

A New 3-D Integral Code for Computation of Accelerator Magnets*

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Abstract

For computing accelerator magnets, integral codes have several advantages over finite element codes; far-field boundaries are treated automatically, and computed fields in the bore region satisfy Maxwell's equations exactly. A new integral code employing edge elements rather than nodal elements has overcome the difficulties associated with earlier integral codes. By the use of field integrals (potential differences) as solution variables, the number of unknowns is reduced to one less than the number of nodes. Two examples, a hollow iron sphere and the dipole magnet of Advanced Photon Source injector synchrotron, show the capability of the code. The CPU time requirements are comparable to those of three-dimensional (3-D) finite-element codes. Experiments show that in practice it can realize much of the potential CPU time saving that parallel processing makes possible.

I. INTRODUCTION

For computing accelerator magnets, integral codes have several advantages over finite element codes; far-field boundaries are treated automatically, and computed fields in the bore region satisfy Maxwell's equations exactly, which is important for optimizing pole shape. Heretofore integral codes have not been able to treat complex iron shapes, nor could they give accurate fields near the iron surface. A new integral code employing edge elements rather than nodal elements has overcome these difficulties better than the earlier codes. The code is described below, and 3-D computations of a hollow sphere and of a dipole magnet for the Advanced Photon Source are presented to compare the new code with earlier codes.

Bossavit [1,2] has popularized the use of edge elements and other Whitney elements in electromagnetic field computation. Subsequently other authors have described integral [3] and finite-element [4] codes employing edge elements.

A vector field F can be approximated inside an element by

$$F = \sum_i f_i w_i \quad (1)$$

For edge elements, the unknown coefficients f are the line integrals of F along each edge and the w_i 's are the edge basis functions. The simplest 3-D element is a tetrahedron with four nodes and six edges. We define the nodes i,j,k,l and

represent edges $e \{i,j\}, \{i,k\}, \{i,l\}$ etc. and the facets $f \{i,j,k\}$, etc. The basis functions are defined as

$$w_e = \lambda_i \nabla \lambda_j - \lambda_j \nabla \lambda_i, \quad e = \{i,j\}, \{i,k\} \quad (2)$$

where the λ_i 's are the barycentric functions.

For first order (4-node) elements, the vector field F varies linearly inside the tetrahedron. But if F is the gradient of a scalar potential, then the line integral of F over a closed loop is zero, and only three of the six edges of the tetrahedron are independent. In that case the vector field F is constant within each tetrahedron.

II. FORMULATION

The magnetic field H is decomposed into the portion H_s produced by the coils and the portion H_m due to the magnetization of the iron:

$$H = H_s + H_m \quad (3)$$

The line integral can be decomposed similarly:

$$\int H \cdot dl = \int H_s \cdot dl + \int H_m \cdot dl \quad (4)$$

We abbreviate (4) as $h = h_m + h_s$, and can express h_m in terms of the scalar potential:

$$h = \phi_i - \phi_j + h_s \quad (5)$$

In terms of the magnetization M , the potential is:

$$\phi = -\frac{1}{4\pi} \int \frac{M \cdot (r-r')}{\|r-r'\|^3} dv \quad (6)$$

where r is the source point and r' the field point. In an integral method, only the iron needs to be discretized. In this code we use a tetrahedral mesh and treat the magnetization as being constant within each tetrahedron. The magnetization and magnetic field are related by the magnetic susceptibility χ , $M = \chi H$, and so are the integrals of field and magnetization:

$$m = \chi h \quad (7)$$

From (6) the potential from a single element can be written:

*Work supported by U.S. Department of Energy, Office of Basic Sciences, under Contract No. W-31-109-ENG-38 and by the Academy of Finland.

$$\phi = -\frac{1}{4\pi} \left(\sum_i m_i w_i \right) \cdot \oint \frac{n}{\|r-r'\|} da, \quad (8)$$

where the w_j 's are the basis functions of the edge elements, defined in (2).

