Emittance Growth in Mismatched Charged Particle Beams*

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Abstract

A new theoretical model of emittance growth in nonstationary charged particle beams has been developed which generalizes the previous theory of nonuniform charge distributions to include rms mismatched and off-centered beams. First the behavior of mismatched uniform beams in linear focusing channels, where no emittance growth occurs, is discussed. Then the results of the new theory are presented and compared with numerical simulation studies for rms mismatched, nonuniform beams.

I. Introduction

Past theoretical studies of mismatched beams used the uniform beam model to calculate the frequencies of the envelope oscillations[1]. Since all forces acting on the particles are linear in this model, the emittance remains constant. In more realistic nonuniform beams, however, the nonlinear space charge forces may cause emittance growth, as has been shown for rms-matched particle distributions in several theoretical[2-5] and experimental investigations[6,7].

Recently, the author developed a new model of emittance growth in nonstationary distributions[8] that extended and generalized the previous theory to include rms mismatched and off-centered beams. The generally nonuniform distribution is modelled by the equivalent uniform beam having the same perveance, rms radius $\tilde{x}$, and rms emittance $\tilde{\epsilon}_x$ according to Lapostolle[9] and Sacherer[10]. In nonstationary beams the total energy per particle is higher than in the equivalent stationary beam by an amount ΔE which constitutes "free energy" that can be thermalized via collisions or nonlinear space charge forces. By assuming that the beam relaxes into a final stationary state at the higher energy and comparing it with the initial stationary state one obtains analytical relations for the increase of the beam radius and for the emittance growth.

In the following we will first describe the behavior of a mismatched uniform beam. Then we will present the results of the new theory for the increase of beam radius and emittance resulting from the thermalization of the free (mismatch) energy and compare them with simulation.

II. Behavior of a Mismatched Uniform Beam

Let us consider a beam with a uniform density profile (K-V distribution) in a linear "smooth" transport channel. The stationary state is characterized by a constant average beam radius a and perfect balance between the external focusing force, $k_0^2 a$, the space charge force, K/a, and the emittance term, ϵ^2/a^3, according to the envelope equation[11]

$$k_0^2 a - \frac{K}{a} - \frac{\epsilon^2}{a^3} = 0. \tag{1}$$

Here, $k_0 = 2\pi/\lambda_0 = \sigma_0/S$ represents the external focusing force, λ_0 the betatron oscillation wavelength without space charge, σ_0 the phase advance per period without space charge and S the length of one focusing period. $K = (I/I_0)(2/\beta^3\gamma^3)$ is the generalized perveance, $I_0 = 120\pi\ mc^2/q$ is the characteristic current, m the particle mass, q the particle charge, c the speed of light, $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$, and v the particle velocity.

Defining the above parameters with space charge by k, λ, σ, respectively, and using the relation

$$k^2 = k_0^2 - \frac{K}{a^2}, \tag{2}$$

one may rewrite Eq. (1) in the form

$$k^2 a - \frac{\epsilon^2}{a^3} = 0, \text{ or } \epsilon = ka^2. \tag{3}$$

The total energy E per particle (transverse kinetic energy E_k + potential energy of the applied field E_p + self-field energy E_s) for the stationary beam is found to be[8]

$$E = \frac{\gamma m v^2}{4}\left[k^2a^2 + k_0^2a^2 + \frac{1}{2}\left(k_0^2 - k^2\right)a^2\left(1 + 4\ell n\frac{b}{a}\right)\right], \tag{4}$$

where b is the radius of the beam tube.

