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Beam Dynamics of Cooled Heavy Ion Beams

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Abstract

Evaluation of the collectively modified Schottky spectra as well as beam transfer functions for Ar^{18+} and Kr^{36+} at 250 rsp. 150 MeV/u are in agreement with theoretical predictions. Maximum observed phase space densities are compared with thresholds of the microwave instability and with intrabeam scattering. The conventional Keil-Schnell threshold is exceeded in the experiment by a factor of 3, with intrabeam scattering as the main limitation. Implications of these results for a heavy ion fusion driver are discussed.

I. INTRODUCTION

As a result of electron cooling in the ESR storage ring [1] we have obtained phase space densities where beam dynamics is influenced by collective effects. Details of the diagnostics set-up are described in Ref. [2, 3 , 4]. The longitudinal Schottky spectra are considerably distorted by these collective effects (double-peaked) as has been shown by measurements of several groups [5, 6, 7]. One might expect that effective cooling leads to the boundaries of longitudinal and transverse stability, if heating by IBS were absent. In addition it is necessary, at least for very small momentum spread, to consider the effect of the cooling friction force on the output signals. In any case a consistent interpretation of the data of Schottky noise and beam transfer function measurements requires comparison with theoretical calculations [8, 9 , 10, 11].

II. SCHOTTKY AND BTF MEASUREMENTS

Schottky Noise

Electron cooling of Ar^{18+} at 250 MeV/u is shown in Fig. 1 for a $(\Delta p/p)_{fwhm} = 10^{-3}$. The cooling equilibrium depends on the electron current. For I_e = 0.1A the spectrum shows the momentum distribution, whereas for I_e = 1A the double peak spectrum reflects anomalous behaviour due to collective effects. In the latter case the momentum spread was reduced to 2.6 10^{-5}. The ion current has been 1 mA (electric), which was obtained by "cooling stacking" [1]. The origin of the Schottky spectrum is the noise from the statistical distribution of particles. This gives rise to current fluctuations, which induce a voltage on a pick-up.

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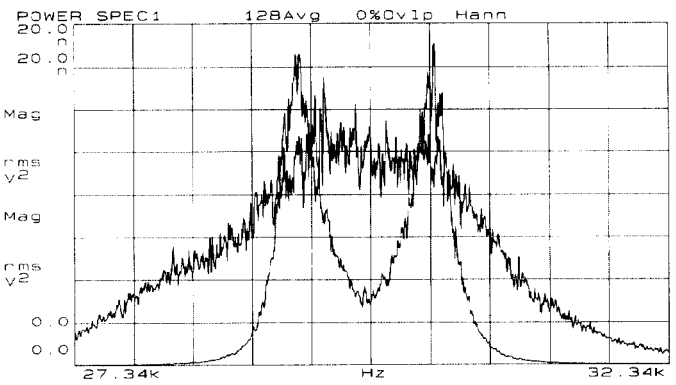

Fig. 1. Schottky spectrum for varying I_e=0.1A / 1A.

The Schottky power is proportional to the square of the current fluctuations, which are proportional to the distribution function of momenta in the low-intensity case [12]. For high phase space densities the collective response of the beam produces an additional electric field, which must be added to the purely statistical fluctuations (of uncorrelated particles) as source term. This results in a total electric field connected with the source term by a dielectric function (see also Ref. [10]):

$$E_{total} = \frac{E_{source}}{\epsilon} \qquad (1)$$

We thus obtain the modified expression for the Schottky power spectrum according to

$$P_{||}(\Omega, n) = \frac{q^2 e^2}{\pi} \frac{N}{n} \frac{\Psi_0(\Omega/n)}{|\epsilon|^2} \qquad (2)$$

The result for negligible correlations among particles, i.e. in the low-intensity limit without cooling is obtained by setting $\epsilon = 1$. The double peak reflects the presence of collective waves in the beam, where the left peak is due to the "slow" plasma wave and the right peak due to the "fast" plasma wave. The consistency of these measurements is supported by a theoretical calculation of the Schottky spectrum for a double-Gaussian, where 80% is contained in the main Gaussian and 20% in a twice as broad Gaussian, with parameters comparable to the experiment. The resulting Schottky spectrum is shown in Fig. 2, along with the result ignoring the collective response, which has the shape of the distribution function Ψ_0 (continuous line).