Because the h_m are expressible in terms of potential differences, only $N-1$ of them are independent, where N is the number of nodes. A tree is formed linking all of the nodes, and then all other h_m can be expressed as sums and differences of the h_m of the tree. Equations (5), (7) and (8) can be combined to form the matrix equation to be solved.

$$(C\chi - I)h = h_s \quad (9)$$

III. IMPLEMENTATION

The solution of a problem with the code progresses step by step.

- A. The magnetic material is discretized into a tetrahedral mesh. For this we have used the pre- and postprocessor OPERA [5].
- B. A tree structure of edges is found, connecting the nodes of the mesh.
- C. Line integrals, h_s , are evaluated along each edge of the tree for the field from coils, using the Biot-Savart law, or equivalently, the potential differences produced by the coils.
- D. The matrix C is found using eq. (8). Line integrals h along edges of the cotree are expressed in terms of the integrals along edges of the tree.
- E. The system of equations is solved, and susceptibilities updated.
- F. The process is iterated until the convergence tolerance is reached.
- G. Fields (either B or H) at points inside or outside the steel are evaluated by a generalization of (9). Other electromagnetic quantities can be found also.

IV. HOLLOW SPHERE IN A UNIFORM FIELD

A hollow sphere of nonlinear steel located in a uniform magnetic field provides a good test of a magnetic field code, because the form of the solution fields is well known and because the magnetic shielding of the interior results from a near cancellation of the applied field by the field from the magnetization [6]. We modeled one octant of a sphere of inner radius 0.1 m, outer radius 0.2 m, and solved it with the new code and with the axisymmetric 2-D finite element code PE2D

[5]. The mesh and tree are shown in Fig. 1., and the two solutions are compared in Fig. 2. Details of the computation are shown in Table 1.

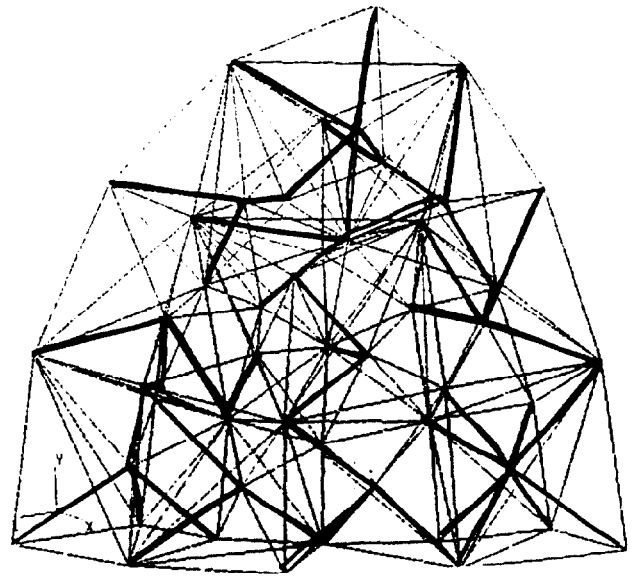


Fig. 1. Mesh for a hollow sphere, with tree accented.

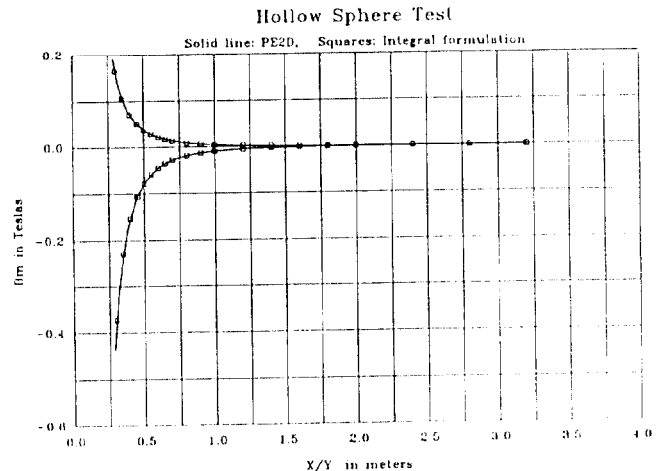


Fig. 2. Computed field for the nonlinear hollow sphere. Variation of field with radius in direction of applied field (lower curve) and perpendicular to applied field (upper curve). Squares: Integral code. Solid curve: Axisymmetric computation with PE2D,