If the beam is mismatched, the beam envelope will perform oscillations about the equilibrium radius a with wave constant[1]

$$k_e = (2k_0^2 + 2k^2)^{1/2} \tag{5}$$

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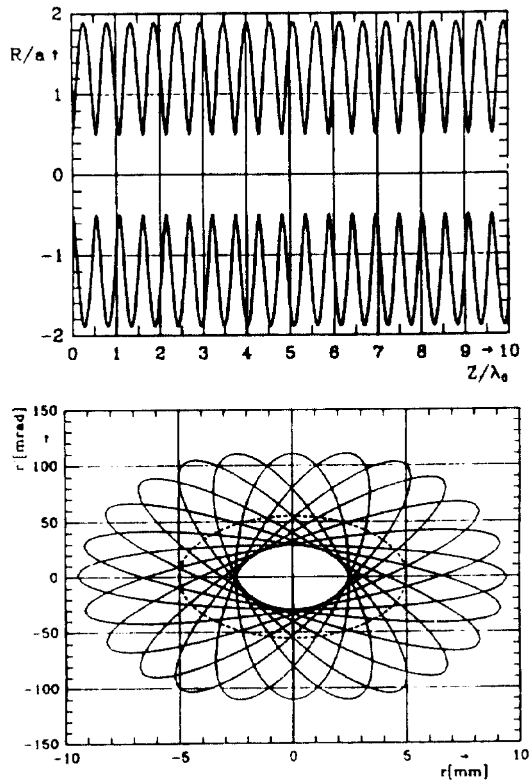

Figure 1: Envelope oscillation (top) and motion of trace-space ellipse (bottom) of a mismatched uniform beam in a smooth linear focusing channel where $k/k_0 = 0.8$, $a_0/a = 0.5$. The dotted ellipse in the bottom picture represents the stationary (matched) beam.

for the axisymmetric in-phase mode. Figure 1 (top) shows the envelope oscillations of a mismatched beam with a "tune depression" of $k/k_0 = 0.8$ and a mismatch defined by $a_0/a = 0.5$, where a_0 is the initial waist radius while a is the radius of the stationary beam. The length of one oscillation period in the figure is in excellent agreement with the value of $\lambda_e = 0.552\ \lambda_0$, calculated from Eq. (5). The bottom of Fig. 1 shows the position of the trace-space ellipse at twelve intervals during one envelope oscillation cycle. The typical rotation of the mismatched beam ellipse is evident. The calculations were performed by numerical integration of the K-V equation.

Figure 2 shows the trace-space picture for a beam with a lower tune depression of $k/k_0 = 0.3$ and $a_0/a = 0.5$. Note that the ellipse motion is qualitatively different from the previous case. It is an oscillation rather than a rotation, with the tips tracing out the two closed curves shown in the figure. Further computer runs and analysis show that this qualitatively different behavior occurs when the beam is dominated by space-charge (plasma oscillations), i.e., when $Ka^2 > \epsilon^2$. For $Ka^2 < \epsilon^2$, the beam is dominated by emittance (betatron oscillations). The transition between the two regimes occurs when $Ka^2 = \epsilon^2$, or, equivalently, from Eqs. (2) and (3), when $k/k_0 = \sqrt{0.5}$.

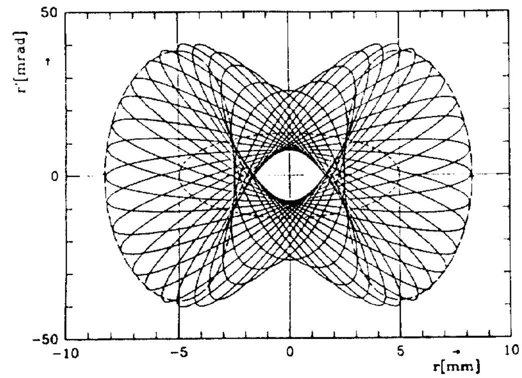

Figure 2: Motion of trace-space ellipse for a space-charge dominated beam where $k/k_0 = 0.3$, $a_0/a = 0.5$. Dotted ellipse at the center of the figure represents the matched beam.

