Divergent Quench Velocity Expression and 4-cm SSC *R&D* Dipole Magnets

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Abstract

A quench velocity expression which has a divergent behavior close to the critical current density is reported here. This quench velocity has the same behavior presented by that of the quench velocity measurements made on the 17-m-long, 4-cm-aperture SSC *R&D* dipole magnets which show a clear departure from the expected theoretical adiabatic expression.

I. Introduction

When a normal zone, with dimensions larger than the minimum propagation zone (mpz), is formed in a superconducting (s.c.) cable [1], it propagates along the conductor with a so-called "axial quench velocity," and the phenomenon itself is called "quench." When the s.c. cable is used to form the coil of a superconducting magnet, the store energy of the magnet is dissipated in the coil in the form of heat generated by the normal zone of the s.c. cable. Special care must be taken to protect the magnet against overheating or major damage may occur in the event of a quench.

The quench velocity measured up to now in the coil (inner-upper turns) of the SSC *R&D* 4-cm dipole magnets has very high value when the operating current is close to the short sample limit (consequently, the hot-spot temperature measured indicates that it is not high [2]), the values are higher than one could expect from the theoretical point of view. This same quench velocity behavior was found in the HERA superconducting dipole magnet [3], and so far there has not been a clear explanation for this phenomenon. The thermal hydraulic quench-back mechanism [3] has been proposed as a possible explanation for these very high quench velocity values. However, in this report, I wish to show that it is possible to obtain a qualitative explanation for this phenomenon using the Fourier conduction mechanism.

II. Usual Quench Velocity

Most of the quench velocity expressions are equivalent to the next one [1]:

$$v = \frac{J_o}{(\delta c)_s}\sqrt{\frac{\rho k}{\theta_s-\theta_o}}\left[1-\frac{2(\theta_s-\theta_o)}{\theta_1-\theta_o}\right]\Big/\sqrt{1-\frac{\theta_s-\theta_o}{\theta_1-\theta_o}}, \quad (1)$$

where J_o is the current density flowing in the copper matrix; δ is the density; c is the specific heat; the quantity (δc) represents the average taken all over the conductor elements and valuated at the temperature θ_s; k is the thermal conductivity; θ_o is the batch temperature; the temperature θ_1 depends on θ_o, the resistivity ρ, J_o, the cross section area A, the perimeter P and the heat transfer coefficient h, as $\theta_1 = \theta_o + \rho J_o^2/hP$; the temperature θ_s depends on the critical temperature, θ_c (at zero current density), and the generation temperature, θ_g as

$$\theta_s = (\theta_g + \theta_c)/2\ . \quad (2)$$

The generation temperature, θ_g, depends on the operating current density, J_o, and the critical current density, J_{co} (at the operating temperature), in the following form:

$$\theta_g = \theta_c - (\theta_c - \theta_o)J_o/J_{co}\ . \quad (3)$$

The heat transfer term corresponds to a very fast transient process having almost no effect in the quench simulations, so it is common to make $h = 0$ in (1) to obtain the adiabatic expression

$$v_{as} = \frac{J_o}{(\delta c)_s}\sqrt{\frac{L_o\theta_s}{\theta_s-\theta_o}}\ , \quad (4)$$

where $L_o = \rho k/\theta_s = 2.45\times10^{-8}W\Omega K^{-2}$ is the Lorentz number. This quench velocity expression depends on the magnetic field through the critical temperature, θ_c, and the critical current density, J_{co} (it is higher for higher magnetic fields). However, this expression does not express the experimental singular behavior of the quench velocity when the operational current approaches the critical current density [4]. In fact, using the relations (2) and (3), the term $\theta_s - \theta_o$ can be written as

$$\theta_s - \theta_o = (\theta_c - \theta_o)(2 - J_o/J_{co})/2\ , \quad (5)$$

which shows a singularity for (4) when $J_o = 2J_{co}$ (note that operating at a current density higher than J_{co} has no physical meaning). There are other expressions [5] which bring about different values than (4), and they depend differently on the operating current density.

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III. SINGULAR QUENCH VELOCITY

Consider one-dimensional quench propagation and assume that the quench appears at the origin $z = 0$ of the reference system S inside the conductor at the time $t = 0$. The conductor is embedded in a magnetic field B, and it drives a current density J_o. The normal zone is propagating on the positive and negative direction with a speed V, which divides the conductor in a normal zone (I) and a s.c. zone (II) regions. The temperature of the normal zone is described by the following equation:

$$(\delta c)\,(\partial\theta/\partial t) = k\,(\partial^2\theta/\partial z^2) + \rho J_o^2 - hP(\theta - \theta_o)/A\ , \quad (6a)$$

and the temperature of the s.c. zone is described by the equation

$$(\delta c)\,(\partial\theta/\partial t) = k\,(\partial^2\theta/\partial z^2) - hP(\theta - \theta_o)/A\ , \quad (6b)$$

where the parameters appearing here have the same meaning as described above.

