One-Dimensional Time-Independent Conduction States and Temperature Distribution along a Normal Zone During a Quench

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Abstract

For a one-dimensional time-independent conduction state, a constant with respect the longitudinal coordinated, z, is associated. This approach contains the cryogenic stabilization criterion as a particular case. Using this constant, the temperature profile along the conductor is studied considering the effect of thermal conductivity and heat transfer to Helium.

1 Introduction

The quench simulations of a superconducting (s.c.) magnet requires some assumptions about the evolution of the normal zone and its temperature profile. The axial evolution of the normal zone is considered through the longitudinal quench velocity. However, the transversal quench propagation may be considered through the transversal quench velocity[1] or with the turn-to-turn time delay quench propagation[2]. The temperature distribution has been assumed adiabatic-like[1, 3] or cosine-like[2] in two differents computer programs. Although both profiles are different, they bring about more or less the same qualitative quench results differing only in about 8% [4]. Unfortunately, there are not experimental data for the temperature profile along the conductor in a quench event to have a realistic comparison. Little attention has received the temperature profile, mainly because it is not so critical parameter in the quench analysis. Nonetheless, a confident quench analysis requires that the temperature distribution along the normal zone be taken into account with good approximation. In this paper, an analytical study is made about the temperature profile. This is deduced from the one-dimensional and time-independent (TI) states approximation of the heat conduction equation. In the approach, a constant associated to the system is deduced, then, using this constant, the temperature profile can be explicitly given for several cases. The cryogenic stability criterion is an immediate consequence of this approach.

2 Conduction Constant.

Once a quench appears in a s.c. cable, the normal zone moves longitudinally with a speed given by the magnitude of the quench velocity. The normal zone becomes resistive, and its temperature, θ, at the point z and at the time t changes in accordance with the heat equation,

$$(\delta c)\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(k(\theta)\frac{\partial\theta}{\partial z}\right)+\rho(\theta)J^2-\frac{PH(\sigma)}{A}\ , \qquad (1)$$

where (δc) represents the product of the density, δ, times the specific heat, c, averaged over all the components of the conductor; k is the thermal conductivity; ρJ^2 is the Joule heating; P is the perimeter of the conductor in contact with He which has a cross section area A; H is the heat transfer function which depends on $\sigma=\theta-\theta_o$, being θ_o the bath temperature (considered constant). A TI state of this system is the time independent solution ($\partial\theta/\partial t=0$) of the equation (1) which is given by

$$\frac{d}{dz}\left(k(\theta)\frac{d\theta}{dz}\right)+\rho(\theta)J^2-\frac{PH(\sigma)}{A}=0\ . \qquad (2)$$

Defining the function v as

$$v=k(\theta)\frac{d\theta}{dz}\ , \qquad (3)$$

the equation (2) can be written as the following dynamical system

$$\frac{dv}{dz}=\frac{PH(\sigma)}{A}-\rho(\theta)J^2 \qquad (4a)$$

and

$$\frac{d\theta}{dz}=v/k(\theta) \qquad (4b)$$

being $k(\theta)$ a positive function.

A constant, K, associated to this system along the longitudinal direction, z, satisfies the following equation

$$\frac{dK}{dz}=0\ , \qquad (5)$$

where the operator d/dz is given by

$$\frac{d}{dz}=\frac{v}{k(\theta)}\frac{\partial}{\partial\theta}+\left(\frac{PH(\sigma)}{A}-\rho(\theta)J^2\right)\frac{\partial}{\partial v}\ . \qquad (6)$$

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The partial differential equation resulting from (5) and (6),

$$\frac{k(\theta)}{v}\frac{\partial K}{\partial\theta}+\left[\frac{PH(\sigma)}{A}-\rho(\theta)J^2\right]\frac{\partial K}{\partial v}=0\ , \qquad (7)$$

can be solved by the characteristics method. The equations for the characteristics are given by

$$\frac{k(\theta)d\theta}{v}=\frac{dv}{\left[\frac{PH(\sigma)}{A}-\rho(\theta)J^2\right]}=\frac{dK}{0}\ . \qquad (8)$$

From the first two terms, the next characteristic curve is obtained

$$C=v^2/2+\int^{\theta}\left[\rho(\xi)J^2-\frac{PH(\xi-\theta_o)}{A}\right]k(\xi)d\xi\ , \qquad (9)$$

so a constant associated to the dynamic system (4) is

$$K=T+V\ , \qquad (10)$$

where the functions T and V are defined as

$$T=v^2/2 \qquad (11a)$$

and

$$V(\theta)=\int^{\theta}\left[\rho(\xi)J^2-\frac{PH(\xi-\theta_o)}{A}\right]k(\xi)d\xi\ , \qquad (11b)$$

Borrowing the language of Classic Mechanics, the first term of the right hand side of (10) represents the "Kinetic" and the second term the "Potential" contribution to the constant of motion of the system.

