Orbits, Tunes and Chromaticities for the BNL SXLS Storage Ring*

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Abstract

The lattice of the BNL SXLS has been analyzed by direct integration of the equations of motion through TOSCA-generated fields of the 180-degree bending magnets to obtain the central trajectory, off-momentum closed orbits, and tunes for small amplitude oscillations. The 1st and higher order chromaticities, betatron functions and synchrotron radiation integrals have been obtained. The present 6-coil configuration exhibits an order of magnitude improvement in higher order chromaticity relative to an earlier 2-coil design by addition of a correction coil. Lattice parameters near the central orbit are in good agreement with earlier calculations.

1. Introduction

The Superconducting X-Ray Lithography Source (SXLS) at BNL is a compact, 8.5 meter circumference, 700 Mev electron storage ring designed to produce synchrotron radiation at a critical wavelength λ_c = 10 Å from two 180° superconducting bending magnets with central field B_0 = 3.87 T. The bending magnets also provide vertically focusing quadrupole and horizontally defocusing sextupole (SD). Since the magnet is iron-free, the field has unavoidable higher order nonlinearities and an extensive fringe field. We have therefore resorted to direct numerical integration of trajectories, with fields evaluated by TOSCA[1], to obtain orbits and tunes for small amplitude oscillations. ORBIT, also contains provision for obtaining off- momentum orbits and thus the chromaticity and dispersion functions. The program was tested[2] with a simple two-coil configuration which yielded lattice properties in excellent agreement with earlier estimates[3] based on a combined function magnet model. The present configuration uses six main coils to decrease the octupole and decapole terms present in the two coil model[4]. The calculated tunes are close to the design values ν_{H0},ν_{V0} =1.415, 0.415 and calculated first-order chromaticities corrected by a sextupole (SF) are positive--$\xi_H^c = \xi_V^c$ = .358. Higher order chromaticities are small compared to the two coil configuration, resulting in improved dynamic aperture estimates[5]. Synchrotron radiation integrals are evaluated, giving damping properties in good agreement with the Conceptual Design Report[3].

2. Formulation

The equations of motion in cylindrical coordinates are[6]

$$\ddot{r} - r(1 + 2\dot{r}_n^2) + (r^2/R_M)[B_z(1 + \dot{r}_n^2) - \dot{z}_n(B_\theta + \dot{r}_n B_r)] \quad (1a)$$
$$[1 + \dot{r}_n^2 + \dot{z}_n^2]^{1/2} = 0$$
$$\ddot{z} - 2r\dot{r}_n\dot{z}_n + (r^2/R_M)[\dot{r}_n\dot{z}_n B_z + \dot{r}_n B_\theta - (1 + \dot{z}_n^2)B_r] \quad (1b)$$
$$[1 + \dot{r}_n^2 + \dot{z}_n^2]^{1/2} = 0$$

where $\dot{r} \equiv dr/d\theta$, $R_M = B\rho$ is the magnetic rigidity and the abbreviations $\dot{r}_n = \dot{r}/r$ and $\dot{z}_n = \dot{z}/r$ are used. The coordinate system is ring-centered as indicated in Fig.1. Eqs. (1) are solved by a 4th order Runge-Kutta procedure[7]. We integrate for 300 equal steps from an initial angle θ_i and $\dot{r}_i = x_i\sin\theta_i/\cos^2\theta_i$ at the edge of the quadrupole QF to the center of the bend at $\theta_f = \pi/2$. Reflectional symmetry is assumed. The double precision integration uses 20 minutes of VAX8600 CPU. To determine the equilibrium orbit we iterate on R_M to attain $\dot{r}(\theta_f = \pi/2) = 0$. To obtain an acceptable orbit we vary the coil length to obtain $r_f = Y_c + R_0$ on the magnet reference circle. For off-momentum ($\delta = \Delta p/p_0$) closed orbits the calculation is started at point o in Fig.1, and we iterate for an offset Δx_0, again to attain $\dot{r}_f = 0$. For the tune calculation, we obtain two additional orbits with small amplitude about the central trajectory; orbit R1 starting at point i with initial coordinates relative to the central trajectory of x_i = XI, $(dx/ds)_i = 0$ giving final coordinates x_{1f} and $(dx/ds)_f$ = XP1, and orbit R2 with initial coordinates $x_i = 0$, $(dx/ds)_i$ = XPI and final values x_{2f} and $(dx/ds)_f$ = XP2. We use XI =.0002 m, XPI =.0002 The matrix through half the magnet is given by the usual expression from Courant and Snyder[8]