Change the variables x and t by ξ and τ defined as

$$\xi = x - Vt \quad \text{and} \quad \tau = t, \quad (7)$$

which are the coordinates of a reference system $\widetilde{S}$ traveling to the right at the speed V. Assuming a stationary state in this reference system (the temperature does not change with time) and using the variable U defined by

$$U = \theta - \theta_o, \quad (8)$$

the equations for the temperature in the normal and s.c. zones are given by

$$k\,(d^2U_{\rm I}/d\xi^2) + (\delta c)\,V\,(dU_{\rm I}/d\xi) - hPU_{\rm I}/A + \rho J_o^2 = 0 \quad (9a)$$

and

$$k\,(d^2U_{\rm II}/d\xi^2) + (\delta c)V\,(dU_{\rm II}/d\xi) - hPU_{\rm II}/A = 0. \quad (9b)$$

The solutions of these equations are, respectively,

$$U_{\rm I}(\xi) = \rho AJ_o^2/Ph + a_1\exp(q_1\xi) + a_2\exp(q_2\xi) \quad (10a)$$

and

$$U_{\rm II}(\xi) = b_1\exp(q_1\xi) + b_2\exp(q_2\xi), \quad (10b)$$

where q_1 and q_2 are the characteristic roots of the equation (9) given by

$$q_1 = \frac{(\delta c)\,V}{2k}\left[\sqrt{1 + (hP/Ak)(\frac{2k}{(\delta c)\,V})^2} - 1\right] \quad (11a)$$

and

$$q_2 = \frac{(\delta c)\,V}{2k}\left[\sqrt{1 + (hP/Ak)(\frac{2k}{(\delta c)\,V})^2} + 1\right]. \quad (11b)$$

Now, consider the following conditions in the solutions (10a) and (10b):

i) The heating temperature is located on the far left hand side of our reference system $\widetilde{S}$, that is,

$$\lim_{\xi\to-\infty} U_{\rm I}(\xi) = U_1 = \rho AJ_o^2/hP. \quad (12)$$

ii) The batch temperature, θ_o, is at the far right hand side, that is

$$\lim_{\xi\to+\infty} U_{\rm II}(\xi) = 0. \quad (13)$$

iii) The solutions (10a) and (10b) are matched at the front ($\xi = 0$) where the boundary of the normal and s.c. zone is, to the trigger temperature, θ_* (which takes account of the mpz required for a quench). This condition is expressed by

$$U_{\rm I}(0) = U_{\rm II}(0) = U_*. \quad (14)$$

Using the above conditions in (10), the solutions take the following form:

$$U_{\rm I}(\xi) = U_1 + (U_* - U_1)\exp(q_1\xi) \quad (15a)$$

and

$$U_{\rm II}(\xi) = U_*\exp(q_2\xi). \quad (15b)$$

Finally, applying the condition of continuity of heat flux at the boundary $\xi = 0$, expressed by

$$-k\left(\frac{dU_{\rm I}}{d\xi}\right)_{\xi=0} = -k\left(\frac{dU_{\rm II}}{d\xi}\right)_{\xi=0}\ , \quad (16)$$

and after making some arrangements, the following expression is obtained:

$$V_{\rm g} = J_{co}\sqrt{L_o}\ F(h)q\Big/\gamma(\delta c)_*\sqrt{1 + \frac{\delta\theta}{\theta_c - \theta_o} - q}\ , \quad (17)$$

where $\delta\theta$ is the shift in the generating temperature, $q = J_o/J_{co}$ is the fraction of critical current density, $L_o = \rho k/(\theta_c - \theta_o)$ is the Lorentz number, $\gamma = cu : sc$ is the copper to s.c. ratio, and the function F(h) is defined by

$$F(h) = \left|\frac{2hP(\theta_* - \theta_o)}{A\rho J_o^2} - 1\right|\Big/\sqrt{1 - \frac{hP(\theta_* - \theta_o)}{A\rho J_o^2}}\ . \quad (18)$$

The square root in the denominator of (18) is well-defined since the requirement of heat propagation implies that the condition $\rho J_o^2 > hP(\theta_* - \theta_o)/A$ is satisfied. The adiabatic quench velocity (h=0) is then given by

$$V_{\rm ag} = J_{co}\sqrt{L_o}\ q\Big/\gamma(\delta c)_*\sqrt{1 + \frac{\delta\theta}{\theta_c - \theta_o} - q}\ , \quad (19)$$

which shows explicitly a singularity at the point $q = 1 + \delta\theta/(\theta_c - \theta_o)$. For q lower than 1, the value obtained with (19) approaches that obtained with (4).[1]

[1] It is possible to choose the approximation $\rho k/\theta_* = L_o$ instead of (20) to obtain a singular quench velocity expression too, but this one has different limit for small q.

