RACETRACK LATTICES FOR LOW-MEDIUM-ENERGY SYNCHROTRONS

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I. INTRODUCTION

In design of magnet lattice for an accelerator the various requirements depending on machine specifications can be met. In this article we discuss the possible lattice design for low-energy ($\gamma_{max} \simeq 3 \div 10$) and medium-energy ($\gamma_{max} \simeq 30 \div 50$) high-intensity synchrotrons. We will be restricted and concentrate on those lattices in which the following requirements are fulfilled:

- the transition energy γ_t is kept high (or low) enough to satisfy many conditions of beam stability;
- the accelerator ring has a racetrack shape with two 180° arcs and two long dispersion-free straight sections desired for rf cavities, injection and extraction elements, Siberian Snakes, etc.;
- the dynamic aperture in presence of chromaticity-correcting sextupoles is sufficiently large.

A careful analysis shows that these requirements are not really contradictory ones[1], but often it is hard to fulfil them together.

II. MAIN TYPES OF MAGNET LATTICES

Since a choice of lattice for arc depends to a large extent on the requirement to avoid the transition crossing, let us remind the methods of creating transitionless lattices. All lattices can be divided in two groups

- lattices with regular arc;
- lattices, in which arc superperiodicity is introduced.

A. Regular lattice

The term — "regular" is concerned to such lattices, in which the arc periodicity coincides with a number of the basic optical building blocks — cells. In such lattices the transition energy $\gamma_t \approx Q_x$ [1], where Q_x is the horizontal betatron tune. The dispersion in long straight sections is canceled by using dispersion suppressors placed at the ends of arcs. As a rule, the dynamic aperture of such rings with full chromaticity correction is a sufficiently large. If it is allowed to keep γ_t not far from the range $[\gamma_{min}, \gamma_{max}]$, then a regular lattice with a high tune Q_x can be adopted for low-energy rings and, with a low tune, for medium-energy (with $\gamma_{min} \simeq 7 \div 10$) synchrotrons. This method has been used in the lattice design of the MKF Main Ring [2].

However, for increasing of the threshold of microwave instability and for the longitudinal matching it is often important to obtain very high or even imaginary γ_t. In the next section we will consider three different methods, by which the γ_t can be altered over a broad range of values.

[1] Based on these requirements we have proposed the lattice designs for MKF and TRIUMF low-energy Boosters.

B. Superperiodic lattice

The basic principle is well known [1,3] — it is necessary to create a perturbation of the dispersion function, that requires to break the natural arc periodicity and to introduce a new periodicity — superperiodicity S. Then, if the horizontal tune of the arc $Q_x \simeq pS$, where p is an integer, a modulation of the dispersion can be achieved, and (for $Q_x < pS$) the transition energy may become high. In the case of a superperiodic lattice the use of special dispersion suppressors is unsuitable, and the dispersion in the long straight sections is canceled by tuning the horizontal tune of each arc to an integer number (the first-order achromat) [4].

The oscillation of the dispersion function corresponding to a high γ_t significantly reduces a number of places in which the chromaticity-correcting sextupoles can be installed. In general, the sextupoles can not be organized in the non-interleaving pairs with phase advance of π in both planes between elements of each pair to cancel the nonlinear effects of the sextupoles. This may result in serious problem of obtaining of a large enough dynamic aperture. The possible way to improve the dynamic aperture is to fulfil the conditions for the arc to be a second-order achromat[2][5]:

- the chromaticities $\xi_x = 0$ and $\xi_y = 0$;
- the tunes of each arc $Q_x = S\nu_x$, $Q_y = S\nu_y$ are equal to integers;
- the tunes of the superperiod are not in resonances $m\nu_x + n\nu_y \neq integer$, where $(m,n) = \{(1,0),(3,0),(1,2),(1,-2)\}$.

In this case all low order resonances introduced by sextupoles are compensated at the ends of each arc. Nevertheless, the compensation has the local nature and the influences of these resonances are still great near the middle of the arcs. Just because one should carry out the analysis of a phase-space distortion and growth of the effective emittances for two different points in the lattice: the first inspection point is located at the entrance of the arc, the second one is located near the middle of the arc. In Fig. 1,2 the phase-space plots $p_{xn} = -1/\sqrt{\beta}(\alpha x + p_x\beta)$ versus $x_n = x/\sqrt{\beta}$ for horizontal betatron motion are shown for these points of the arc. One can see the large phase-space distortion in the middle of the arc corresponding to the strong 1/3 resonance.