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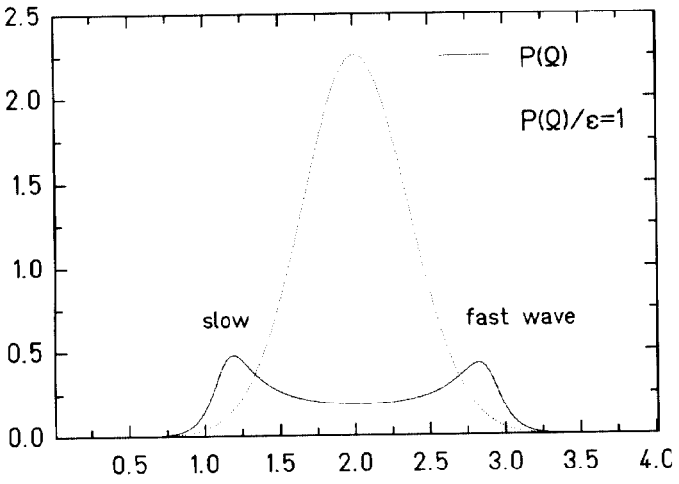

Fig. 2. Calculated Schottky spectrum.

Impedance

The beam transfer function $r_{||}$ is defined as ratio of beam response to the excitation voltage on a kicker, where we have to include the additional collective response as above by means of the dielectric function. It is usual to compare the collective response with a feedback loop given by the impedance. The relationship can be written in the same fashion as Eq. 2, where the source term is the response in the absence of the impedance, rsp. intensity. Hence we have

$$r_{||} = \frac{r_{||,0}}{\epsilon} \qquad (3)$$

The inverse response is the stability diagram, which is written as

$$\frac{1}{r_{||}} = \frac{1}{r_{||,0}} + Z_{||} \qquad (4)$$

hence the shift of the stability diagram gives directly the impedance. The shift of the enlarged lower part of the diagram is shown on the r.h.s. of Fig. 3, whereas the l.h.s. shows the full diagram [2].

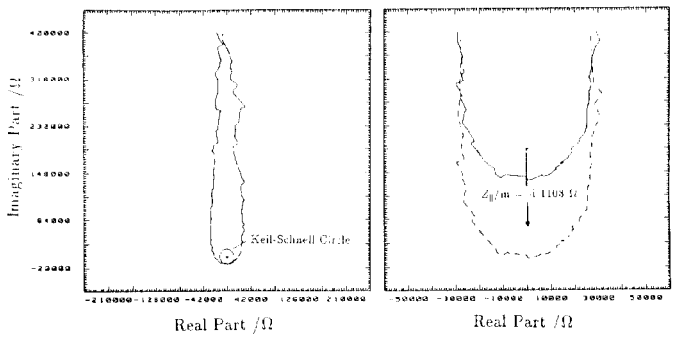

Fig. 3. Stability diagram (full and enlarged).

In this diagram the shift is directly proportional to the current, or inversely proportional to $(\Delta p/p)^2$ for fixed current. We note that the measured impedance agrees within 10% with the theoretical value for the space charge impedance, which is given by

$$\frac{Z_{||}}{n} = -i\frac{1 + 2ln(R_p/R_b)}{2\beta\gamma^2} 377 \ \ (Ohm) \qquad (5)$$

In order to identify the measured impedance with the space charge impedance we note that the latter is independent of the harmonic number n. We thus compare in Fig. 4 measured results for different n and find a good constancy within the error bar.

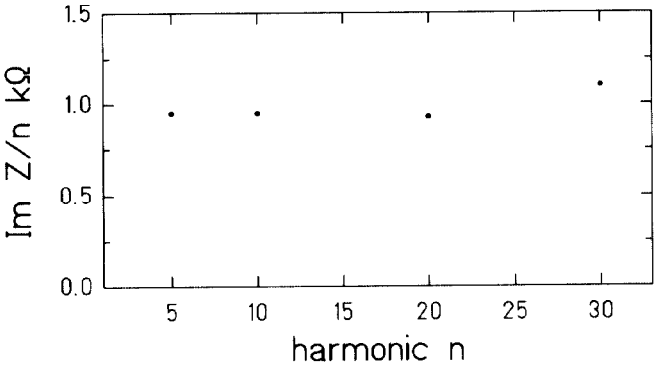

Fig. 4. Impedance at different harmonics.

A peculiarity was found at n=5, where the Schottky signal showed a large central peak absent in the results for n=20,30. This might be a result of a weak "self-bunching" effect due to an inductive impedance of the rf cavity, which is resonant near n=2.