The horizontal tune is given in terms of these matrix elements and the strength $SQ = L(G/R_M)^{1/2}$ of the assumed ideal quadrupole by the phase-amplitude form of the transport matrix[8] and the reflectional properties of the magnet[9] for one-half turn of the ring starting at point o (where $\alpha_0 = 0$ is assumed)

$$\cos\pi\nu_H = (M_{11} + LM_{21})\cos^2SQ + (K^2LM_{12} - M_{22})\sin^2SQ + [M_{21}/K - K(LM_{11} + LM_{22} + M_{12})]\sin SQ\cos SQ \quad (2)$$

We solve Eq.(2) by Newton's iteration for the strength SQ which yields the desired tune ν_H. Then we get the vertical tune by a similar method

$$\cos\pi\nu_V = (M_{11} + LM_{21})\cosh^2SQ + (K^2LM_{12} + M_{22})\sinh^2SQ + [M_{21}/K + K(LM_{11} + LM_{22} + M_{12})]\sinh SQ\cosh SQ \quad (3)$$

The transverse position of the conductors is then moved until we obtain a vertical tune near the design value. The optimum equilibrium orbit for the final conductor configuration is obtained by an iteration in which the initial electron position $x_i(=R_0)$ and R_M are varied simultaneously to satisfy two final conditions $r_f(\pi/2) = R_0$, $\dot{r}_f(\pi/2) = 0$. We choose the optimum central trajectory as that offset orbit (relative to the reference axis of Fig.1) with circumference most nearly equal to the reference axis circumference C =8503.209236 mm which corresponds to the 6th harmonic of the RF cavity frequency. The position coordinate S along the trajectory is evaluated by ORBIT as $S(\theta) = \int_0^\theta d\theta\, (r^2+\dot{r}^2)^{1/2}$ and the circumference is then $C = 4S(\pi/2)$. The value β_0 at point 0 can then be evaluated from the matrix elements and the known matrix from points 0 to i to obtain the matrix from 0 to f, M_{0f}. Using the Courant- Snyder phase amplitude matrix form and the elements of M_{0f} we obtain

$$\beta_{o(H,V)} = M_{12}/M_{11}\tan(\pi\nu_{H,V}/2) \quad (4a)$$

$$\beta_{f(H,V)} = M_{11}M_{12}/\sin(\pi\nu_{H,V}/2)\cos(\pi\nu_{H,V}/2) \quad (4b)$$

The betatron functions β, α, phase ϕ and dispersion η and η' (= $d\eta/ds$) are computed from the well known Twiss parameter transformation[10] ($\alpha_0 = 0$). In practice we find that α is more

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accurately computed by $\alpha = -\beta'/2$. For the Twiss parameter transformations the matrix elements from point o to P along central trajectory R0 are evaluated as described above with the added feature that the perpendicular distance from the central orbit to R1 and R2, x_{1p} and x_{2p} are measured along the line perpendicular to R0 at point P

$$y(x) = y_p - (x - x_p)(\frac{\dot{r}}{r}\cos\theta_p - \sin\theta_p)/(\frac{\dot{r}}{r}\sin\theta_p + \cos\theta_p) \quad (5)$$

The η function is obtained from definition $\eta = x_p(\delta)/\delta$ where $x_p(\delta)$ is the perpendicular distance from the central orbit R0 to an orbit RD computed by ORBIT for an off-momentum particle starting at point $x_0(\delta)$ as described above. η' is obtained directly by numerical differentiation of η. The betatron functions are used to evaluate synchrotron radiation integrals I_1 through I_5 as given in the lucid presentation of Helm,et.al.[11].

When the correction sextupole is energized with strength $S_F = B''L/R_M$, where the median plane sextupole field is $B_z = B''x^2/2$, the closed orbit for off momentum particles is computed using a point kick $x_0^2 S_F/2$ and, for the tune calculation, a point gradient $G_z = B''x_0$ which produces an additional kick of $-S_F x_0(\Delta x_0)$ for particles displaced by Δx_0 from the closed orbit. The sextupole strength required to yield equal corrected chromaticity in both planes is given to first order by $S_F = 2\pi(\xi_V^0 - \xi_H^0)/(\beta_{0H} + \beta_{0V})\,\eta_0$ where ξ_H^0 and ξ_V^0 are chromaticities for $S_F = 0$. The corrected chromaticity is $\xi^c = \xi_H^c = \xi_V^c = (\xi_H^0\beta_{0V} + \xi_V^0\beta_{0H})/(\beta_{0H} + \beta_{0V})$.

3. Results

The conductor configuration adopted for the SXLS is shown in Fig.2. The main coils are labeled A and B and operate at current density of 18346.9 Amp/cm^2. A third correction coil labeled C, at the same current density, has been added to decrease the octupole and decapole components. Additional trim coils, not shown, will provide adjustment capability for the gradient and sextupole component. Integration of Eqs.(1) with ORBIT yields equilibrium orbits in Fig.3a for several offsets of the straight section axis. Orbit #3 is the optimum central trajectory; however, the dispersion in Fig.3b and tune vs momentum in Figs.4a and 4b are referred to orbit #1. The chromatic variation of tune is seen to be reasonably linear for sextupole correction $S_F = 0$, in contrast to an earlier two coil model[2] without coil C which gave 2nd and 3rd order chromaticities more than an order of magnitude larger than the present results. In Figs. 5a and b we show the β, α and η functions calculated as described above. They are given in Table I with the synchrotron integrals and resultant damping times, emittance, energy spread and damping partitions.

