Non Similarity Solution Approximation to the Thermal Hydraulic Quenchback in Superconductors

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Abstract

Using an approximation to the conduction fluid equations, a solution is given using the direct method of integration of a differential equation. This solution establishes a possible increasing relation between the quench velocity and length of the conductor.

I. Introduction

When a normal zone appears in a superconducting (s.c.) cable, it propagates axially with a so called "quench velocity," V_q, which depends mainly of the conductor characteristics and very little on the cooling effect of Helium [1]. However, if the ratio of the longitudinal to transversal dimensions of the conductor is quite large, the expansion of the heated Helium in the normal zone may drive fluid elements far from the initial normal zone, which could induce other normal zones in the conductor because of the compression of these fluid elements and their friction with the strands of the cable. The quench velocity would have much higher value than the pure Fourier conduction mechanism; this is called thermohydraulic quench back (THQB) mechanism. After the numerical discovery [2] of the THQB mechanism, an analytical approximation to this phenomenon appeared [3] using the similarity method for the differential equations. Although this solution suggests high quench velocities for the THQB mechanism, it cannot be accepted since the solution predicts much faster quench velocities for short s.c. cables than for long ones. In addition, an experiment [4] seems to contradict its predictions. In this paper, a direct method of integration is used in the differential equation (derived from approximations) in order look for a different solution.

II. Approximations

For one-dimensional fluid (Helium) moving in a long tube pipe of diameter D and length L, satisfying the relation $D/L << 1$, and being heating up (quench appears in the s.c. surounding the tube pipe) at the origin, $z = 0$, the equations which govern the state of the fluid are the continuity equation

$$\frac{d\rho}{dt} + \rho\,\frac{\partial v}{\partial z} = 0\ , \tag{1}$$

the momentum conservation equation

$$\rho\,\frac{dv}{dt} = -\frac{\partial p}{\partial z} - \rho F\ , \tag{2}$$

the mechanical energy conservation equation

$$\rho\,\frac{d}{dt}(e + \frac{1}{2}v^2) = -\frac{\partial (pv)}{\partial z} + q\ , \tag{3}$$

the state equation

$$d\rho = \frac{1}{c^2}dp - \frac{B\rho}{c_p}T\ ds\ , \tag{4}$$

and the second law equation

$$de = Tds + d\omega\ , \tag{5}$$

where c is the speed of sound, B is the volume coefficient of thermal expansion, c_p is the specific heat, T is the temperature, e is the specific internal energy, s is the specific entropy, q is the power density entering the Helium, p is the pressure, ρ is the density, v is the velocity of the fluid, and F is given in terms of the Fanning friction factor, f, as

$$F = \frac{2fv^2}{D}\ . \tag{6}$$

The operator d/dt is defined as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v\,\frac{\partial}{\partial z}\ . \tag{7}$$

In the model, the hot-zone (Helium gas), which is located where the quench appears, is separated from the cold-zone (Helium fluid) by a hot-cold interface front which has a velocity $\dot{Z}$ at its location, $z = Z$. The initial condition for any fluid element is

$$v(z,0) = 0\ , \tag{8}$$

and the boundary conditions (assuming an infinity cable for the moment) are

$$v(\infty,t) = 0 \tag{9a}$$

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and

$$v(z = Z, t) = \dot{Z} \ . \tag{9b}$$

Using (2) and (3), and identifying $p(\partial v/\partial z)/\rho$ with the rate of change of the external work, $d\omega/dt$, the expression (5) is written as

$$Tds = \frac{q}{\rho} + vF \ , \tag{10}$$

where vF represents the entropy production due to the irreverible conversion by friction of kinetic energy to internal energy [5]. Using (10) and (7) in (4), it follows that

$$\frac{d\rho}{dt} = \frac{1}{c^2}\left(\frac{\partial p}{\partial z} + v\,\frac{\partial p}{\partial z}\right) - \frac{B\rho}{c_p}\left(\frac{q}{\rho} + Fv\right) \ . \tag{11}$$

