

## Modification of LAMPF's Magnet-Mapping Code for Offsets of Center Coordinates

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### Abstract

One of the magnet measurements performed at LAMPF is the determination of the cylindrical harmonics of a quadrupole magnet using a rotating coil. The data are analyzed with the code HARMAL [1] to derive the amplitudes of the harmonics. Initially, the origin of the polar coordinate system is the axis of the rotating coil. A new coordinate system is found by a simple translation of the old system such that the dipole moment in the new system is zero. The origin of this translated system is referred to as the magnetic center. Given this translation, the code calculates the coefficients of the cylindrical harmonics in the new system. The code has been modified to use an analytical calculation to determine these new coefficients. The method of calculation is described and some implications of this formulation are presented.

### I. INTRODUCTION

In linear and circular accelerators, quadrupoles are used in an alternating-gradient configuration for strong beam focusing [2]. The force on a particle due to an ideal, square-edged quadrupole is linearly proportional to the radial distance of the particle from the center of the magnet. Real quadrupoles have perturbations that introduce higher harmonics in the field [3]. It is desirable to limit the higher-order field components of the magnetic elements as much as possible, except in special cases such as sextupoles used for chromaticity corrections. For LAMPF's 750-keV proton transport, higher-order fields must be measured and corrected to better than 0.1% of the quadrupole field in the region of the beam [4]. If higher-order fields are present, tails will be introduced causing an effective emittance growth (the fields are conservative so there is no real emittance growth).

After assembly and as part of the fabrication and quality assurance process, magnets are measured with rotating coils [5]. The data are analyzed with the code HARMAL. This code gives the magnetic center offset, the harmonic amplitudes, and the integral of the quadrupole gradient. These results are used to determine the quadrupole field quality and to allow suitable first-order modeling of the magnet in codes such as TRACE [6] or TRANSPORT [7].

The magnetic field center and mechanical center are often misaligned. Relative misalignments of greater than 0.25 mm are often due to fabrication errors and can be corrected. Smaller errors can be corrected by aligning the magnetic center instead of the mechanical center to the beam axis. In addition, the harmonics must be calculated

about the magnetic center instead of the mechanical center. The algorithm for this transformation has been derived and will be shown.

### II. FIELD DESCRIPTION

#### General Field

The equations for a static magnetic field in a current free region can be expressed as a scalar potential satisfying Laplace's equation

$$\nabla^2 \Phi = 0 \quad (1)$$

Cylindrical coordinates are used, with the z-axis in the beam direction. Using separation of variables, one solution to Laplace's equation is

$$\Phi(r, \varphi, z) = - \sum_{n=0}^{\infty} \int_0^{\infty} dk \left( \sum_{j=0}^{\infty} \frac{1}{j!(j+n)!} r^{2j+n} \left(\frac{k}{2}\right)^{2j} \right)$$

$(F_n(k) \cos(kz) + G_n(k) \sin(kz)) \cos(n\varphi + \varphi_n)$ . (2)  
 $F_n(k)$ ,  $G_n(k)$ , and  $\varphi_n$  are the coefficients determined to fit the field in the region of interest. The magnetic field is defined by

$$\vec{B} = -\nabla \Phi \quad (3)$$

The nth harmonic is the term associated with  $\cos(n\varphi + \varphi_n)$ . Note that in the full three-dimensional field, the radial dependence of the nth harmonic is  $r^{2j+n-1}$ ; in particular, the quadrupole ( $n=2$ ) harmonic has a radial dependence  $r^{2j+1}$ , where only the  $j=0$  term is linear. A pure  $n=2$  quadrupole has non-linear terms, especially in the fringe fields where there is a strong z-dependence.

