Coil Shapes Towards Pure Multipoles in Circular Regions (A Numerical Approach)

V. Thiagarajan
Accelerator Design and Operations Division
Superconducting Super Collider Laboratory*
2550 Beckleymeade Avenue
Dallas, Texas 75237

Abstract

Coil shapes to produce pure multipole fields in circular regions have been studied and numerically evaluated. Coil shapes have been assumed as functions of unknown coefficients (and prescribed constraining parameters) and cosine functions. The coefficients have been numerically determined to produce the required multipole, simultaneously reducing the other multipoles to zero, or to negligible values.

I. INTRODUCTION

Coil shapes to produce approximately pure multipole fields in circular regions were studied earlier, with cosine expressions for the shapes of coils [1]. The required multipole was found to be of the order of Δ (with $\Delta < 1.0$); the multipole impurities were found to be of the order of higher powers of Δ. It is possible to reduce the impurities further by assuming coil shapes as a linear combination of the profiles (used in [1]) along with weight coefficients; these weight coefficients could be determined to reduce the multipole impurities to zero in a numerical sense. Such a procedure has been studied here, and the numerical method used to obtain these coefficients is described.

II. MULTIPOLE EXPANSION

Let there be a current source $I = ja(da)(d\phi)$, located at the source co-ordinates a, ϕ with the current density being j. Let the coil be bounded by the constant radius a_1 of the circular field region on the inside and a radius $a_2(\phi)$ on the inside. The multipole expansion for the 2-D potential A_z at the field co-ordinates (r, θ) for $r < a$ is [1,2]:

$$\begin{aligned} A_z = \frac{\mu_o}{2\pi}\int_0^{2\pi} j \Bigg[& -ra_1\left[1-\left(\frac{a_1}{a_2}\right)^{-1}\right]\cos(\phi-\theta) \\ & -\frac{r^2}{2}\ln\left(\frac{a_1}{a_2}\right)\cos 2(\phi-\theta) \\ & +\frac{r^3}{3a_1}\left[1-\left(\frac{a_1}{a_2}\right)\right]\cos 3(\phi-\theta)+\ldots \\ & +\frac{r^4}{8{a_1}^2}\left[1-\left(\frac{a_1}{a_2}\right)^2\right]\cos 4(\phi-\theta) \\ & +\frac{r^5}{15{a_1}^3}\left[1-\left(\frac{a_1}{a_2}\right)^3\right]\cos 5(\phi-\theta) \\ & +\frac{r^6}{24{a_1}^4}\left[1-\left(\frac{a_1}{a_2}\right)^4\right]\cos 6(\phi-\theta) \\ & +\ldots\ldots\ldots\ldots \Bigg] d\phi. \end{aligned} \quad (1)$$

The 2-dimensional magnetic field is given by

$$\vec{B} = \nabla X A. \quad (2)$$

The components of $\vec{B}$ in the cylindrical co-ordinate system are

$$\begin{aligned} B_r &= \frac{1}{r}\frac{\partial A_z}{\partial \theta} \\ B_\theta &= -\frac{\partial A_z}{\partial r}. \end{aligned} \quad (3)$$

The cartesian components of the B field can be obtained from the following equation:

$$\begin{aligned} B_x &= B_r\cos\theta - B_\theta\sin\theta \\ B_y &= B_r\sin\theta + B_\theta\cos\theta. \end{aligned} \quad (4)$$

III. COIL SHAPES

It was shown [1] that the following coil shapes and current distributions lead, approximately, to the required multipole:

Dipole:

$$\begin{aligned} a_2 &= a_1(1+\Delta|\cos\phi|) \\ j &= j_o\frac{\cos\phi}{|\cos\phi|}. \end{aligned} \quad (5)$$

Quadrupole:

$$\begin{aligned} a_2 &= a_1 e^{\Delta|\cos(2\phi)|} \\ j &= j_o\frac{\cos 2\phi}{|\cos 2\phi|}. \end{aligned} \quad (6)$$

*Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC02-89ER40486.

Sextupole and higher order poles:

$$
\begin{aligned}
a_2 &= a_1 [1. - \Delta |\cos m\phi|]^{\frac{-1.}{m-2}} \\
j &= j_o \frac{\cos m\phi}{|\cos m\phi|}.
\end{aligned} \tag{7}
$$

In Equation (7), $m = 3$ leads to a sextupole, $m = 4$ leads to an octupole, etc.

The parameter Δ in Equations (5) to (7) must be < 1.0 to keep the design multipole predominant (of the order of Δ), and the multipole impurities small (of the order of higher powers of Δ).