III. Emittance Growth in a Nonuniform Mismatched Beam

If the beam has a nonuniform profile, the space charge forces are nonlinear and the free energy ΔE due to mismatch or other effects can be thermalized. Assuming that the nonlinear forces due to the collective self fields have the same effect as collisions[12] one expects from thermodynamic considerations that the beam relaxes towards a stationary Maxwell-Boltzmann distribution with a density profile[13]

$$n(r) = n(0)\exp[-q\phi(r)/kT]. \tag{6}$$

Here $\phi(r) = \phi_e(r) + \phi_s(r)$ is the sum of the external focusing potential given by $\phi_e(r) = \frac{1}{2}\gamma m v^2 k_0^2 r^2$ for a beam with nonrelativistic transverse velocities and the potential $\phi_s(r)$ due to the self fields, and kT is the transverse temperature in the laboratory frame.

Since $\phi_r(r)$ depends on the density $n(r)$, this distribution has an analytic form only for the low-temperature ($T \to 0$) and zero space charge ($\phi_s \to 0$) limits, where the profiles are uniform ($n = n_0$) and Gaussian [$n(r) = n(0)\exp(-\gamma m v^2 k_0^2 r^2/2kT)$], respectively:

Approximating the initial and final nonuniform distributions by the equivalent uniform beams, the author obtained the following relations for the effective radius $a = 2\tilde{x}$ and emittance $\epsilon = 4\tilde{\epsilon}$ of an initially nonstationary beam[8]:

$$\left(\frac{a_f}{a_i}\right)^2 - 1 - \left(1 - \frac{k_i^2}{k_0^2}\right)\ell n\frac{a_f}{a_i} = h, \tag{7}$$

$$\left(\frac{\epsilon_f}{\epsilon_i}\right)^2 = \frac{a_f}{a_i}\left\{1 + \frac{k_0^2}{k_i^2}\left[\left(\frac{a_f}{a_i}\right)^2 - 1\right]\right\}^{1/2}. \tag{8}$$

The subscript i denotes the equivalent initial stationary state for the mismatched beam, the subscript f denotes the final stationary state and the parameter h relates to the free energy ΔE of the nonstationary (mismatched) initial beam by $h = 2\Delta E/(\gamma m v^2 k_0^2 a_i^2)$. The case of an rms matched beam having a nonuniform density profile

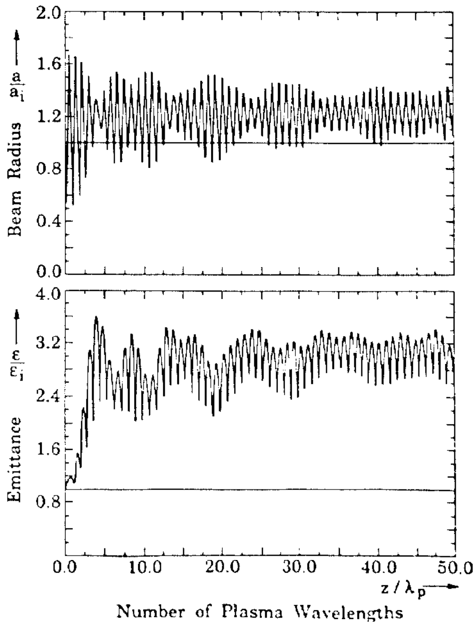

Figure 3: Simulation results for effective beam radius a/a_i and emittance ϵ/ϵ_i in the case $k/k_0 = 0.3$, $a_0/a_i = 0.5$ for an rms-mismatched initial Gaussian distribution.

has been treated previously[2,3] with the assumption that the radius $a_f = a_i =$ const. The free energy parameter in this case relates to the field-energy difference $U = w_n - w_u$ between the nonuniform and equivalent uniform beam and is given by[8]

$$h = h_s = \frac{1}{4}\left(1 - \frac{k_i^2}{k_0^2}\right)\frac{U}{w_0}, \tag{9}$$

where $w_0 = \mu_0 I^2/16\pi\beta^2 = I^2/(4\times 10^7\beta^2)[J/m]$ and U/w_0 is a dimensionless quantity that measures the nonuniformity of the distribution.