III. INTERPRETATION OF STABILITY REGION

Friction Force

We first have to examine whether the observed stability diagram is only a consequence of the distribution function and impedance, or whether its shape is also influenced by the cooling friction force. This is conceivable if one notices that the detailed shape of the stability region depends on the Landau damping provided by the slopes of the momentum distribution. Electron cooling is also a damping mechanism with a rate given by the cooling rate ν_{cool}. The friction force can be ignored, if the ratio of cooling rate to phase mixing rate (given by the spread of angular frequency) is small. This criterion can be approximately written as (for a more detailed model see Ref. [9])

$$\frac{\nu_{cool}}{n\delta\omega} << 1 \qquad (6)$$

It is noted that the influence of the friction force is largest for small n. By comparing results for the stability diagram at n=5 and n=30 of the example of Fig. 1 we have found no difference, however. This agrees with the estimated value of the ratio of Eq. 6 of 0.015 for n=2 and 0.0025 for n=30, which are both

small. At much smaller momentum spread of the order of 10^{-6}, however, we expect an influence on the stability region, which is predicted theoretically for the above ratio of 0.01 or larger [9].

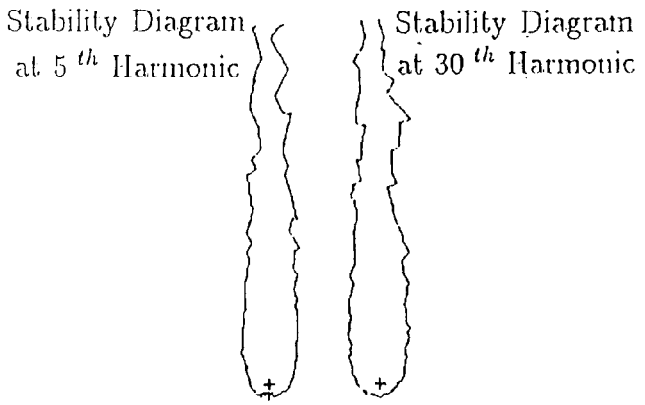

Fig. 5. Stability diagram at different harmonics.

Shape of Distribution Function

The experimental distribution function can be directly determined from the beam transfer function by integration. The result is shown in Fig. 6 for the cooled Ar^{18+} beam.

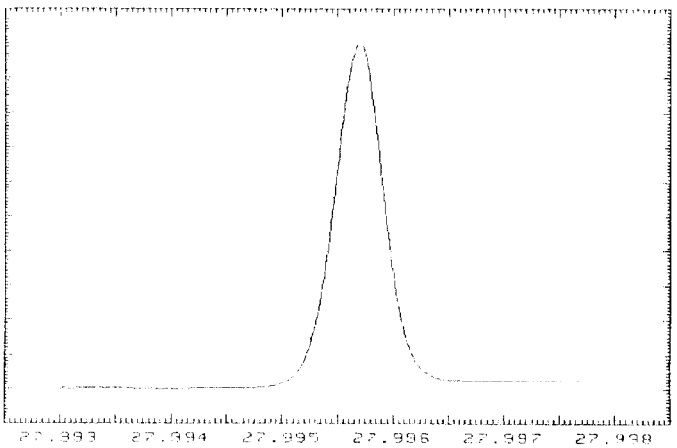

Fig. 6. Measured distribution function.

This function has more extended tails than a Gaussian, which can explain the large extension of the measured stability diagram towards reactive impedances (upward in Fig. 5, which is the direction relevant for the space charge impedance below transition energy). This argument is confirmed in Fig.7 by a theoretical calculation of the stability diagram for different distribution functions normalized to the same $(\Delta p/p)_{fwhm}$: a quadratic $(1-x^2)^2$ (dotted); a Gaussian e^{-x^2} (dashed); and a bi-Gaussian $\epsilon^{-x^2}+\frac{\alpha}{2}e^{-(x/2)^2}$ (dashed-dotted), where a fraction α (here $\alpha = 0.2$) of the main distribution is contained in the broader Gaussian.

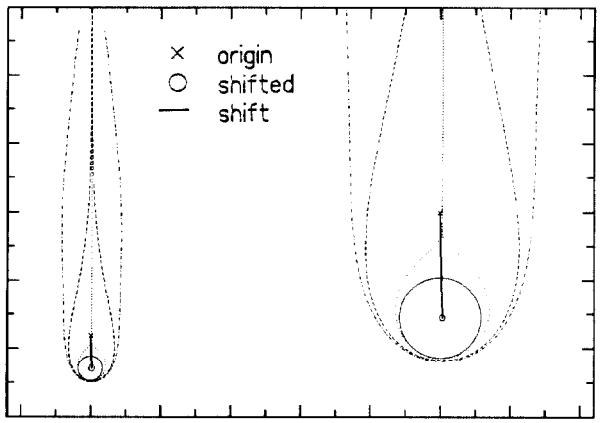

Fig. 7. Theoretical stability regions (full and enlarged).

