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HOW TO GET A SEPARATRIX BRANCH WITH LOW DIVERGENCE AT A 1/3-INTEGER RESONANT BEAM SLOW EXTRACTION

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I. INTRODUCTION

After the invention by H.Hereward of the resonant slow extraction scheme [1], this method is widely used to almost all existing synchrotrons. An analytical treatment of the resonant extraction based upon 1/2 or 1/3-integer resonances are proposed in numerous works [2,3,4]. This article deals with the particular problem concerned with the separatrix dependent on chromaticity in 1/3-integer resonance. It means that the outgoing separatrices do not overlap for different momentum in other words they occur to be smeared. It is desired to get a separatrix branch with low divergence at the septum position in order to minimize the particle losses on it. Hardt [5] proposed the method of compensation of this effect by introducing nonzero dispersions in the beam extraction straight section. But in his formula only global chromaticity is included and not local. In this article some handy formulas are proposed using analytic techniques for solving this problem including both global and local chromaticity effects. These formulas have been applied to the design of the resonant slow extraction system for the Extender Ring of the Moscow Kaon Factory [6].

II. DERIVATION OF THE FORMULAS

In this Section we will use the formalism of the resonance canonical perturbation theory expounded in [7]. We start with the Hamiltonian describing the particle dynamics near 1/3-integer resonance:

$$H = \Delta e J + \varepsilon J^{3/2}\cos 3\varphi\,. \qquad (1)$$

Here $\Delta e = Q_x - p/3$ is the frequency difference, $Q_x = Q_x^{(0)} + Q_x'\delta$ is the horizontal betatron tune, Q_x' is the horizontal chromaticity, $\delta = \Delta P/P_0$ is the fractional difference of the particle momentum P from that of the reference momentum P_0, ε is the resonance force which is determined by the strength S and location of the sextupoles as follows:

$$\begin{aligned}\varepsilon &= (A^2+B^2)^{1/2}\\ A &= \frac{1}{24\pi}\sqrt{\frac{2}{1+\delta}}\int_0^C S\beta_x^{3/2}\cos 3(\mu_x - \Delta e s/R)\,ds\,,\\ B &= -\frac{1}{24\pi}\sqrt{\frac{2}{1+\delta}}\int_0^C S\beta_x^{3/2}\sin 3(\mu_x - \Delta e s/R)\,ds\,.\end{aligned}\qquad (2)$$

Here C is the circumference of the ring, α_x, β_x and μ_x are the Twiss parameters and the phase advance in the horizontal space along the closed nonlinear equilibrium orbit at s location. The action J and angle φ variables are transformed to the Cartesian coordinates p_x and x by formulas:

$$\begin{aligned}x &= \delta D_x + \sqrt{\frac{2\beta_x J}{1+\delta}}\cos\Phi\\ p_x &= \delta D_x' - \sqrt{\frac{2(1+\delta)J}{\beta_x}}\cos\Phi\Big[\tan\Phi + \alpha_x\Big]\,,\end{aligned}\qquad (3)$$

where

$$\begin{aligned}\Phi &= \varphi + \mu_x - \Delta e s/R - \chi/3\\ \chi &= -\arctan(B/A)\,.\end{aligned}\qquad (4)$$

The phase portrait looks very simply in the coordinates $(2J)^{1/2}\cos\varphi$ and $(2J)^{1/2}\sin\varphi$ — it is a stable triangle limited by straight separatrises. The fixed points of the hamiltonian system with the Hamiltonian (1) for $\Delta e < 0$ are:

$$\begin{aligned}J_i^u &= \Big(\frac{2\Delta e}{3\varepsilon}\Big)^2\\ \varphi_i &= i\frac{2\pi}{3}\,,\quad i=0,1,2\,.\end{aligned}\qquad (5)$$

and for $\Delta e > 0$ are:

$$\begin{aligned}J_i^u &= \Big(\frac{2\Delta e}{3\varepsilon}\Big)^2\\ \varphi_i &= \frac{\pi}{3} + i\frac{2\pi}{3}\,,\quad i=0,1,2\,.\end{aligned}\qquad (6)$$