Assuming now $dv/dt = 0$,

$$\frac{\partial p}{\partial z} = -\rho F \tag{12}$$

in (2), and using this in (11) along with the assumtion that $\rho = constant$ and $c^2 = constant$, the following expression is obtained after some rearragements:

$$\frac{\partial v}{\partial z} + \frac{1}{\rho c^2}\frac{\partial p}{\partial t} = \frac{B}{c_p}\left[\frac{q}{\rho} + \frac{4mf}{D}\,v^3\right] \ , \tag{13}$$

where m is defined as

$$m = \frac{1}{2}\left(1 + \frac{c_p}{Bc^2}\right) \ . \tag{14}$$

Consider now that the Joule power heating density does not depend on z, $\partial q/\partial z = 0$. Then using the expression

$$\frac{\partial^2 p}{\partial t \partial z} = -\frac{4\rho f v}{D}\frac{\partial v}{\partial t} \tag{15}$$

in the partial differentiation with respect to z of the equation (13), it follows that

$$\frac{\partial^2 v}{\partial z^2} = \frac{4fv}{Dc^2}\frac{\partial v}{\partial t} + \frac{3B}{c_p}\left(\frac{4mf}{D}\right)v^2\frac{\partial v}{\partial z} \ . \tag{16}$$

Finally, using (12) and (16) and rearranging terms, the following equation results:

$$\frac{\partial^2 v}{\partial z^2} = (m\beta - \alpha/3)\frac{\partial v^3}{\partial z} \ , \tag{17}$$

where α and β are given by

$$\alpha = \frac{4f}{Dc^2} \tag{18a}$$

and

$$\beta = \frac{4Bf}{c_p D} \ . \tag{18b}$$

The solutions of the equation (17) are given by the following quadrature:

$$\int \frac{dv}{a(t) + (m\beta - \alpha/3)v^3} = z + b(t) \ , \tag{19}$$

where the functions $a(t)$ and $b(t)$ are determined by the boundary and initial conditions.

III. Conditions And Solution

It is not difficult to see that in order to satisfy the condition (9a), the function $a(t)$ must be zero. Applying the condition (9b) to this, the following solution results:

$$v(z,t) = \frac{\dot{Z}}{\sqrt{1 - g(z - Z)\dot{Z}^2}} \ , \tag{20}$$

where g is defined as

$$g = 2(m\beta - \alpha/3) = \frac{8f}{D}\left[\frac{B}{2c_p} + \frac{1}{6c^2}\right] \ . \tag{21}$$

Note that in order for the solution to have physical meaning, the relations $z \geq Z$ and $1 - g(z - Z)\dot{Z}^2 > 0$ must be satisfied. The simplest way to satisfy the condition (8) is to ask for Z the dependence

$$Z(t) = X_o t^n \ , \tag{22}$$

where $n > 1$ and X_o is a constant. This relation is in accordance with the experiments [3]. The expression (20) suggests the velocity of the fluid elements farther apart from the hot-cold interface front. This fact, together with the above observation, imposes the following restriction in the space-time coordinates for the validity of the expression (20):

$$0 \leq z - Z \leq \frac{1}{g\dot{Z}^2} \ , t > 0 \ , \tag{23}$$

i.e., the size of the region, ahead of the hot-cold interface front where the expression (20) is applicable, is inversely proportional to the square of the hot-spot interface front velocity. For $g = 0$, (20) is valid everywhere in the fluid, and each element has the same velocity, $\dot{Z}$.

IV. Temperature And Pressure

Making use of the relation (12), the pressure rise in the fluid at the point z and at the time t is obtained by integrating the equation

$$p(z,t) = -\frac{2\rho f}{D}\int v^2 dz \ . \tag{24}$$

Substituting (20) in (24) and integrating, the next expression arises:

$$p(z,t) = \frac{2f\rho}{Dg}\log\left[1 - g(z - Z)\dot{Z}^2\right] + d(t) \ , \tag{25}$$

where $d(t)$ is an arbitrary function determined by the boundary conditions. Assume that at the distance $z = L$, the pressure drops to zero. Then the pressure is given by

$$p(z,t) = \frac{2f\rho}{Dg}\log\left[\frac{1 - g(z - Z)\dot{Z}^2}{1 - g(L - Z)\dot{Z}^2}\right] \ . \tag{26}$$

