NONLINEAR DYNAMICS IN THE BOOSTER OF THE MOSCOW KAON FACTORY

N. I. Golubeva, A. I. Iliev and Yu. V. Senichev, INR, Moscow, USSR

I. INTRODUCTION

The proposed lattice for the MKF Booster low-energy (7.47 GeV) synchrotron [1] has a racetrack shape with two 180° arcs and two long dispersion-free straight sections. To obtain the desirable high transition energy $\gamma_t \simeq 20$ each arc has a 'missing magnet' *FODO* structure with relatively small modulation of the β-functions [2]. The horizontal betatron tune of each arc Q_x equals to 3 to provide zero dispersion in long straight sections. In order to reduce the tune spread 32 sextupoles are placed in the arcs for chromaticity correction. In contrary to sextupoles introduced intentionally, in the Booster lattice there are casual and, generally speaking, unavoidable sources of nonlinearities like multipolar field imperfections in the magnets. Since, the Booster is high-intensity synchrotron (average current $= 250\,\mu A$) with large transverse beam size, the nonlinear dynamics of the particle motion is a matter of serious interest. In order to be sure that the lattice is satisfied the design performance in presence of chromaticity-correcting sextupoles we have to estimate the phase-space distortion, the growth of the effective emittances[1], nonlinear tune dependence on the amplitudes, etc. We should also determine the acceptable multipole contents of the Booster magnets.

In general, the study of nonlinear effects is very complicated problem and it has various aspects. And, we are not stupid enough to have a hope to solve the problem completely in one 'jump'. The present note is an attempt to understand the phase-space topology and rather the plan for future work should be done than a full description of nonlinear properties of the lattice.

As a first step, to be sure that the lattice allows for chromaticity correction, we have studied the nonlinear actions on particle motion provided by sextupoles for the perfect machine, i.e. without misalignments and multipolar imperfections in the magnets.

II. NONLINEARITY DUE TO SEXTUPOLES

The oscillation of the dispersion on the arcs, corresponding to high γ_t, significantly reduces a number of places in which the sextupoles can be installed. Consequently, the chromaticity correction requires relatively strong sextupoles. This is the general disadvantage of any racetrack high transition energy lattice with relatively small circumference.

A. The principles of analysis

Let us give some remarks concerned the principles of analysis of nonlinear dynamics. In this study we have attempted to obtain a clear view of the following regions of different particle's behavior:

- a region of quasi-linear motion that is often called "linear aperture";
- a region of chaos;
- a boundary region beyond which motion becomes unbounded — "dynamic aperture".

Linear aperture

Since we suppose the particle motion would be in the range of linear aperture we try to estimate the size of this region. To do this it is essential to have a criterion of "linearity". The good candidates for such figure of merit are

— amplitude depended tune shifts from the nominal values $Q_{x0}\,, Q_{y0}$;

— some kind of "SMEAR";

In analysis of tracking data the set of particle positions (x, p_x, y, p_y) obtained after each turn is used to compute the quantities ε_x,ε_y and w, where $\varepsilon_z\,,\, z = \{x, y\}$ is invariant of linear betatron theory ('emittance'): $\varepsilon_z = \gamma_z z^2 + 2\alpha_z p_z z + \beta_z p_z^2$. And $w = (\varepsilon_x^2 + \varepsilon_y^2)^{1/2}$. Then we have computed the average and and *rms* values of them: $\bar{\varepsilon}_x$,$\bar{\varepsilon}_y$, $\overline{w}$, σ_x, σ_y. σ_w. The three kinds of smear S_x, S_y, S_w are defined as

$$S_x = \frac{\sigma_x}{\bar{\varepsilon}_x}\,,\; S_y = \frac{\sigma_y}{\bar{\varepsilon}_y}\,,\; S_w = \frac{\sigma_w}{\overline{w}}\,. \qquad (1)$$

At this stage of the study the phase-space region in which the following criteria (both or one) are fulfilled:

- $|\Delta Q| \leq 0.005$;
- $S_x\,, S_y\,, S_w \;\leq 15\%$,

we call "linear aperture".

Dynamic aperture

The most general definition of dynamic aperture (DA) is

Def. 1 *The DA is the phase-space volume (set of initial conditions $\{x_0, p_{x0}, y_0, p_{y0}, \tau_0, \delta\}$) for which the particle motion is stable (remains finite) over a large enough number of turns N_t i.e. $|z(n)| \leq A_z\,,\; z = \{x, y\}\,,\; n = 1 \ldots N_t\,,\; A_z$ is a boundary.*

Thus to obtain the DA one should test many particles with initial conditions within a range of the phase-space in which the particles are injected. In practice, since a number of the testing particles and number of turns for tracking are limited by computer power and 'staying power' of scientist, one should accept some different and less general definition of the dynamic aperture:

Def. 2 *The DA is a set of initial conditions of actions $\{J_{x0}, J_{y0}\}$ $(J_z = \varepsilon_z/2)$ and δ at fixed initial betatron phases ϕ_{x0}, ϕ_{y0} for which the particle motion remains finite over a fixed number of turns N_t i.e. $|z(n)| \leq A_z\,,\; z = \{x, y\}\,,\; n = 1 \ldots N_t\,,\; A_z$ is a boundary.*

Obviously, the finite number of testing particles and a particular choice in the initial betatron phases ϕ_{x0}, ϕ_{y0} leads to that DA defined by Def. 2 becomes

- in dependence on the azimuthal position of the inspection point IP (i.e. starting point for tracking) and

[1] It is important to provide some safety factor in the aperture definition.