Henceforward for definition without violation generality we suppose that $\Delta e < 0$. As far as we are interested in functions at the septum location so also we will put the origin of the beam axis at this location. Then in coordinates x and p_x the formula (5) looks as:

$$\begin{aligned}x_i &= \delta D_x + \sqrt{\frac{2\beta_x}{1+\delta}\frac{2|\Delta e|}{3\varepsilon}}\cos\Big(i\frac{2\pi}{3} - \frac{\chi}{3}\Big)\\ p_{xi} &= \delta D_x' - \sqrt{\frac{2(1+\delta)}{\beta_x}\frac{2|\Delta e|}{3\varepsilon}}\cos\Big(i\frac{2\pi}{3} - \frac{\chi}{3}\Big)\\ &\quad\times\Big[\tan\Big(i\frac{2\pi}{3} - \frac{\chi}{3}\Big) + \alpha_x\Big]\,,\quad i=0,1,2\,.\end{aligned}\qquad (7)$$

As seen from (7) the linear dependence of the stable triangle position on δ is determined by the dispersions D_x and D_x' and the horizontal tune chromaticity Q_x'. Meanwhile the nonlinear dependence is determined by $\beta_x(s,\delta)$, $\alpha_x(s,\delta)$, $\mu_x(\delta)$, $e(\delta)$, $\chi(\delta)$ and impede the further analytical treatment. We simplify the problem by consideration only the linear theory in δ. We expand the right sides in (7) in a series in power of δ and omit the terms of power higher than 1. It leads to

$$\begin{aligned}x_i &= \delta D_x + \frac{2\sqrt{2\beta_x^{(0)}}}{3}\frac{|\Delta e_0|}{\varepsilon_0}\cos\chi_i^{(0)}\\ &\quad\times\Big\{1 + \Big[\frac{\beta_x^{(1)}-1}{2} + \frac{Q_x'}{\Delta e_0} - \varepsilon_1 + \frac{\chi_1}{3}\tan\chi_i^{(0)}\Big]\delta\Big\}\end{aligned}$$

$$\begin{aligned}p_{xi} &= \delta D_x' - \frac{2}{3}\sqrt{\frac{2}{\beta_x^{(0)}}}\frac{|\Delta e_0|}{\varepsilon_0}\cos\chi_i^{(0)}\Big\{\tan\chi_i^{(0)} + \alpha_x^{(0)}\\ &\quad + \Big[\Big(\tan\chi_i^{(0)} + \alpha_x^{(0)}\Big)\Big(\frac{Q_x'}{\Delta e_0} - \frac{\beta_x^{(1)}-1}{2} - \varepsilon_1\Big) + \alpha_x^{(1)}\\ &\quad - \frac{\chi_1}{3}\Big(1 - \alpha_x^{(0)}\tan\chi_i^{(0)}\Big)\Big]\delta\Big\}\,,\quad i=0,1,2\end{aligned}\qquad (8)$$

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where $\chi_i^{(0)} = i(2\pi - \chi_0)/3$ or in short form:

$$\begin{aligned} x_i &= x_i^{(0)} + (D_x + x_i^{(1)})\delta \\ p_{xi} &= p_{xi}^{(0)} + (D'_x + p_{xi}^{(1)})\delta \end{aligned} \quad (9)$$

Here we used the next notations:

$$\begin{aligned} \beta_x(\delta) &= \beta_x^{(0)}(1 + \beta_x^{(1)}\delta + \ldots) \\ \alpha_x(\delta) &= \alpha_x^{(0)} + \alpha_x^{(1)}\delta + \ldots \\ \varepsilon(\delta) &= \varepsilon_0(1 + \varepsilon_1\delta + \ldots) \\ \chi(\delta) &= \chi_0 + \chi_1\delta + \ldots \end{aligned} \quad (10)$$

Let x_s is a septum location and p_{xs} is a value of straight separatrix crossing two fixed points, e.g. M_0 and M_2 at $x = x_s$. We get for p_{xs}:

$$p_{xs} = \frac{(p_{x2} - p_{x0})}{(x_2 - x_0)}(x_s - x_0) + p_{x0} \quad (11)$$