At this point the temperature of the differential fluid element "dz" has two contributions: one is due to the pressure itself, $p(z,t)\left(\frac{\partial T}{\partial p}\right)_V$, and the other arises from the wall shear stress ($f_w = D(\rho F)/4$) at the point z, $\frac{2f}{c_p D}\int_0^t v^3(z,\tau)d\tau$ (the mass of the fluid element in consideration is $m = \pi\rho D^2 dz/4$, and the volume generated in the differential dz-displacement is $dV = \pi D v dz dt$). The total temperature rise is given by

$$\Delta T(z,t) = \frac{2f}{c_p D}\int_0^t v^3(z,\tau)d\tau + p(z,t)\left(\frac{\partial T}{\partial p}\right)_V \quad . \quad (27)$$

V. THQB Finish Time

The time at which the entire conductor becomes normal is called the "finish time," and it is given by

$$\Delta T(z = L, t_f) = (\Delta T)_{on} \ , \quad (28)$$

where $(\Delta T)_{on}$ is the change needed to reach the current-sharing threshold temperature. This time is explicitly expressed as the solution of the following equation:

$$\frac{2fn^3X_o^3}{c_p D}\int_0^{t_f}\frac{\tau^{3n-3}d\tau}{\sqrt{1-g(L-X_o\tau^n)n^2X_o^2\tau^{2n-2}}} = (\Delta T)_{on} \ . \quad (29)$$

As Figure 1 shows, for two different s.c. cables of lengths L and l such that $L \geq l$, the finish time satisfies the relation

$$t_f(L) \geq t_f(l) \ , \quad (30)$$

so the THQB induced quench velocity is never higher for shorter s.c. cables. In order for the THQB mechanism to be observed, the finish time must be smaller than the normal quench delay time, $t_q = L/V_q$, and the time of the end-quench phenomenon, t_{end}. This last time is the time taken for the devise to fall off its stored energy. Then the following relation must be satisfied:

$$t_f \leq min\{t_q, t_{end}\} \ . \quad (31)$$

For the experiment in Reference 3 (ORNL), the following parameters are considered: $n = 4/3$, $f = 0.02$, $D = 0.707mm$, $B = 0.1176\ K^{-1}$, $c_p = 8400\ J/K_gK$, $c = 131\ m/s$, and $L = 25\ m$. Then it follows $g = 3.782\times10^{-3}$ and $\frac{2f}{c_pD} = 6.735\times10^{-3}\ Ks^2m^{-3}$, and from Figure 1, the time to see the entire cable-in-conduit normal is about 15 seconds, so a THQB mechanism is not expected here. However, if the diameter, D, is reduced by about one order of magnitude, this mechanism is very likely to appear.

The above approach may not be applicable to the 4-cm R&D full-length dipole magnets, since the parameter D is not well-defined. In addition, the contribution of the Fourier conduction mechanism has not yet been clarified [6].

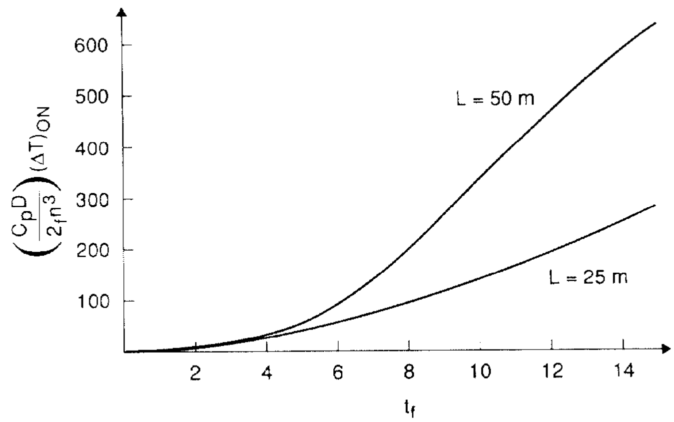

Figure 1: Normalized Change of Temperature as a Function of the Finish Time.

VI. Conclusion

Using the direct integration approach in the one-dimensional differential equations, a restrictive fluid velocity expression was obtained which satisfies the initial and boundary conditions. This expression brings about a finish time which is nonincreasing with the length of the conductor. In particular, if a THQB mechanism is established in a magnet, the expression suggest that the normal zone will grow faster for longer magnets in a quench event. More experimental and theoretical work is required to fully understand this THQB phenomenon.

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