The constant (10) can be specified at any point, z_*, obtaining the following relation

$$\frac{1}{2}(v^2-v_*^2)=\int_{\theta(z_*)}^{\theta(z)}\left[\frac{PH(\xi-\theta_o)}{A}-\rho(\xi)J^2\right]k(\xi)d\xi\ . \qquad (12)$$

If there is a region in the conductor such that at given point, $z_*=0$, the temperature is the bath temperature, $\theta(z_*=0)=\theta_o$, and at other point, $z=l$ corresponding to the normal zone, the temperature is $\theta(z=l)=\theta_l$, and at both points the magnitude of heat flux has the same value, $|v(z_*=0)|=|v(z=l)|$, then from the relation (12), Maddock et al's cryogenic stabilization theorem [5] appears as a particular case in this approach,

$$\int_{\theta_o}^{\theta_l}\left[H(\xi-\theta_o)-\frac{A\rho(\xi)J^2}{P}\right]k(\xi)d\xi=0\ . \qquad (13)$$

3 Cosine-like Profile.

Considering the normal zone region where the temperatures are higher than 25 K (the resistivity is not constant), assuming that the thermal conductivity is constant, k_o, and that linear growing of the resistivity with the temperature, $\rho=\rho_1\theta$, the constant associated to this system is given by

$$K=v^2/2+k_o\rho_1J^2\theta^2/2\ . \qquad (14)$$

Rearranging terms in (14), it follows

$$k_o\int\frac{d\theta}{\sqrt{2K-k_o\rho_1J^2\theta^2}}=-z \qquad (15)$$

which can be integrated to obtain the following temperature profile

$$\theta(z)=\sqrt{\frac{2K}{k_o\rho_1J^2}}\ cos\left(\sqrt{\frac{\rho_1J^2}{k_o}}z\right)\ , \qquad (16)$$

where the new constant of integration has been set equal to zero. The constant associated to the system may be valuated by assuming that at the point $z=0$, the flux of heat is zero, $v=0$, and the temperature at this point is the hot-spot temperature, $\theta(z=0)=\theta_x$. Thus, K is given by

$$K=\frac{1}{2}\rho_1\theta_oJ^2\theta_x^2\ , \qquad (17)$$

and the temperature profile is given as[1]

$$\theta(z)=\theta_x\ cos\left(\sqrt{\frac{\rho_1J^2}{k_o}}\ z\right)\ . \qquad (18)$$

4 Polynomial Approximation.

Assume that in a given normal zone region of the conductor, the resistivity, the thermal conductivity, and the heat transfer function can have a polynomial approximation,

$$\rho=\rho_\epsilon\theta^\epsilon\ ,k=k_\gamma\theta^\gamma\ ,\text{and}\quad H=h(\sigma)+H_o \qquad (19)$$

where the powers and coefficients depend upon the intervale of temperatures in consideration. Therefore, the constant (10) has eight possible expressions which can be summarized as follows

$$K=\frac{1}{2}v^2+\begin{cases}\frac{\rho_\epsilon k_\gamma J^2}{1+\gamma+\epsilon}\theta^{1+\gamma+\epsilon} & \text{if } 1+\gamma+\epsilon\neq0\\ \rho_\epsilon k_\gamma J^2Log(\theta) & \text{if } 1+\gamma+\epsilon=0\end{cases}+$$

$$-\frac{Pk_\gamma}{A}\begin{cases}\frac{h\theta^{\gamma+2}}{2+\gamma} & \text{if } 2+\gamma\neq0\\ hLog(\theta) & \text{if } 2+\gamma=0\end{cases}+$$

$$-\frac{Pk_\gamma}{A}\begin{cases}\frac{H_o-\theta_oh}{1+\gamma}\ \theta^{1+\gamma} & \text{if } 1+\gamma\neq0\\ (H_o-\theta_oh)Log(\theta) & \text{if } 1+\gamma=0\end{cases} \qquad (20)$$

[1] In the SSC-RR computer program (see reference 2), the following approximation is used: For the normal zone where the temperatures are lower than about 25 K, the resistivity is constant, ρ_{RRR}. For temperature higher than these, the profile of the resistivity in this normal zone of length L is taken as $\rho=\rho_x cos(\omega z)$, where ρ_x is the resistivity in the hot-spot temperature, and ω is given by $\omega=arcos(\rho_{RRR}/\rho_x)/L$.