Certainly p_{xs} is a function of δ and is of our interest. We demand of this function $p_{xs}(\delta)$ that its first derivative with respect to δ equals to zero at $\delta = 0$:

$$\left.\frac{dp_{xs}}{d\delta}\right|_{\delta=0} = 0 \quad (12)$$

For small δ which is a common case for large synchrotrons this demand provides approximately constant value of the function $p_{xs}(\delta)$ and what is actually need. So from (9), (12) it follows that

$$\left.\frac{dp_{xs}}{d\delta}\right|_{\delta=0} = F_0 D_x + D'_x + F_1 = 0\,, \quad (13)$$

where

$$\begin{aligned} F_0 &= -\frac{p_{x2}^{(0)} - p_{x0}^{(0)}}{x_2^{(0)} - x_0^{(0)}} \\ F_1 &= p_{x0}^{(1)} - \frac{(p_{x2}^{(0)} - p_{x0}^{(0)})}{(x_2^{(0)} - x_0^{(0)})}x_0^{(1)} + (x_s - x_0^{(0)}) \\ &\times \frac{(p_{x2}^{(1)} - p_{x0}^{(1)})(x_2^{(0)} - x_0^{(0)}) - (p_{x2}^{(0)} - p_{x0}^{(0)})(x_2^{(1)} - x_0^{(1)})}{(x_2^{(0)} - x_0^{(0)})^2}. \end{aligned} \quad (14)$$

From the mathematical point of view we have one equality which binds the values of 11 functions D_x, D'_x, Q'_x, $\beta_x^{(0)}$, $\beta_x^{(1)}$, $\alpha_x^{(0)}$, $\alpha_x^{(1)}$, ε_0, ε_1, χ_0, χ_1 at the septum location and so have 10 independent variables. But the practical design of a synchrotron may impose additional demands. For example, the Extender ring of the Moscow Kaon Factory has a racetrack lattices structure with two long straight sections one of which provides beam extraction and has fixed lattices excluding matching lines. To get the extraction section with the periodic dispersion functions a multiple of 2π horizontal phase advance is required. So if μ_x is a phase advance on the line matching with the arcs then the dispersion functions at the septum location must satisfy the following equation:

$$D'_x = -\tan(\mu_x)\frac{D_x}{\beta_x^{(0)}} \quad (15)$$

The procedure of lattices design for the Extender ring of the MKF allows to vary parameters of the optic elements only in the dispersion supressors, in the matching lines of the extraction straight section and in the second straight section. As far as properly the lattices with the extraction straight section are fixed so the Twiss parameters — $\beta_x^{(0)}$, $\beta_x^{(1)}$, $\alpha_x^{(0)}$, $\alpha_x^{(1)}$ — at the septum location are fixed too. Moreover such procedure disturbs the other parameters — ε_0, ε_1, χ_0, χ_1 — only slightly. So two or three iterations of such procedure with the successive applying of the given formulas lead to the success.

Thus only one independent variable — Q'_x — is left and it is allowed to introduce any pure mathematical requirement for simplification of the above equations. Such requirement is dictated only by the user's needs and taste and finally is not obliged.

III. RESULTS

The actual divergence slightly differs from those which Hardt's and the present theory supply because they do not consider the high order perturbations at all. Consequently one should take care of the minimizing these perturbations before using these theories. From the other side the more absolute value of the chromaticity Q'_x as well as δ correspond to the more difference between the results due to Hardt and the present theory. In the present time the designed lattice structure for the Extender Ring of the Moscow Kaon Factory can support only achromatic mode when $Q'_x = 0$. In that case Hardt theory proposes $D_x = D'_x = 0$ and as a result zero beam divergence at the septum position. The present theory with the same dispersion functions gives non zero magnitude of the beam divergence equal to 0.002 $mrad$ at the septum position for $|\delta| \leq 0.00035$. The actual divergence has been executed by means of the DIMAD code [8] which provides the exact values of the fixed points at a 1/3 - integer resonance. The value of the divergence has been found at 0.005 $mrad$ for the same rang of δ. The advantage of the present approach over the other is hardly visible in this example but expected more considerable in the chromatic mode of the resonant slow extraction.

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