4. Acknowledgements

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5. References

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Table I Parameters From ORBIT, NOV29 coil, 0. 9 mm Offset

	ORB11(TOSCA)	CDR(2/89)
Energy (Mev)	699.29114	696
Mag. Rigidity $B\rho$ (T-m)	2.33258416	2.3216061
Central Dipole B_0 (T)	3.87001259	3.85
Initial Orb.Pos. X_i (m)	.604572135	.6037
Orbit Circumference (m)	8.50348864	8.503
RF frequency (Mhz)	211.5383378	211.54
Horizontal Tune ν_H	1.415	1.415
Vertical Tune ν_V	0.406217	0.415
Quad QF Strength SQ	.290519868	
Quad Gradient (T/m)	8.19455929	7.471
H-Chromaticity ξ_H^0 (SF=0)	-1.16815	
V-Chromaticity ξ_V^0 (SF=0)	1.34504	
Corr.H-Chromaticity ξ_H^c	0.35880	
Corr.V-Chromaticity ξ_V^c	0.35697	
Corr.Sext. Strength(m^{-2})	2.74722497	
Horizontal β_0 (m)	2.47847713	2.292
Vertical β_0 (m)	1.57069436	1.580
Horizontal β_f (m)	0.43633064	0.386 2
Vertical β_f (m)	7.31096202	7.101
Dispersion η_0 (m)	1.40917	1.290
Dispersion η_f (m)	0.26754955	0.3055
Synchro.Integral I_1	2.55968069	
Synchro.Integral I_2	9.88411480	
Synchro.Integral I_3	15.89931656	
Synchro.Integral I_4	3.93927893	
Synchro.Integral I_5	4.59946020	
Mom. Compaction α	.301015359	.32
Energy Loss/turn U_0 (keV)	33.66	34.4
Damped Energy Spread σ_ε	6.93x10-4	6.9x10-4
Damped Emittance ε_x (m-r)	5.5520x10-7	7.17x10-7
Damping Partition J_x	.60145354	.53
Damping Partition J_ε	2.39854646	2.47
H-Damp. Time τ_x (msec)	1.98209140	2.16
V-Damp. Time τ_z (msec)	1.19213590	1.15
E-Damp. Time τ_ε (msec)	0.49702431	.46

Table II: Lattice Parameters for SXLS COILNOV29 ORBIT results using TOSCA and ROY2 fit coefficients from TOSCA data

	ORB11(w/TOSCA)	ORB12(ROY2)
ν_H	1.415	1.415
ν_V	0.40043464	0.39991641
$B\rho$ (T-m)	2.32986495	2.32986384
$r(\pi/2)$(m)	1.78078450	1.78078385
S_Q=KL	0.29021396	0.29019028
β_{oH} (m)	2.47931038	2.47880288
β_{oV}(m)	1.60096606	1.60380121
β_{fH}(m)	0.43374180	0.43364157
β_{fV} (m)	7.36071701	7.36598389
ν_{0H}	1.41496	1.41498
ν_{1H}	-1.16857	-1.16833
ν_{2H}	2.10806	1.80049
ν_{3H}	-43.8374	-48.9103
ν_{4H}	147.023	799.899
ν_{5H}	-24219.3	-16337.2
ν_{0V}	0.400434	0.399916
ν_{1V}	1.34504	1.34490
ν_{2V}	4.25356	4.21889
ν_{3V}	190.081	200.015
ν_{4V}	-6311.85	-6084.37
ν_{5V}	132570.	119862.