In this way, knowing the state (θ, v) in one point of the normal zone, the parameters of (21) and the constant (23) can be known, and solving the integral

$$\int^{\theta} \frac{k(\xi)d\xi}{\sqrt{2K - 2V(\xi)}} = \pm z \ , \qquad (21)$$

the temperature profile can be known in the length of the normal zone where the parametrization is valid. In principle, this procedure may be made section by section of the normal zone, matching the solutions at the boundaries, to obtain the entire temperature profile of the whole normal zone. But, unfortunately it is not always easy to obtain the explicit solution from (21). Two particular cases will be presented below.

4.1 Thermal Conductivity Effect.

Consider the normal zone region where the resistivity is constant($\epsilon = 0$), the heat transfer is neglected ($H_o = 0, h = 0$), and the thermal conductivity power satisfies $1 + \gamma \neq 0$, the constant associated to the system is

$$K = \frac{1}{2}v^2 + \frac{\rho_o k_\gamma J^2}{1+\gamma}\,\theta^{1+\gamma} \ . \qquad (22)$$

The integration of (21) brings about the following solution

$$\theta(z) = \left(\frac{1+\gamma}{2\rho_o k_\gamma J^2}\right)^{\frac{1}{1+\gamma}} \left[2K - (\rho_o J^2 k_\gamma)^2 (z+a)^2\right]^{\frac{1}{1+\gamma}} \ , \qquad (23)$$

where a is a constant of integration. It is clear that the profile when k increases with the temperature ($1+\gamma > 0$) is different when k is decreasing ($1+\gamma < 0$). Even though, for copper, the transition from $1+\gamma > 0$ to $1+\gamma < 0$ occures just about when the copper resistivity starts to increase ($\epsilon > 0$), the above approximation point out the fact that the thermal conductivity has a qualitative change in the normal zone temperature profile. The case $\epsilon > 0$ can be studied following the same procedure.

4.2 Heat Transfer Effect.

Assume that the thermal conductivity and the resistivity are constants($\gamma = 0$ and $\epsilon = 0$), in the normal zone region where the heat transfer is taken place. From (10), the constant associated to this system is

$$K = \frac{1}{2}\,v^2 + [\rho_o k_o J^2 + (h\theta_o - H_o)]\theta - \frac{Phk_o}{2A}\,\theta^2 \ , \qquad (24)$$

and the temperature profile, after integrating (21) and rearranging terms, is

$$\theta(z) = \frac{A}{Phk_o}\left[\rho_o k_o J^2 + h\theta_o - H_o\right] +$$

$$+\frac{A}{Phk_o}\sqrt{\frac{2KPhk_o}{A} + (\rho_o k_o J^2 + h\theta_o - H_o)^2} \ \times$$

$$sinh\sqrt{\frac{Phk_o}{A}}\,(z+\phi) \ , \qquad (25)$$

where ϕ is the constant of integration determined by the condition of the state of the system, (θ, v), at some point. This profile contrasts with that one obtained when no heat transfer is considered

$$\theta(z) = \frac{K}{\rho_o k_o J^2} - \frac{1}{2}(\rho_o k_o J^2)(z+a)^2 \ , \qquad (26)$$

being a a constant of integration. So, the heat transfer also has a qualitative effect in the temperature profile.

5 Conclusion.

Using TI states of the one-dimensional heat conduction equation, some analytical approximation have been used to study the temperature profile of the normal zone of the s.c. cable. The analysis points out that the heat transfer as well as the conductivity may have a non trivial contribution on the temperature distribution during a quench. A computer numerical calculation may give more information if care is taken in considering correctly the non linear element involve in the equation. But even doing this, experimental data are required to compare them with a numerical or any analytical approximations.

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