IV. NUMERICAL DETERMINATION OF COIL SHAPES

We will label each term in the integral (1) as I_1, I_2, I_3, etc.; further, each one of these terms can be decomposed into a term containing $\cos\theta$ and a term containing $\sin\theta$; we will label these terms (or, their integrals over ϕ) I_1C, I_1S, I_2C, I_2S, etc. If I_1C were non-zero and the rest were zero, pure dipole field would result. Similarly, if we keep I_2C non-zero and the rest zero, pure quadrupole field results. Higher order multipoles can be achieved in a similar fashion by suitably specializing on the terms in integral (1). We can try to achieve the required multipole design, by using a linear combination of the profiles given in Equations (5) to(7) as follows:

$$
\begin{aligned}
\frac{a_2}{a_1} &= A_1 + A_2|\cos\phi| + A_3 e^{A_4|\cos 2\phi|} \\
&+ \frac{A_5}{(1 - A_6|\cos 3\phi|)} + \frac{A_7}{(1 - A_8|\cos 4\phi|)^{1/2}} \\
&+ \frac{A_9}{(1 - A_{10}|\cos 5\phi|)^{\frac{1}{3}}} + \frac{A_{11}}{(1 - A_{12}|\cos 6\phi|)^{1/4}} \\
&+ \frac{A_{13}}{(1 - A_{14}|\cos 7\phi|)^{1/5}} + \ldots.
\end{aligned} \tag{8}
$$

The task is to determine the coefficients A_1, A_2, etc., to keep one of the integrals non-zero and simultaneously reducing the others to zero. We will address the design of a quadrupole as an example. The current distribution is given by Equation (6). We know from the approximate solution of coil profile for a quadrupole in Equation (6), that it will consist of four crescent-type shapes. Therefore, we need to force the coil profile at three points in a quadrant and calculate the rest of the coefficients. Let us use A_1, A_2, A_3 to force the profile to required values at $\phi = 0, \frac{\pi}{4}$, and $\frac{\pi}{2}$. We will input A_4, A_6, A_8, A_{10}, and A_{12}, and calculate A_5, A_7, A_9, and A_{10} to reduce four undesired integrals to zero. If we substitute Equations (8) and (6) into Equation (1) and carry out the integration, we find that all the terms except I_2C, I_4C, I_6C, I_8C, ..., etc., are zero. In order to preserve the quadrupole field, we need I_2C to be non-zero and the rest to be zero. Hence, we will evaluate A_5, A_7, A_9, A_{11} so as to reduce I_4C, I_6C, I_8C, and $I_{10}C$ to zero.

The first two equations are obtained from the following conditions:

$$\frac{a_2}{a_1} = \beta \text{ at } \phi = 0 \text{ and } \frac{\pi}{2} \text{ (where } 1.0 < \beta < 2.0).$$

The third equation is obtained from the following condition:

$$\frac{a_2}{a_1} = \alpha \text{ at } \phi = \frac{\pi}{4} \quad (\alpha = 1.0, \text{usually}).$$

We need four more equations; they are:

$$
\begin{aligned}
\int_0^{2\pi} I_4C\, d\phi &= 0 \\
\int_0^{2\pi} I_6C\, d\phi &= 0 \\
\int_0^{2\pi} I_8C\, d\phi &= 0 \\
\int_0^{2\pi} I_{10}C\, d\phi &= 0.
\end{aligned}
$$

These seven equations are nonlinear in the unknown coefficients A_1, A_2, A_3, A_5, A_7, A_9, A_{11}, and A_{13}. They can be linearized about an assumed solution, and the resulting linear equations can be iteratively solved with initial guesses for the unknown coefficients. Further details of linearization and numerical solution can be found in [3].

V. RESULTS AND DISCUSSION

The equations detailed in the previous section were solved for $\beta = 1.2, 1.3$ and 1.4, $\alpha = 1.0$. The resulting coefficients for the coil profile are shown in Table 1. Field computations were performed using the code PE2D [4] with $a_1 = 2.65$ cm and with a current density of 3.5E8 amps/m^2, for case 2 (with $\beta = 1.3$). The coil shape is shown in Figure 1, and the potential distribution is shown in Figure 2. The field at a radius of 1.0 cm was used to conduct a harmonic analysis using the following equation:

$$B_y + iB_x = \sum_{n=o}^{\infty} (b_n + ia_n)(x + iy)^n.$$

The resulting coefficients are shown in Table 2. If the quadrupole were pure, $b(1)$ will be nonzero and the rest will be zero. It is seen from Table 2 that $b(1) = 0.5218$, and $b(5) = 0.8048\text{E}{-5}$. The rest of the coefficients are still less. The impurities are found to be less than 0.02%. Additional solutions for quadrupole, dipole, and sextupole, along with details of the solutions can be found in [3].

VI. ACKNOWLEDGEMENTS

The author acknowledges encouragement from Richard York, John Skaritka, Michael Syphers, and Don Edwards.