Thus the fact that arcs form the second-order achromats does not work out completely the problem of the improvement of the dynamic aperture. The only reliable method to minimize the effects of the nonlinear aberrations is to reduce them — keeping the betatron amplitudes as low as possible and decreasing strength of sextupoles. And a work should be done along this line.

There are following different methods to create a superperiodicity S [3]:

- to change the field gradients in arc's quadrupoles (β-function modulation) or/and

[2] We suppose that the periodicity in the sextupole's distribution in the arcs coincides with the number of arc's superperiods.

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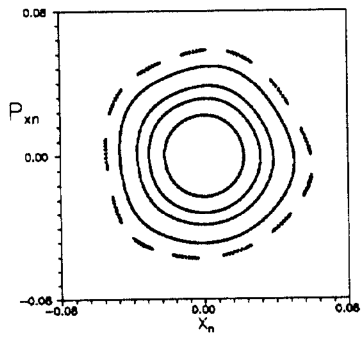

Figure 1: The MKF Booster turn-by-turn tracking data displayed in normalized phase space. Plot corresponds to the entrance of the arc.

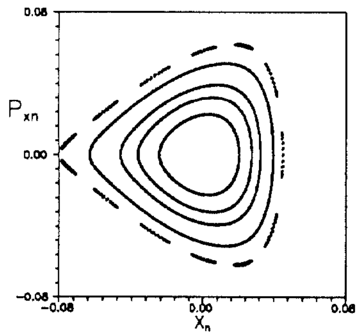

Figure 2: The same phase-space plot as in Fig. 1, but the indicated particle positions correspond to the middle of the arc.

- to violate a regular dipole arrangement (modulation of curvature radius ρ).

Let us consider the first method.

Perturbations in quadrupole strengths

In the past this method had been used only for 'circular' lattice design. For the fist time the racetrack lattice with perturbations in quadrupoles was realized for design of TRIUMF KAON Factory medium-energy rings [6]. Some later the same idea was used in the first proposal of the racetrack lattice for MKF 45 GeV Main Ring [7].

Starting with a regular lattice, the arc superperiodicity S is created by introducing perturbations in field gradients of some quadrupoles. These perturbations result in a modulation of the β-function, and the dispersion becomes negative in some regions of the arc so that the transition energy γ_t is increased to a high (or imaginary) value. In Fig. 3 the maximum fractional changes in β-function and dispersion computed by using analytic formulas [8], for case $\gamma_t \simeq \infty$, versus tune of a superperiod ν_x are shown. We can see to avoid high peaks of the β-function one should put ν_x near 1. But to prevent a high dispersion, ν_x should be chosen far from 1. Since a high peak value of the β-function is more dangerous, the optimal values for ν_x lie in the range $0.8 \div 0.875$. Thus the large undesirable changes in β-function can be reduced by making the horizontal tune of each arc equals to $S-1$ so that $\nu_x = 1-1/S$. For large rings, in particular for medium-energy synchrotrons, consisted of a large number of cells, it is possible to create a high arc periodicity $S = 5 \div 8$ ($\nu_x = 0.8 \div 0.875$) and, as consequence, to obtain a high γ_t using relatively small modulation

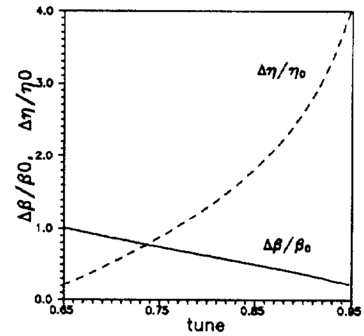

Figure 3: Maximum fractional changes in β-function and dispersion for $\gamma_t \simeq \infty$.

of the β-function. The analysis of resonance properties of such lattices shows that the dynamic aperture is large enough.

After successful progress of lattices for medium-energy synchrotrons the same idea was realized for lattice designs of low-energy boosters of TRIUMF and Moscow KAON Factories [7,9]. Since the typical value of arc superperiodicity is $3 \div 4$, then $\nu_x \simeq 0.67 \div 0.75$. Then, as we can see from Fig. 3, for obtaining a high γ_t it is required more modulation of the β-function. And since the resonance strengths excited by sextupoles are proportional to $\beta^{3/2}$, the dynamic aperture should be more sensitive to chromaticity correction. As it was to be expected, dynamic aperture for such lattices with full chromaticity correction is unacceptable reduced [9,10]. Besides in this lattice there are not enough room in arcs to accommodate desirable diagnostic devices, collimators, correctors etc.

Thus this method seems to be unsuitable for low-energy rings and one should consider some different approach.