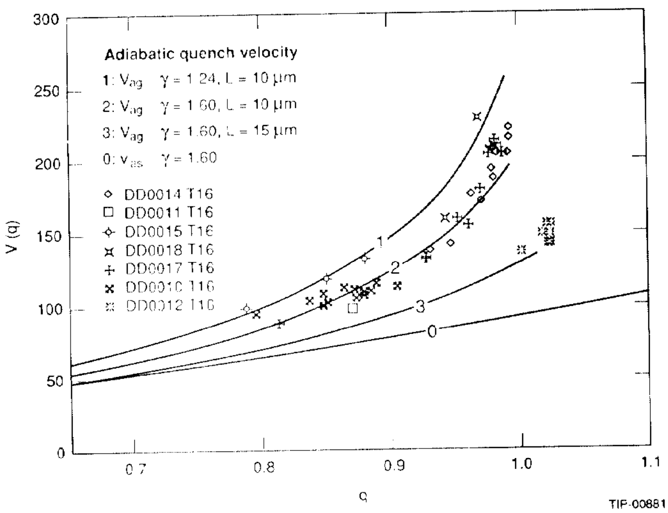

Figure 1: Comparison of the Measured Quench Velocities with both Theoretical Expressions (4) and (19). The length needed for the Normal Zone to propagate is $10\mu m$ or $15\mu m$.

IV. V_{ag} In The SSC $R\&D$ 4cm Dipole Magnets

In order to make the calculation of the adiabatic quench velocity as a function of q, the magnetic field in the conductors is needed as a function of the operating current. Turn 16 is given approximately by the relation [2] $B = 0.7505 + 0.947 \times 10^{-3}\mathbf{I}$, where the current, **I**, is given in amperes and the magnetic field, B, is given in Teslas. The critical current density of the conductor, at the bath temperature $\theta_o = 4.35K$, can be deduced from the reference [6]. Figure 1 shows the result of the calculations comparing the values of both expressions (4) and (19) as well as some experimental results. The length L used to fit the experimental data is between $10\,\mu m$ and $15\,\mu m$ (these values are consistent with the mpz length condition) which gives us a $\delta\theta$ of the order of $10^{-6}K$.

V. Conclusion

Using the Fourier conduction mechanism, it was possible to obtain an adiabatic quench velocity expression (19) which reproduces the quench behavior of the SSC $R\&D$ dipole magnets with respect the fraction of critical current. For its derivation, it was important to notice that the current sharing [7] was neglected and that the heat generation started at the temperature θ_g, but the normal zone initiated its propagation (quench) at the temperature θ_*, which was the temperature that the normal zone front carried away. The quench velocity is an important parameter for confidence on the quench simulation studies to find the appropriate quench protection system for these magnets. The determination of the quench velocity for the other turns in the coil required a more elaborate computer program, now in progress. Without the term $\delta\theta$, expression (19) is singular exactly at $J_o = J_{co}$ and reproduces only qualitatively the experimental data. The term $\delta\theta$ is used to fit the experimental data which, in turn, gives us information about the mpz length needed to trigger a quench (according to the model used here). The singularity obtained in expression (17) appears due to the approximation of not sharing current. It is expected that a better model will remove this singularity, while retaining very high quench velocity values when $q \to 1$. Although a constant specific heat has been used to derive the adiabatic quench velocity expression (19), it is possible to remove this restriction using the concept of Hentalpy [8] and making $h = 0$ in equation (6). The Fourier conduction mechanism suggests a possible qualitative and quantitative explanation to the quench velocity behavior with respect to the fraction of short sample current limit. More detailed experiments are required to rule it out from the connection of the very high quench velocity experimental values observed.

VI. Acknowledgements

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VII. References

[1] M. M. Wilson, *Superconducting Magnets*, Oxford Science Publications 1986, Sections 5.5 and 9.

[2] "Investigation of Heater-Induced Quenches in a Full-Length SSC *R&D* Dipole," SSCL Publication SSC-N-666.

[3] F.K.-H. MeB and P. Schmiiers, "Superconducting Accelerator Magnets," DESY, HERA 89-01, page 49.

[4] A. Devred, "Quench Characteristics of Full-Length SSC *R&D* Dipole Magnet," SSC-234 (1989).

[5] V. Altov *et al.*, *Cryogenics* **13**, 420 (1973). L. Dresner, *Advances in Cryogenic Engineering* **26**, 647 (1980). A. Devred, "Investigation of the Normal Zone....," SSC-218 (1989).

[6] G. Morgan and W.B. Sampsom, "New Coefficients for a $J_{co}(T, B)$ Analytic Form," SSC-N-519 (1988).

[7] L. Dresner, "Analytic Solution...," *IEEE Trans. of Mag.*, Vol. **MAG-15**, No. 1, 328 (1979).

[8] A. Devred and G. López, internal communication, SSC Laboratory, May 1990.