#### Field in Two Dimensions

In many cases the full three-dimensional problem may be reduced to a two-dimensional problem in polar coordinates. For example, if the field is constant along the z-axis, except at sharp boundaries, the problem can be approximated by a two-dimensional solution. Also, at high energies, where the particle's transverse position is approximately constant as it traverses a magnet, the force on the particle can be treated as an impulse proportional to the integral of the field over z. This integral reduces determination of the field to a two-dimensional problem. Solving Laplace's equation in two dimensions or integrating over the potential given by equation 2 leads to a solution

$$\Phi(r, \varphi) = - \sum_{n=0}^{\infty} F_n(k=0) \frac{2\pi}{n!} r^n \cos(n\varphi + \varphi_n) \quad (4)$$

This two-dimensional potential can be equivalently expressed as

$$\Phi(r, \varphi) = - \sum_{n=0}^{\infty} A_n r^n \cos(n\varphi) + B_n r^n \sin(n\varphi). \quad (5)$$

The radial term of the  $n$ th harmonic of the field given by equation 3 is now proportional to  $r^{n-1}$ , and the quadrupole  $n=2$  term is then linear in  $r$ .

### Definition of Field Center

The center of the field is defined as that point at which the field is identically zero. This point, if it exists, will be represented  $(r_c, \varphi_c)$ . There may not be a point where the field is zero; a pure dipole field is not zero at any point. For a quadrupole magnet, a good approximation can be made by finding the point  $(r_c, \varphi_c)$  such that the quadrupole component is opposite to and cancels the dipole moment. If the origin of the coordinate system is at  $(r_c, \varphi_c)$ , then the  $n=1$  dipole coefficients are equal to zero. If the dipole coefficients are equal to zero, then the field is zero at this origin.

### Coordinate Translation

One can translate the coordinate system such that the origin of the new coordinate system is at the point  $(r_c, \varphi_c)$  of the old coordinate system. Fig. 1 represents the translation between the two coordinate systems. The translated coordinate system is represented by a prime. Given any translation of the origin by  $(r_c, \varphi_c)$ , the potential in the this coordinate system is

$$\Phi(r', \varphi') = - \sum_{n=0}^{\infty} A'_n r'^n \cos(n\varphi') + B'_n r'^n \sin(n\varphi'). \quad (6)$$

The coefficients in the new coordinate system,  $A'_n$  and  $B'_n$ , and the coefficients in the old coordinate system are related by

$$A'_n = \sum_{p=n}^{\infty} \frac{p!}{n!(p-n)!} r_c^{p-n} \left[ A_p \cos((p-n)\varphi_c) + B_p \sin((p-n)\varphi_c) \right] \text{ and} \quad (7)$$

$$B'_n = \sum_{p=n}^{\infty} \frac{p!}{n!(p-n)!} r_c^{p-n} \left[ -A_p \sin((p-n)\varphi_c) + B_p \cos((p-n)\varphi_c) \right]. \quad (8)$$

This relation was derived by expressing the potential in Cartesian coordinates and translating to the new coordinate system. The polynomial was expanded and the coefficients of each order determined. The system was transformed back to polar coordinates and the relations 7 and 8 were found when the potential was written in a form such as equation 6.

The old harmonic amplitudes that determine a new harmonic amplitude are equal or greater in order than the new harmonic amplitude being generated. For example, the quadrupole amplitudes for  $n=2$  in the new coordinate system are derived from the  $n=2$  and all higher harmonic amplitudes in the original coordinate system, but are independent of the dipole,  $n=1$  harmonics in the original system. If, in the original coordinate system, there are only dipole,  $n=1$ , and

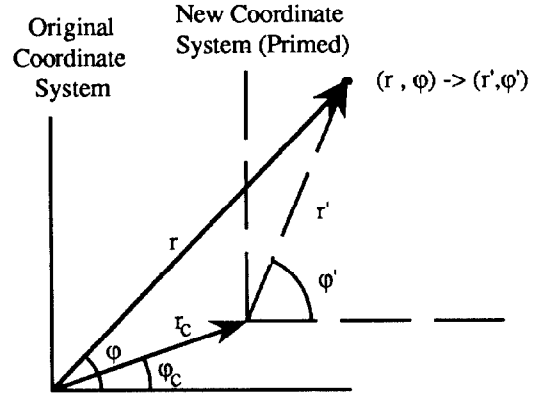


Figure 1. Representation of a general coordinate translation. The origin of the new coordinate system is translated from the old coordinate system by the vector described by  $(r_c, \varphi_c)$ .