Scheme with modulated orbit curvature

The first proposal for eliminating of transition energy [11] have consisted in using of dipoles with reversed curvature. The evident defect of the method — unacceptable large increase of ring's length. A less extreme variant is the use of a regular focussing lattice, in which some of cells have not dipoles — 'missing magnet' scheme. The method was used for design of many lattices having a 'circular' shape. In such lattice the empty cells have a double meanings: they are needed for high γ_t and they form the straight sections used for injection,extraction, etc. The application of this method for a racetrack design has essential difficulties. As it can be shown [3,12] to obtain a high γ_t one should have the horizontal tune of a superperiod very close to 1: $\nu_x \simeq 0.9$. Obviously, this requirement can not be satisfied in a racetrack low-energy rings with $Q_x = S - 1$. And one should choose the horizontal tune of a superperiod ν_x so close to 1 as it is required. In this case, evidently, the condition of the first-order achromat can not be fulfilled, and we have to renounce our requirement of dispersion cancelation in both straight sections. Thus in design of a racetrack low-energy synchrotron we have met with two oppositions: high γ_t and zero dispersion in straight sections. Keeping within the bounds of *only* 'missing magnet' scheme we should

1. either to require the dispersion cancelation in only one straight section. This condition can be written as [10,13] $\nu_{x\,str1} = \frac{2m+1}{2} - \frac{Q_x}{2}$, where $\nu_{x\,str1}$ – horizontal tune of dispersion-free straight section, $Q_x/2 \neq$ an integer, m

– an integer. However, in such lattice it is so hard to accommodate all rf cavities in only one dispersion-free straight section. The one superperiodicity of whole ring is not so friendly for spin control and should result in exciting of strong structure resonances;

2. or finally say 'good-bye' to hope of obtaining zero dispersion in straight sections and put $\nu_{x\,str1} = \nu_{x\,str2}$. In this case the dispersion would have nonzero although small value in the straight sections, and a careful analysis of synchro-betatron coupling in cavities should be done.

Now we are in the position to consider a scheme that seems to be the most convenient for a racetrack design of low-energy synchrotrons.

Scheme with both modulated ρ and β

As far as we know the fist proposal of using a scheme with two modulations for 'circular' lattice design was done in [3]. In that paper a superperiodicity had been created by modulating drift spaces between quadrupoles and dipoles. It was shown that by increasing drift length, for example, near the ends of each superperiod and by increasing magnet density around center of the one, the variations in both β and ρ, and as a result a high γ_t may be achieved. But it is not difficult to see that the scheme is hardly adopted for racetrack designs. Since to obtain a high value of γ_t it is needed a large difference between long and short drifts. It leads to considerable increase of arc length and large beatings of the lattice functions.

The alternative approach is the use of 'missing magnet' and β-modulation schemes simultaneously. As an example of the method let us consider the low-energy (7.5 GeV) booster of MKF [10,13]. The lattice has a racetrack shape. Each arc has superperiodicity $S = 4$. The superperiod contains four $FODO$ cells, two central halve-cells have not dipoles ('missing magnets'). The horizontal tune of arc equals to 3, that gives zero dispersion in long straight sections. It is clear that γ_t is not so high, but using small perturbations in quadrupoles it is possible to obtain any γ_t without significant modulation of the β-function (see Fig. 4). As long as,

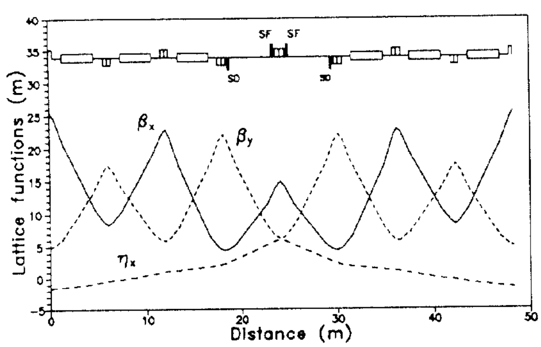

Figure 4: Lattice functions for a superperiod of MKF booster (a scheme with both β and ρ modulation).

- the analysis shows the dynamic aperture is sufficiently large when the full chromaticity correction is made;
- empty cells do not considerable increase the length of the ring (the superperiodicity of arc is not so great), moreover the short straight sections can be useful for accommodation of collimators, diagnostic devices, correctors and injection systems,

it seems that proposed lattice is the best solution for low-energy synchrotrons.

III. CONCLUSION

We have described an alternative transitionless lattices for low-medium-energy synchrotrons, which have a racetrack shape and a large enough dynamic aperture. We have shown that a lattice based on β-modulation is a good solution for medium-energy synchrotrons, and a lattice with modulation of β-function together with 'missing magnet' scheme is most suitable for low-energy rings.

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