V. DIPOLE MAGNET FOR THE APS INJECTOR SYNCHROTRON

The code was also applied to the dipole magnet of the injector synchrotron for the Advanced Photon Source (APS), now under construction at Argonne National Laboratory (ANL), [7]. Earlier computations, performed by the codes GFUN and TOSCA [5], are described elsewhere [8]. The mesh and tree are shown in Fig. 3., and the solutions are compared in Fig. 4. Details of the computation are shown in Table 1.

VI. PARALLEL COMPUTING

The code described here runs quickly on serial computers; as noted above in Table 1, a computation with 165 nodes and 648 elements required only 20 CPU minutes on the smallest SUN SPARCstation SLC.

Nonetheless, an important feature of the code is that it can take full advantage of parallel computation. In evaluating the geometrical factors in the matrix elements every element is independent of every other; hence parallel computation is very effective. In going from one processor to two, the computation time decreased by a factor of about 1.85. Adding more processors yielded comparable improvement, up to 36 processors were tested.

In solving systems of equations with dense matrices, Gaussian elimination lends itself to parallel computation. Adding a second processor of a shared memory computer was found to decrease the computation time by a factor 1.65 or more, but adding several processors yielded diminishing improvements. For a distributed memory machine, an LU or QR decomposition would be the solution method of choice, but we have not yet used the code on such a machine.

VII. REFERENCES

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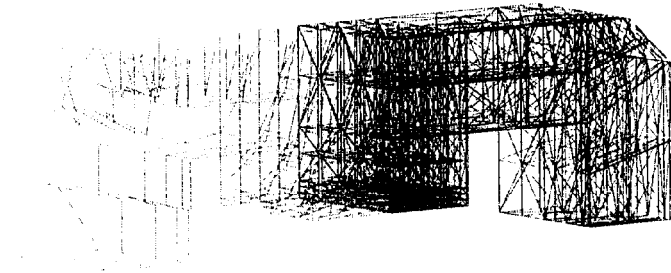


Fig. 3. Tree (left) and mesh (right) for the dipole magnet of the APS injector synchrotron. 1600 elements, 522 nodes.

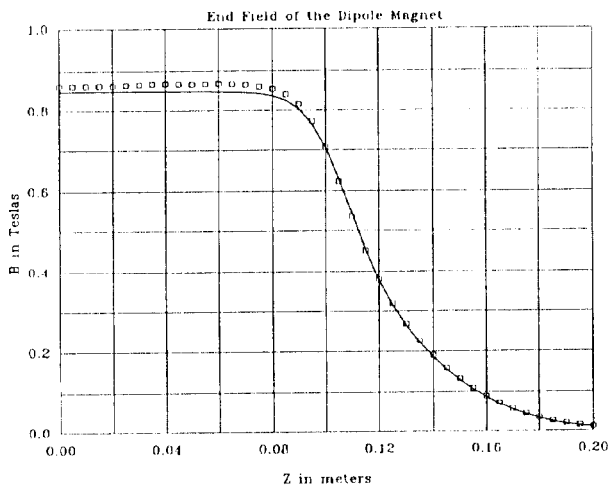


Fig. 4. Computed field for the APS injector synchrotron dipole. Solid line: TOSCA [6], Squares: Integral code.

Table 1
Parameters for the computations

Parameter	Hollow Sphere	APS Inj. Dipole
<i>Linear//Nonlinear</i>	nonlinear	nonlinear
<i>Elements</i>	648	1600
<i>Nodes</i>	165	522
<i>Equations</i>	164	521
<i>Iterations</i>	30	42
<i>CPU-time on SUN SPARCstation SLC</i>		
<i>Field from coils</i>	<0.01 sec	20 min
<i>Matrix setup</i>	10 min	92 min
<i>Tree generation</i>	0.07 sec.	1.06 sec.
<i>Solution of equations</i>	10 min	3 hours
<i>Total:</i>	20 min	5 hours