quadrupole,  $n=2$ , harmonics, then there cannot be any higher-order terms introduced by a translation of coordinates, and the magnitude of the highest non-zero harmonic cannot be reduced by a translation of coordinates. These conclusions all follow from an inspection of the relations given by equations 7 and 8.

## III. MEASUREMENTS

### Rotating Coil Method

One method to measure the harmonics as expressed in equation 5 is with the use of a rotating coil as represented in Fig. 2. By making the coil long compared to the field, the results are essentially the integral of the field over  $z$ . As the coil is rotated it cuts through the  $B_\varphi$  component of the

field, generating a voltage proportional to  $\frac{d}{dt} \left( \int_{-\infty}^{\infty} dz B_\varphi \right)$ .

By measuring the voltage across the coil, the integrated  $B_\varphi$  field can be determined as outlined below.

### Data Analysis

At LAMPF the code used to analyze the harmonic data generated by the rotating coil is HARMAL. The coil steps through a series of angles, and at each angle a voltage is measured proportional to the integrated current out of the

coil. The voltage is also proportional to the integrated field through the coil at each angle

$$V(\varphi) \propto \int dz B_{\varphi}$$

$$= \sum_{n=0}^{\infty} -nA_n r^{n-1} \sin(n\varphi) + nB_n r^{n-1} \cos(n\varphi). \quad (9)$$

The code determines the coefficients  $A_n$  and  $B_n$  by a Fourier analysis of the voltage  $V(\varphi)$ .

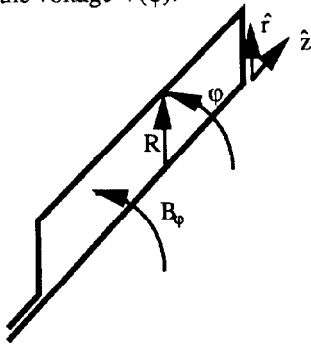


Figure 2. Representation of a rotating coil showing its relation to the cylindrical coordinates. The coil is of radius  $R$  and rotates about one of the wires.

Measurement and analysis of a quadrupole magnet usually have a small  $n=1$  dipole amplitude caused by some misalignment. HARMAL determines the field center  $(r_c, \varphi_c)$  at which the field is zero [8]. This is done by determining the harmonic representation of the field in the original coordinate system and then using Newton's method to determine where the field is zero. The starting point for application of Newton's method is the coordinate at which the dipole moment is canceled by the quadrupole moment.

Once the translation is determined by HARMAL, the harmonic amplitudes must be redefined in the new coordinate system. HARMAL was modified in 1990 to transform the coefficients using the relations specified by equations 7 and 8. The relations and the code were checked against numerically generated fields and simple analytical fields that could be translated independent of the code's translation. The code was then required to translate the fields back and obtain the same result as the original field. Agreement was found to the numerical accuracy of the code.

#### IV. CONCLUSION

To reduce effective emittance growth and tails, higher-order harmonics must be reduced to about 0.1% of the fundamental quadrupole component. To reduce steering effects of quadrupoles, the quadrupole field center must be aligned to the beam axis to better than 0.25 mm. The harmonic components and field center can be measured with a long rotating coil. At LAMPF, the data are analyzed with the code HARMAL. HARMAL determines the coordinate translation that puts the origin at the field center where the field is zero. A new algorithm for determining the harmonic amplitudes in this translated coordinate system has been

derived and incorporated into HARMAL. This transformation has been tested with a few simple cases and has been found to be correct in these cases.

#### IV. REFERENCES AND ACKNOWLEDGEMENTS

##### Acknowledgements

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##### References

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