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Lattice Properties of the Phase I BNL X-Ray Lithography Source Obtained from Fits to Magnetic Measurement Data*

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Abstract

The orbit, tune, chromaticity and β values for the Phase I XLS ring were computed by numerical integration of the equations of motion using fields obtained from the coefficients of the 3-dimensional solution of Laplace's Equation evaluated by fits to magnetic measurements. The results are in good agreement with available data. The method has been extended to higher order fits of TOSCA generated fields in planes normal to the reference axis using the coil configuration proposed for the Superconducting X-Ray Lithography Source. Agreement with results from numerical integration through fields given directly by TOSCA is excellent. The formulation of the normal multipole expansion presented by Brown and Servranckx has been extended to include skew multipole terms. The method appears appropriate for analysis of magnetic measurements of the SXLS.

1. Introduction

The Phase I X-Ray Lithography Source (XLS) is a conventional magnet prototype of the compact Superconducting X-Ray Lithography Source (SXLS) under construction at BNL. The two 180-degree bending magnets of these storage rings incorporate dipole, vertically focusing quadrupole, and defocusing sextupole (SD) fields as well as unavoidable higher order multipole components. The expected lattice properties of the XLS were obtained by Murphy and Vignola[1] using a conventional combined function bending magnet model. Subsequently we performed more detailed calculations using numerical integration of the equations of motion by a 4th order Runge-Kutta technique (program ORBIT) through fields generated by the TOSCA program[2] with dipole and quadrupole components similar to the CDR model. The resulting tunes, betatron functions and synchrotron radiation integrals[3] were in remarkably good agreement with the earlier work. We have subsequently used program ORBIT with TOSCA generated fields from more complex conductor configurations to evaluate the lattice properties of the SXLS.[4] However, a major obstacle in the use of ORBIT has been the excessive computer time required by TOSCA. The CPU time constraint has also precluded the use of ORBIT in multiturn tracking studies. Finally, we required a method to obtain the field components directly from the magnetic measurements data.[5] We therefore developed a procedure to obtain the expansion coefficients for the scalar potential following the lucid presentation of Brown and Servranckx[6] (BS). The XLS magnetic measurements of median plane B_z transverse to the reference axis were fitted by least squares at 101 longitudinal positions along the axis for the m = 0 coefficients; higher order coefficients were generated by a recursion relation up to 2nd order. The procedure was confirmed by fitting to TOSCA generated "data" and comparing to lattice functions produced by ORBIT using TOSCA. The values for orbit, tune and chromaticity calculated for the XLS are in reasonable agreement with initial measurements.[7] The method has been extended to obtain all expansion coefficients by performing two dimensional least squares fitting of vertical component (B_z) simulated data in planes normal to the reference axis. Lattice properties obtained by ORBIT using TOSCA generated fields and fields obtained from the fit coefficients are in excellent agreement. We have also extended the BS formulation to include the asymmetric (skew) solution of Laplace's equation with excellent fits to simulated data.

2. Formulation of Three Dimensional Solution for Field

The solution of Laplace's Equation in curvilinear coordinates (x normal to the central trajectory) which has odd parity $\phi(x,z,s) = -\phi(x,-z,s)$ and thus yields a solution for B_z which is symmetric about the median plane z = 0 is given by BS,

$$\phi(x,z,s) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+1,n}(s)\,\frac{x^n}{n!}\,\frac{z^{2m+1}}{(2m+1)!} \tag{1}$$

where $h=1/\rho(s)$ and $\rho(s)$=radius of curvature. The coordinate system is illustrated in the preceding paper[4] on a central trajectory about the XLS reference axis. The B_z component is given by BS from Eq.(1) and $\vec{B} = \vec{\nabla}\phi$ as

$$B_z = \frac{\partial\phi}{\partial z} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+1,n}(s)\,\frac{x^n}{n!}\,\frac{z^{2m}}{(2m)!} \tag{2}$$

To 2nd order, where $\dot{A} = dA/ds$, BS gives,

$$B_x = A_{11}z + A_{12}xz \tag{3a}$$

$$B_z = A_{10} + A_{11}x + A_{12}x^2/2! + A_{30}z^2/2! \tag{3b}$$

$$B_s = (\dot{A}_{10}z + \dot{A}_{11}xz)/(1 + hx) \tag{3c}$$

The coefficients with m > 0 are written in terms of the m = 0 coefficients by use of the recursion relation given by BS from substitution of Eq.(1) into Laplace's Equation. In particular, $A_{30} = -(\ddot{A}_{10} + hA_{11} + A_{12})$.