For stability with respect to the longitudinal microwave mode the point $-Z_{\|}$ must be inside the respective stability curve. Due to its sharp edge the quadratic distribution has the smallest stability region, whereas the extended tails of the bi-Gaussian lead to an enlarged stable area near the positive imaginary axis (reactive impedance).

The Keil- Schnell circle criterion describing the largest circle of stability in the complex impedance plane is also shown in Fig. 7. It is obviously overrestrictive in the case of mainly reactive impedance. Our shift is a factor 3 bigger than allowed by the circle criterion, hence we have exceeded the Keil-Schnell current threshold by this factor.

Here we observe that above transition energy the space charge impedance effectively changes sign and the circle criterion is strictly applicable (negative mass instability).

It is worth noting that (below transition energy) the Keil-Schnell threshold marks the beginning of a dip in the Schottky spectrum: the dielectric function has just doubled its value at the center of the frequency spectrum (i.e. $\epsilon = 2$), which indicates that collective motion becomes more significant (see also ref. [10]).

IV. INTRABEAM SCATTERING AND MICROWAVE INSTABILITY

For decreasing momentum spread the shift in Fig. 5 would become bigger until the stability boundary is reached. The question can be raised why this has actually not happened and why we have achieved - in the best case - only a factor of 3 above the circle criterion, whereas the actual size of the stability region would allow even a factor 10 or more. To answer this question it is necessary to consider intrabeam scattering as a heating mechanism competing with cooling. Calculations carried out for the ESR have indicated that

for small enough emittance the intrabeam scattering puts a limit to further cooling before the threshold for the longitudinal microwave instability is reached [13].

In Fig. 8 we plot the momentum spread versus intensity for two series of measurements: Ar^{18+} at 250 MeV/u and Kr^{36+} at 150 MeV/u. Both series confirm the observation made at the TSR Heidelberg [14] as well as earlier at the LEAR and Novosibirsk experiments [6, 15], where an approximate scaling was found according to:

$$\Delta p/p \propto N^{1/3} \tag{7}$$

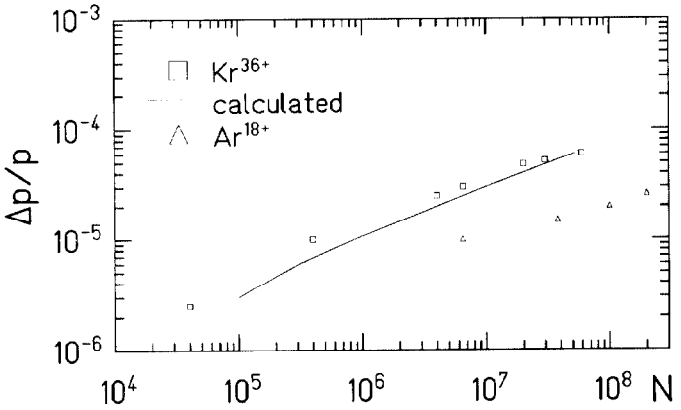

Fig. 8. Measured and calculated equilibria.

For the case of Kr^{36+} at 150 MeV/u we have calculated theoretically equilibrium solutions for the ESR based on the Piwinski theory and the following asumptions: (1) constant emittance of 0.5 π mm mrad horizontally and vertically; (2) constant longitudinal cooling time of 100 msec (see continuous line in Fig. 8). For our parameters intrabeam scattering leads predominantly to longitudinal heating related with the dispersion of the machine. There is a much slower (by several orders of magnitude) horizontal heating and even slower vertical cooling, hence we assume that the emittances are determined in the experiment by some other effects (nonlinear resonances, for higher intensity transverse instability). The cooling time is estimated from the experimental observation for cooling of Ar^{18+} [2]. The comparison of the calculated equilibrium with the measured one can only be made qualitatively since the asumption of constant emittance and cooling time is not justified for lower intensity. Nonetheless there is a good agreement supporting the assumption that intrabeam scattering determines $\Delta p/p$. Measurements of the emittance by analyzing the angles of recombined ions (i.e. radiative electron capture [1]) have shown that for reduced intensity the emittance decreases as well. This would enhance longitudinal heating and lead to a larger equilibrium momentum spread. Such a trend is also recognized in Fig. 8.