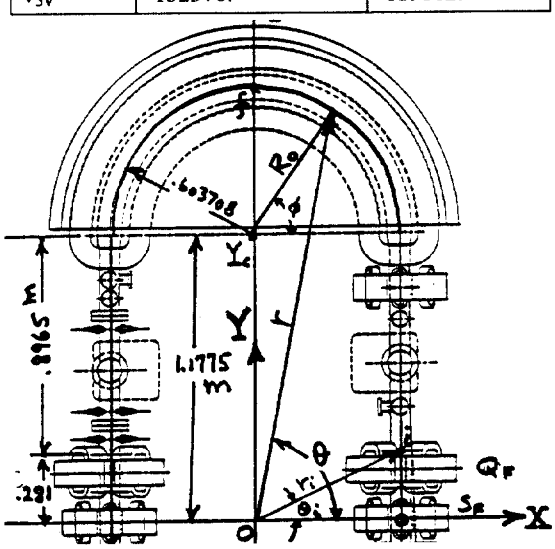
Figure 1: Plan View of Half SXLS

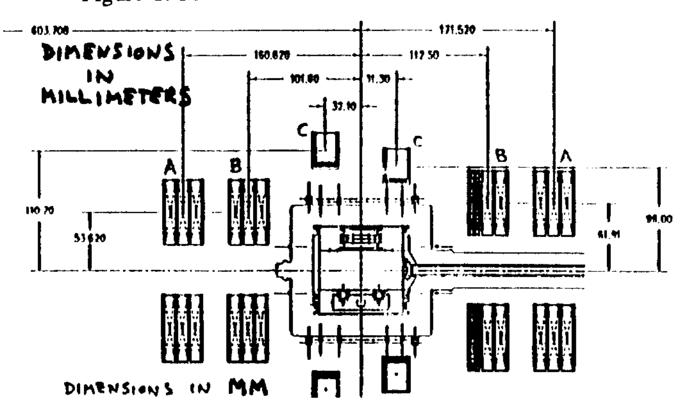

Figure 2: SXLS Coil Configuration

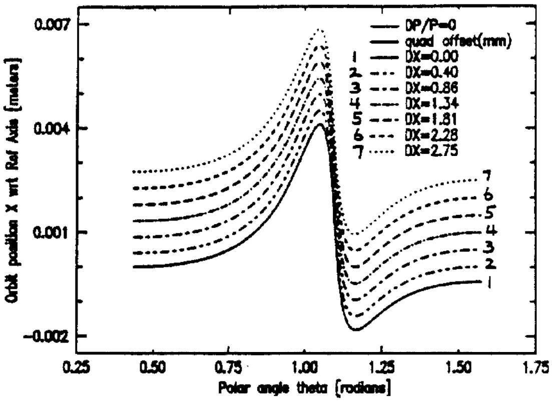

Figure 3a: Equilibrium Orbits for Various Quad Offsets

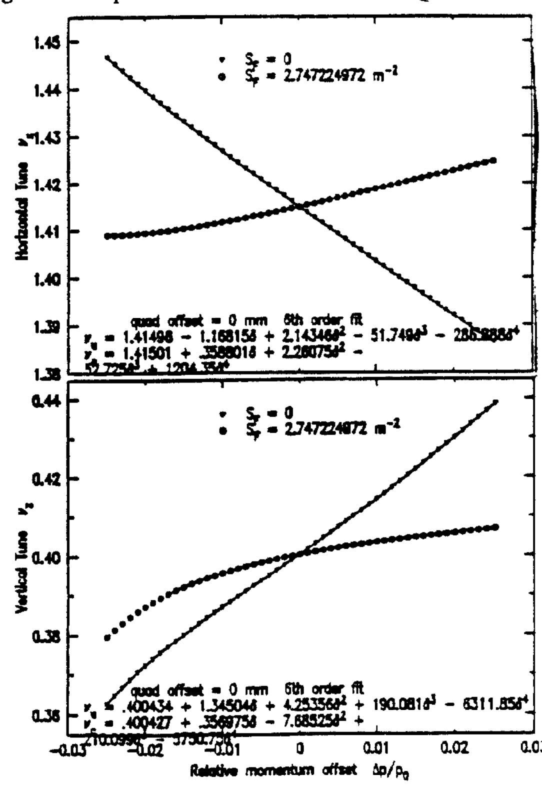

Figure 4: Tune vs. Momentum Offset for Orbit #1

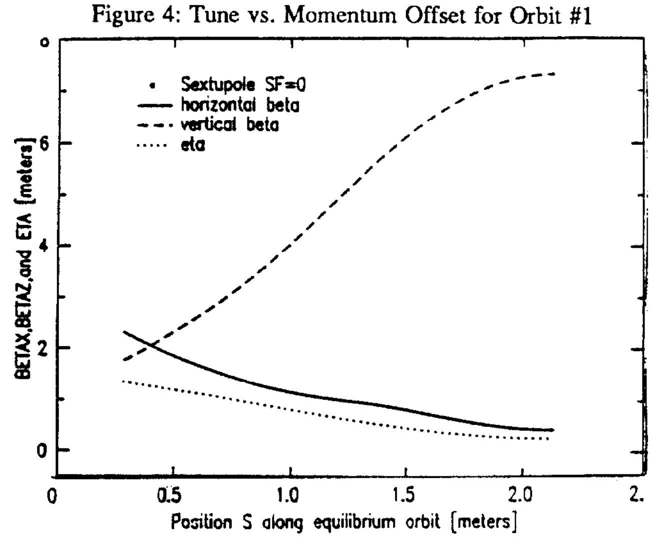

Figure 5: β, η and α functions about orbit #3, S_F = 0