The even parity solution to Laplace's Equation $\phi(x,z,s) = \phi_E(x,-z,s)$ can be written as

$$\phi_E(x,z,s) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+2,n}(s)\,\frac{x^n}{n!}\,\frac{z^{2m+2}}{(2m+2)!} \tag{4}$$

and leads to expressions for the field components in which B_z is antisymmetric about the z=0 plane.

$$B_x^E = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+2,n+1}(s)\,\frac{x^n}{n!}\,\frac{z^{2m+2}}{(2m+2)!} \tag{5a}$$

$$B_z^E = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+2,n}(s)\,\frac{x^n}{n!}\,\frac{z^{2m+1}}{(2m+1)!} \tag{5b}$$

$$B_s^E = \frac{1}{(1+hx)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \dot{A}_{2m+2,n}(s)\,\frac{x^n}{n!}\,\frac{z^{2m+2}}{(2m+2)!} \tag{5c}$$

Again, a recursion relation can be derived by substitution of Eq.(4) in Laplace's Equation. It is

$$-A_{2m+4,n} = \ddot{A}_{2m+2,n} + nh\ddot{A}_{2m+2,n-1} - \dot{n}hA_{2m+2,n-1} \tag{6}$$

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$$+ A_{2m+2,n+2} + h(3n+1)\, A_{2m+2,n+1} + nh^2(3n-1)\, A_{2m+2,n}$$
$$+ nh^3(n-1)^2 A_{2m+2,n-1} + 3nh\, A_{2m+4,n-1} +$$
$$3nh^2(n-1)\, A_{2m+4,n-2} + nh^3(n-1)(n-2)\, A_{2m+4,n-3}$$

For ORBIT we use a cylindrical coordinate system (r,θ,z) shown in the preceding paper [4]. Further, we assume that the central trajectory is sufficiently close to the reference axis that we can evaluate the components of Eq.(2) and (5) relative to that axis. B_z is the same in both systems. The transverse components required by ORBIT are, for $y > y_c$

$$B_r = B_x \cos(\theta - \phi) + B_s \sin(\theta - \phi) \qquad (7a)$$
$$B_\theta = B_s \cos(\theta - \phi) - B_x \sin(\theta - \phi) \quad \text{for } Y > Y_c \qquad (7b)$$

3. Ring Parameters from 3-D Expansion Coefficients

For the XLS analysis we first tested the above procedure by generating "data" from TOSCA for the same grid used in the magnetic measurements and for a particular conductor configuration designated FEB16. Vertical field components in the median plane, $B_z(x,0,s)$ were computed at 101 positions along the reference axis and transversely at 13 points in the interval $-27 \le x \le 27$ mm. The transverse data were fitted by a least squares program out to 4th order in x to obtain the coefficients A_{10}, A_{11}, A_{12}, A_{13}, and A_{14} at each azimuth. The transverse field components B_x and B_s were evaluated only to 2nd order as in Eq.(3). The coefficients were fitted longitudinally using a cubic spline routine to obtain the derivatives $\dot{A}_{10}$, $\dot{A}_{11}$, and $\ddot{A}_{10}$ of Eq.(3). The tables of spline coefficients were incorporated into a subroutine, MIKE613, which replaced TOSCA in ORBIT and the resulting orbit, tune, chromaticity and β values are compared in Table I to similar numbers produced by ORBIT with TOSCA generated field components. The agreement of the central orbit position and other parameters related to small amplitude oscillations about the central orbit such as tune, β, and horizontal chromaticity are excellent. The vertical chromaticity is off by about 10%. Higher order chromaticities are in poor agreement and suggest that we need higher order terms in B_x and B_s to accurately calculate the tunes for off momentum orbits far from the central trajectory. We also verify in Table I that increasing the number of data points transversely and longitudinally by a factor of two does not significantly affect the results.