For the rms mismatched beam the initial effective radius a_0 differs from the equilibrium radius a_i of the equivalent stationary beam and one obtains[8]

$$h = h_m = \frac{1}{2}\frac{k_i^2}{k_0^2}\left(\frac{a_i^2}{a_0^2} - 1\right) - \frac{1}{2}\left(1 - \frac{a_0^2}{a_i^2}\right) + \left(1 - \frac{k_i^2}{k_0^2}\right)\ell n\frac{a_i}{a_0}. \tag{10}$$

Given h from (9) and (10), one can calculate the radius and emittance increase of a nonuniform and/or mismatched beam using Eqs. (7) and (8).

To check the new theory, extensive systematic numerical simulation studies have been performed at Los Alamos[14]. Work is also in progress at the University of Maryland to compare theory and simulation with experiments in a periodic solenoidal focusing channel[15]. In all cases studied so far, the theoretical predictions from Eqs. (7) and (10) are in remarkably good agreement with the simulation results (generally better than 10%). As an example, one finds for a Gaussian beam ($U/w_0 = 0.1212$) with an initial tune depression of $k_i/k_0 = 0.3$ and a radius mismatch of $a_0/a_i = 0.5$ the results $h = h_m + h_s = 0.39 + 0.03 = 0.42$, $a_f/a_i = 1.28$, $\epsilon_f/\epsilon_i = 3.64$. The simulation results for such a mismatched beam using a Gaussian density profile are shown in Fig. 3. The top curve shows the radius and the bottom the emittance versus distance in plasma wavelengths, z/λ_p, where $\lambda_p = 2\pi/k_p$ is defined by the relation $k_p = [2(k_0^2 - k_i^2)]^{1/2}$. Most of the emittance growth occurs in about four plasma periods which roughly correspond to the average betatron wavelength. The final mean radius and the emittance peak are in good agreement with the theoretical values. Further results of the simulation studies can be found in Ref. [14]. As will be discussed in [14] and [15], most of the emittance growth appears to occur due to the formation of a large halo surrounding the beam core. Thus one can draw the important conclusion that beam mismatch is apparently one of the main causes for the halos observed in many experiments.

IV. Acknowledgments

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V. References

[1] J. Struckmeier and M. Reiser, Part. Accel. **14**, 227 (1984).

[2] J. Struckmeier, J. Klabunde, and M. Reiser, Part. Accel. **15**, 47 (1984).

[3] T.P. Wangler, K.R. Crandall, R.S. Mills, and M. Reiser, IEEE Trans. Nucl. Sci. **32**, 2196 (1985).

[4] I. Hofmann and J. Struckmeier, Part. Accel. **21**, 69 (1987).

[5] O.A. Anderson, Part. Accel. **21**, 197 (1987).

[6] C.M. Celata, A. Faltens, D.L. Judd, L. Smith, and M.G. Tiefenback, Proc. 1987 Particle Accelerator Conference, edited by E.R. Linstrom and L.S. Taylor, IEEE N.Y., p. 1167.

[7] M. Reiser, C.R. Chang, D. Kehne, K.Low, T. Shea, H. Rudd, and I. Haber, Phys. Rev. Lett. **61**, 2933 (1988).

[8] M. Reiser, "Free Energy and Emittance Growth in Nonstationary Charged Particle Beams," CPB Technical Report #90-034, Oct. 25, 1990, to be published.

[9] P.M. Lapostolle, IEEE Trans. Nucl. Sci. **18**, 1101 (1971).

[10] F.J. Sacherer, IEEE Trans. Nucl. Sci. **18**, 1105 (1971).

[11] M. Reiser, Part. Accel. **8**, 167 (1978).

[12] G. Schmidt, *Physics of High Temperature Plasmas*, Second Edition, Academic Press, 1979, p. 317.

[13] J.D. Lawson, *The Physics of Charged Particle Beams*, Second Edition, Oxford Science Publications, 1988, p. 201.

[14] A. Cucchetti, T.P. Wangler, and M. Reiser, Paper ERA 21, this proceedings.

[15] D. Kehne, M. Reiser, and H. Rudd, Paper ERA 20, this proceedings.