Table 1: Coefficients for Quadrupole Coil Profile.

	CASE 1 $\beta = 1.2$	CASE 2 $\beta = 1.3$	CASE 3 $\beta = 1.4$
A_1	0.174461	−0.180633	−0.518068
A_2	0.225794E−3	0.372139E−3	0.52793E−3
A_3	0.981432	1.53405	2.11447
A_4	0.2	0.2	0.2
A_5	−0.905029E−3	−0.136362E−2	−0.178955E−2
A_6	0.2	0.2	0.2
A_7	0.13077	0.303228	0.531166
A_8	0.2	0.2	0.2
A_9	0.599738E−5	−0.404481E−3	−0.104305E−2
A_{10}	0.2	0.2	0.2
A_{11}	−0.30121	−0.690685	−1.18746
A_{12}	0.2	0.2	0.2

Table 2: Harmonic Coefficients for Quadrupole Case 2.

n	b(n)	a(n)
0	−0.1807E−7	−0.5889E−14
1	−0.5218	−0.3400E−6
2	−0.2913E−6	−0.2848E−12
3	−0.2864E−6	−0.3733E−12
4	−0.3178E−6	0.5178E−12
5	−0.8048E−5	−0.1573E−10
6	−0.4006E−6	−0.9138E−12
7	0.4440E−6	0.1157E−11
8	−0.487E−6	0.1428E−11
9	0.6660E−6	−0.2170E−11
10	−0.5691E−6	0.2040E−11

VII. REFERENCES

[1] V. Thiagarajan, "Coil Shapes Towards Pure Multipoles in Circular Regions," SSCL-N-733, SSC Laboratory, Dallas, Texas.

[2] P. Schmuser, "Superconducting Accelerator Magnets," DESY HERA 89-01, January 1989.

[3] V. Thiagarajan, "Coil Shapes Towards Pure Multipoles in Circular Regions (A Numerical Approach)," SSCL-399 Rev., SSC Laboratory, Dallas, Texas.

[4] *PE 2D Reference Manual*, Version 8.2, Vector Fields Limited, 24, Bankside, Kidlington, Oxford OX5 1JE, England.

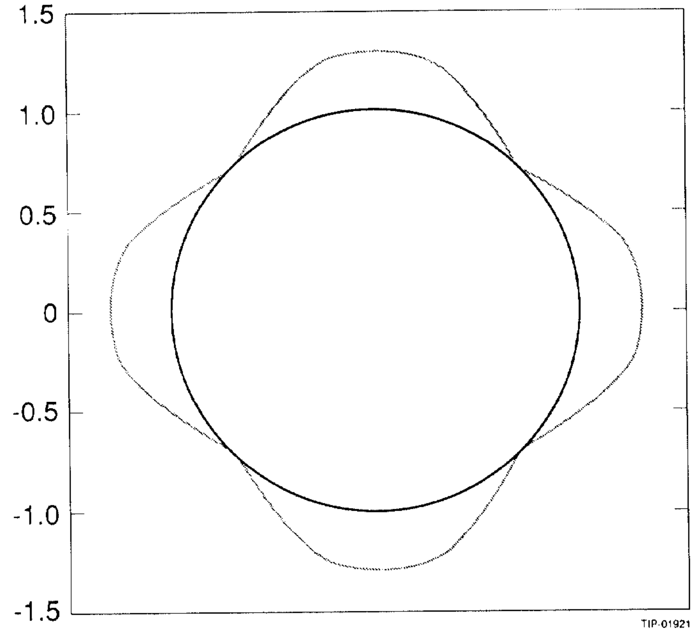

Figure 1: Quadrupole Coil Profile ($\beta = 1.3$).

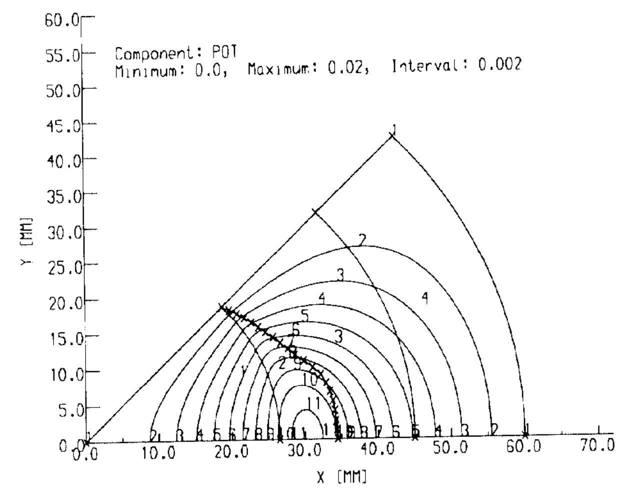

Figure 2: Quadrupole Field Distribution.