The same procedure outlined above was applied to the magnetic measurements data. The resulting equilibrium orbit shown in Fig. 1 indicates that the magnet is too long on either end by about 4 mm, i.e. the orbit position at $\theta = \pi/2$ is about 4 mm inside the reference axis when the electron trajectory starts on the reference axis at the straight-section quadrupole QF. Alternatively an outward dispacement of the orbit initial coordinate at QF by 4 mm, i.e. an offset of the QF quadrupole axis, gives an orbit on the reference axis at $\theta = \pi/2$. This displacement has been made in the machine and the resultant closed orbit deviation is now within ±1 mm. The tune vs. momentum calculation is shown in Fig.2 and indicates that the central vertical tune about the offset equilibrium orbit is $\nu_{V0} = 1.317$ when the horizontal tune is fixed at the $\nu_{H0} = 1.415$ design value. This value for ν_V is significantly lower than the design value of 0.415. Measurements of ν_V with the correction poleface quadrupole windings set to nearly zero give $\nu_V = 0.34$, in good agreement with the calculation. Similarly, chromaticity measurements with the correction sextupole poleface windings set to nearly zero give $\xi_H = 1.79$ and $\xi_V = 1.94$, in reasonable agreement with the predicted value of corrected (SF on) chromaticity $\xi_v^c = \xi_v^c = 1.779$.

4. Three-Dimensional Representation of SXLS Field

The results of the above analysis indicate that we require higher order terms to accurately represent the field far from the central orbit. Also we will require accurate m > 0 coefficients to reproduce the field components far from the median plane for multiturn tracking. Inspection of the recursion relation given by BS for the normal (odd-parity) potential and Eq.(6) for the skew solution shows that the expressions for these coefficients become rapidly more complex with increasing order. Moreover, higher order introduces higher derivatives; 6th derivatives d^6A/ds^6 etc. appear for 6th order and impose formidable requirements on smoothness for the interpolating function f(s). To avoid these problems and make possible fitting to high order, we have developed a 2-dimensional least squares procedure to obtain all of the expansion coefficients directly without recourse to recursion. The procedure requires that we fit $B_z(x,z,s)$ in planes normal to the reference axis at longitudinal positions s_i. Then it is only necessary to interpolate for the coefficients and first derivatives required by B_z in Eq. (2) and B_s. We have tested this procedure by generating TOSCA "data" with a double precision version which yields $B_z(x,z)$ on a grid of 465 points in the interval $-30 \le x \le 30$ mm and $-14 \le z \le 14$ mm at 2 mm spacing for 151 planes between $\theta = .4350$ and $\pi/2$ radians, using the most recent SXLS conductor configuration designated NOV29. RMS errors of the calculated B_z's are estimated to be at most 3×10^{-10} T. As expected, the multipole content of this conductor dominated field is high, especially in the extensive fringe field region, and the least square goodness-of-fit criterion χ^2/N minimized at 14th order with $\chi^2/N \approx 0.6$. We further find that χ^2/N is a factor of 3 smaller by fitting to a given order rather than evaluating the rectangular array of coefficients which results from a constant upper limit on the sums of Eq.(2); we therefore use an upper limit of NR-2m on the n-sum and NR/2 on the m-sum. This results in $(NR+2)^2/4$ terms which, for 14th order gives NP = 64 coefficients. Comparison of lattice parameters from ORBIT in Table II for the TOSCA generated components and the components generated by the fit coefficients are in excellent agreement up to 3rd order chromaticity terms. We have used polynomial (cubic) interpolation longitudinally in the present calculation and anticipate improved accuracy with a smoother function such as a spline or fitted polynomial. Also, the longitudinal spacing of points in the fringe field region is not optimal in the present calculation.

5. Acknowledgements

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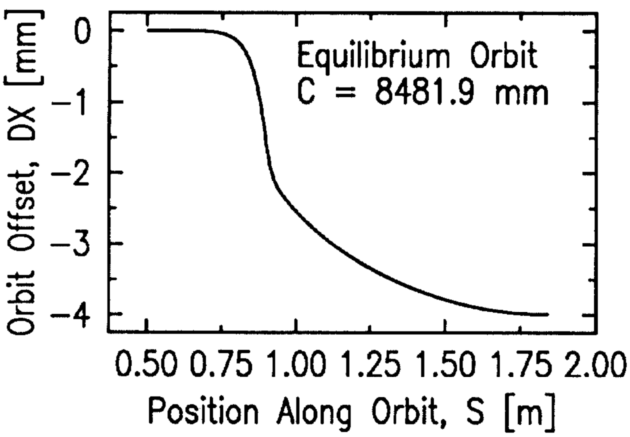

