}{\sqrt{\left(1-\mathrm{r}^{2}\right)}}, \text { where } \mathrm{r}=\frac{\Sigma_{\mathrm{x}, 12}}{\sqrt{\Sigma_{\mathrm{x}, 11} \Sigma_{\mathrm{x}, 22}}}
$$

and similar expressions for the vertical case. The $\Sigma$ matrix transforms as

$$
\Sigma=\mathrm{M} \Sigma^{0} \mathrm{M}^{\mathrm{T}} .
$$

Proceeding in the reverse direction, that is starting with the desired horizontal and vertical phase space ellipses at injection, the $\Sigma$ matrix was calculated before the $\mathrm{B}_{3}$ bending magnet using the previously obtained solutions for the strengths and location of the dispersion matching quadrupole. The calculated H and V beam sizes are also plotted on Figs. 4 as a function of $L_{1}$ with $L_{2}$ as parameter for $\Theta=142^{\circ}, B=3 T, 1_{q}=.15 \mathrm{~m}$ and $\mathrm{n}_{1}=\mathrm{n}_{2}=.6$. One can see, that only solutions with $\mathrm{L}_{2} \geq .2 \mathrm{~m}$ and $L_{1} \geq .1 \mathrm{~m}$ yield reasonably small horizontal and vertical beam sizes at the same time.


Figs. 4: $\theta_{4}, k_{q}$, and the horizontal/vertical beam sizes, calculated to match the dispersion vector are shown as a function of $\mathrm{L}_{1}$, while $\mathrm{L}_{2}$ is used as parameter. $\Theta=142^{\circ} \mathrm{n}_{3}=.8, \mathrm{n}_{4}=.6$, and $\mathrm{l}_{\mathrm{q}}=.15 \mathrm{~m}$.
One can see from the above analysis, that at $B=3 T$ either the beam size can be kept small $\left(\approx 1 \mathrm{~cm}^{2}\right)$ or all beam lines can be cleared, but it is difficult to satisfy both criteria at the same time. The solutions which clear the beam lines require stronger quadrupoles (therefore the resulting beam size is larger). By increasing the length of the quadrupole the strengths is decreasing, (but of course, at the same time the length of the bend is also increasing). By lowering the bending field to $\mathrm{B}=2 \mathrm{~T}$, weeker ( $\delta \approx 1 / \rho$ ) quadrupole is needed to match the dispersion vector, and therefore the horizontal focusing/vertical defocusing will be less. However, this is more then compensated for by the increase in the length ( $1 \approx \rho$ ) of the bending magnet. As a net result, the $\sigma_{x, y}$ beam size will even slightly increase. However, the gap area of the noniron dipoles can be increased by the same ratio as the bending field is decreased.

In Table-1, three cases of the $142^{\circ}$ bend are presented. The first case cuts one beam line, the second barely and the third safely avoids all of them.

Table-1
Parameters of different dispersion matching bends with $\Theta=142^{\circ}$.

| $\mathrm{L}_{1,2}$ <br> m | $\mathrm{B}_{3,4}$ <br> T | $\theta_{4}$ | $\mathrm{n}_{3,4}$ | $\mathrm{B}^{\prime}$ <br> $\mathrm{T} /$ | $\mathrm{l}_{\mathrm{Q}}$ <br> m | $\sigma_{\mathrm{x}}$ <br> cm | $\sigma_{\mathrm{y}}$ <br> cm | $(3 \sigma)^{2}$ <br> $\mathrm{~cm}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $.10-.3$ | 3 | 48 | $.8-60$ | 29 | .25 | .28 | .43 | 1.08 |
| $.15-.2$ | 3 | 58 | $.8-.60$ | 51 | .15 | .34 | .38 | 1.16 |
| $.10-.2$ | $2-3$ | 62 | $.8-.55$ | 54 | .15 | .42 | .45 | 1.70 |

## 4. Achromatic bend / Momentum dispersion matching

The $29^{\circ}$ achromatic bend serves to match the original $1 \%$ momentum dispersion to that of $.1 \%$ in the ring. It consists of two $14.5^{\circ}$ bending magnets operating at .582 T bending field ( $1=29 \mathrm{~m}, p=1.146 \mathrm{~m}$ ) and two 15 cm long quadrupoles separated from each other by 30 cm to accomodate the aperture (on which the beam is focused). The quadrupoles are separated from the dipoles by 25 cm drift spaces.

The beam is focused at the middle of the achromatic bend with the help of the $\mathrm{Q}_{1,23}$ quadrupole triplett after the linac.

## 5. Phase space ellipse matching

After the achromatic bend, a quadrupole quintuplett is used to match the horizontal and vertical phase space ellipses from the slit through the second half of the achromat and through the $151^{\circ}$ dispersion matching bend into the ring.

## 6. Beam envelope and accuracy of matching

The location and the strengths of the quadrupoles as well as the beam envelope through the whole transport line for the three cases presented in Table-1 are shown on Figs. 5. The quadrupole strengths are somewhat different for the three cases, but the quadrupole locations were kept the same. In all three cases the match of the dispersion vector and the phase space ellipses into the ring are good to the third decimal place The dotted line on Fig. 5 b shows the effect of $\delta \mathrm{p} / \mathrm{p}=1 \%$ in the $29^{\circ}$ achromatic bend and the effect of the $\delta p / \mathrm{p}=.1 \%$ in the $151^{\circ}$ final bend. The dispersion and its derivative is shown of Fig. 5d. The effect of $\delta_{p} / \neq \neq 0$ and the behaviour of $\eta$ and $\eta^{\prime}$ are very similar for all cases.


Figs.5: Horizontal and vertical beam envelopes through the whole transport line, for cases presented in Table-1.

## 7. References

[1] J.Murphy et all.,"Comissioning of the SXLS ring at Brookhaven", These Proceedings.
[2] P.Letellier et all., "Comissioning of the 200 MeV Injector Linac", Proc. IEEE Conf. on Accelerator Physics, 1989.


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