Keil-Schnell Factor

It is interesting to compare for various ions and energies the factor by which the actual current exceeds the Keil-Schnell circle threshold:

$$\frac{I}{I_{KS}} = \frac{4eqIZ_{\|}\ln 2}{\pi|\eta|^2\beta^2\gamma Amc^2(\frac{\Delta p}{p})^2_{fwhm}} \tag{8}$$

with $Z_{\|}$ the coupling impedance, $\eta = 1/\gamma^2 - 1/\gamma_t^2$ and $(\frac{\Delta p}{p})^2_{fwhm}$ the fwhm momentum spread. This dimensionless factor can be regarded as "figure of merit" of longitudinal cooling [10]. Results are plotted in Fig. 9, which indicates that the excess factor becomes the higher the larger the intensity. This can be understood quantitatively by noting that for increasing intensity the momentum spread increases only according to Eq. 7, hence we readily obtain from Eq. 8 the following simple scaling:

$$\frac{I}{I_{KS}} \propto N^{1/3} \tag{9}$$

which is confirmed approximately by Fig. 9. The fact that for Kr^{36+} we have not even reached $I/I_{KS} = 1$ (in contrast with Ar^{18+}) can thus be explained by the increase of the intrabeam scattering rate, which is proportional to Z^4/A^2.

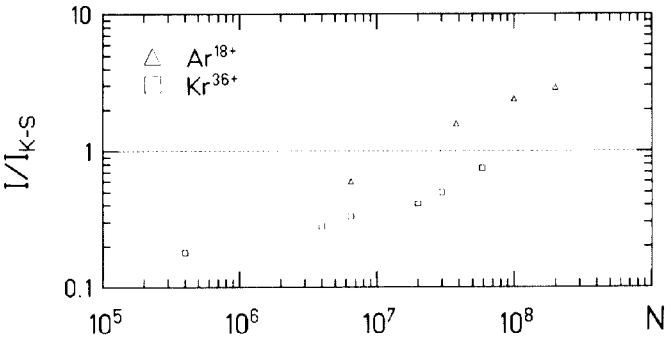

Fig. 9. I/I_{KS} as function of intensity.

In Fig. 10 we show the Schottky spectrum for the

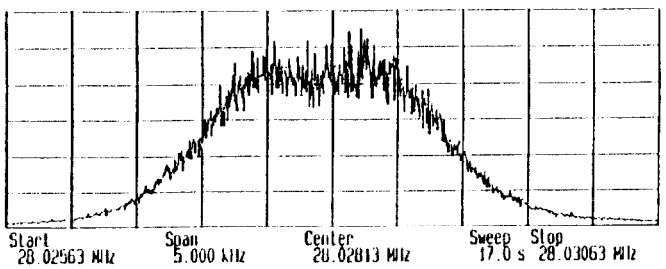

Fig. 10. Schottky spectrum: highest intensity of Kr^{36+}.

highest intensity case of Fig. 9 for Kr^{36+} where I/I_{KS} = 0.75. A slight indication of a dip in the spectrum is recognized in this case, which is typical for I/I_{KS} near unity.

V. APPLICATION TO A HEAVY ION FUSION DRIVER

One of the major concerns of heavy ion fusion has been the limitation in the storage rings due to the longitudinal microwave instability. In the HIBALL study it was assumed that due to stabilizing tails the factor I/I_{KS} can be as large as 50, without loosing stability with respect to the longitudinal microwave mode. In a more recent design for an advanced driver scenario using a non-Liouvillean stacking by means of photoionization (see Ref. [16, 17]) this factor has been kept below 10.

The present measurements, supplemented by results from other rings at different energies give experimental confirmation that for coasting beams such a factor can indeed be achieved (Fig. 11).

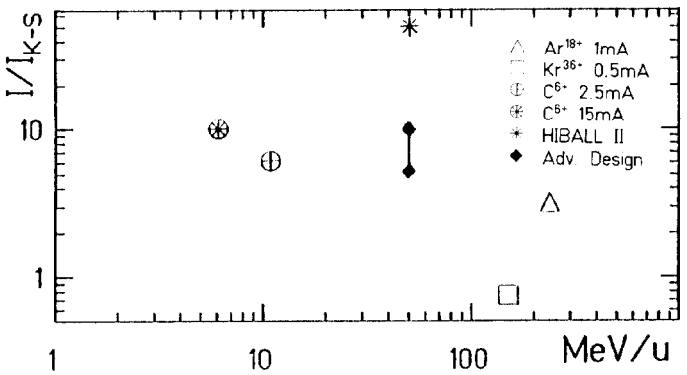

Fig. 11. Application of measured I/I_{KS} to heavy ion fusion.

Of relevance in this context are in particular also measurements at the TSR Heidelberg with C^{6+} at lower energy, where this factor has been as large as 6-10, depending on the intensity. The comparison between the 2.5 mA and 15 mA case confirms the trend indicated by Eq. 9. Hence this answers one of the key issues of the rf linac / storage ring approach to heavy ion fusion.

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