Figure 1: XLS Equilibrium Orbit With No Quadrupole Offset

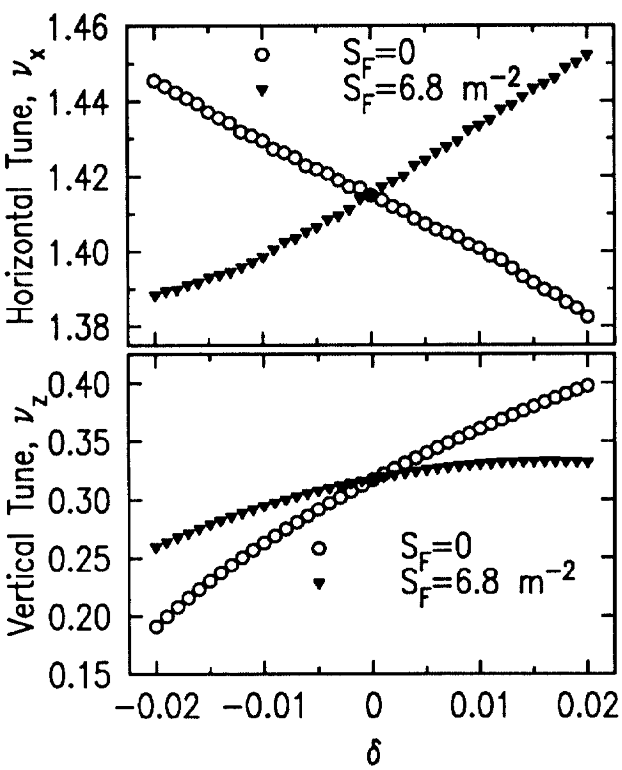

Figure 2: XLS Tune vs. Momentum Offset $\delta = \Delta p/p_0$, 2nd Order Fit to Magnetic Measurement Data, Quad Offset = 4 mm ($\nu = \sum \nu_i\delta^i$).

(2a): $\nu^0 = 1.4148 - 1.3729\delta - .8142\delta^2 - 744\delta^3$, $\nu^{6.8}=1.4149 + 1.7729\delta + 7.0798\delta^2 - 393\delta^3$,

(2b): $\nu^0 = .3168 + 4.8348\delta - 49.73\delta^2 + 610\delta^3$, $\nu^{6.8}=.3168 + 1.7843\delta - 53.8\delta^2 + 72.04\delta^3$

Table I: Comparison of FEB16 TOSCA results with Cubic Spline Fit of TOSCA field.

	Feb16 TOSCA	FIT 615
ν_H	1.415	1.415
ν_V	.4111	0.4148
$B\rho$ [T-m]	2.3276	2.3276
$\Delta r_{\pi/2}$ [mm]	.07637	.07566
β_{0H} [m]	2.4911	2.4911
β_{0V} [m]	1.5427	1.5238
S_Q	.2911	.2913
β_{fH} [m]	.4381	.4391
β_{fV} [m]	7.2671	7.2352
η_0 [m]	1.4197	1.4217
ν^{u}_{1H}	-1.2824	-1.2614
ν^{u}_{2H}	.9124	2.5875
ν^{u}_{3H}	-43.343	-73.635
ν^{u}_{1V}	1.7342	1.5203
ν^{u}_{2V}	8.9081	-2.1797
ν^{u}_{3V}	154.6	-22.61
R Points		13
Angles		101

Table II: Parameters for SXLS COILNOV29, ORBIT results using TOSCA and 14th order fit coefficients generated from TOSCA data. ($\nu_{H,V} = \sum \nu_i\delta^i$)

	ORB11(w/TOSCA)	ORB12(ROY2 coef.)
ν_H	1.415	1.415
ν_V	0.40043464	0.39991641
$B\rho$ [T-m]	2.32986495	2.32986384
$r(\pi/2)$ [m]	1.78078450	1.78078385
S_Q =KL	0.29021396	0.29019028
β_{0H} (m)	2.47931038	2.47880288
β_{0V} [m]	1.60096606	1.60380121
β_{fH} [m]	0.43374180	0.43364157
β_{fV} [m]	7.36071701	7.36598389
ν_{0H}	1.41496	1.41498
ν_{1H}	-1.16857	-1.16833
ν_{2H}	2.10806	1.80049
ν_{3H}	-43.8374	-48.9103
ν_{4H}	147.023	799.899
ν_{0V}	0.400434	0.399916
ν_{1V}	1.34504	1.34490
ν_{2V}	4.25356	4.21889
ν_{3V}	190.081	200.015
ν_{4V}	-6311.85	-6084.37