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- in dependence on the particular choice in the initial phases.

This dependencies require to analyze larger than one point in the lattice. In section B. you will see why we have chosen two inspection points: the first point is localized at the entrance of the arc, the second one is placed in the middle of same arc.

At this stage of the research we have found by tracking over $N_t = 1000$ turns only cuts of the stable volume corresponding to on-momentum particles $\delta = 0$ with full coupling, i.e. $\varepsilon_x = \varepsilon_y = \varepsilon$. We have tested only one particle with particular choice $p_{x0} = p_{y0} = 0$ ($\phi_x = \phi_y = 0$) for each value of ε. The boundary A_z was chosen equals to 1 m for both transverse planes.

B. Linear arc's optic and geometric aberrations

In proposed lattice design for the Booster the conditions for the arcs to be a second-order pseudo-achromat are fulfilled [3]:

- the horizontal betatron tune of each arc Q_x equals to the integer ($Q_x = 3$), the vertical tune of the arc Q_y is only slightly differed from integer number $Q_y = 3.125$ (it is needed for acceleration of polarized proton beam);
- the tunes of the superperiod ($\nu_x = 0.75\,, \nu_y = 0.78125,$) are not satisfied the resonance conditions: $m\nu_x + n\nu_y \neq integer$, $(m,n) = \{(1,0),(3,0),(1,2),(1,-2)\}$.

In this case all low order resonances introduced by sextupoles are compensated after passing of each arc. But, in any second-order achromat there is a point $\pi\times$ *an odd integer* distant from the entrance of the achromat (in our case this point is the middle of the arc), in which the influences of these resonances are still great. Really, in the first order in sextupole's strength the perturbation of the linear invariant of motion (or action) J after passing N superperiods having identical sextupole's arrangement is expressed by

$$|\Delta J| \simeq |\Delta J_{sup}| \cdot \frac{\sqrt{1-\cos(2\pi N(m\cdot\nu_x + n\cdot\nu_y))}}{\sqrt{1-\cos(2\pi(m\cdot\nu_x + n\cdot\nu_y))}}, \qquad (2)$$

where ΔJ_{sup} is an integral over one superperiod. It is clear from (2) that at the ends of the arc ($N = 4$) there are not the phase-space distortions provided by low order sexdtupole's resonances, but near the middle of the arc ($N = 2$, $N\cdot\nu_{x,y} \simeq 1.5$ the influences of that resonances are still great. Just because we have carried out the analysis for two different points in the lattice: the first inspection point (IP1) is the entrance of the arc, the second one (IP2) is the middle of the same arc. In Fig. 1,2 the phase-space plots $p_{xn} = -1/\sqrt{\beta}(\alpha x + p_x\beta)$ versus $x_n = x/\sqrt{\beta}$ for horizontal betatron motion are shown for these points of the arc. One can see the large phase-space distortion in the middle of the arc corresponding to the strong 1/3 resonance.

C. Results

Following the principles described above we have used the program DIMAD [4] for tracking particles. The phase-space coordinates of the particle, obtained for each turn in points of the arc IP1 and IP2, are used to calculate all kinds of smear defined by (1), the average and *rms* values of the betatron tunes. The results are summarized in Fig. 3-7. In Figures 3-5 the smears S_x, S_y and S_w versus initial amplitude of the testing particles ε are shown. As you can see, in all cases the

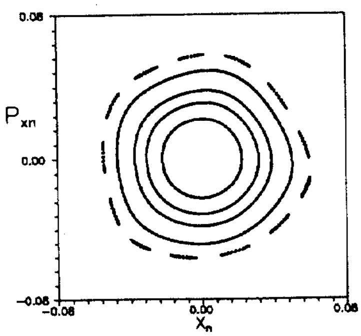

Figure 1: The turn-by-turn tracking data displayed in normalized phase space. Plot corresponds to the entrance of the arc.

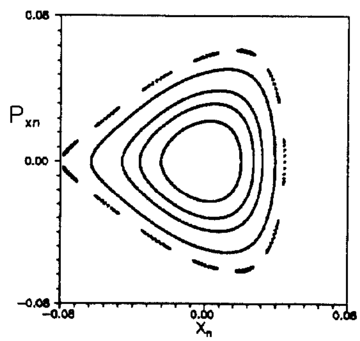

Figure 2: The same phase-space plot as in Fig. 1, but the indicated particle positions correspond to the middle of the arc.

smear computed for inspection point IP2 considerably larger than smear for IP1. The same is true for *rms* values of the tunes (Fig. 6). Based on these results we derive that the linear aperture is about $250\,\pi\,mm\,mrad$. The dynamic aperture in sense described above, is approximately $2300\,\pi\,mm\,mrad$. Near the DA the smear becomes $\simeq 100\%$. As it is seen from Fig. 3,7 at the amplitudes about $1000\,\pi\,mm\,mrad$ there is a strong coupling resonance.

III. THE EFFECT OF HIGH-ORDER MULTIPOLE IMPERFECTIONS ON DA

In order to estimate the effect of multipole imperfections of magnets on DA, the systematic multipole components have been taken into consideration. Two thin-multipoles have been used to simulate the field errors in each dipole magnet. They have been placed in the both ends of the dipoles. At present stage, we use the following integrated strengths of the multipoles: $K_2 = \beta = 102.81, K_4 = 7.68, K_6 = 2495.5, K_8 = -6235.5$ [2]. To estimate the importance of each multipole, we plot DA versus number of the field harmonic (Fig. 8). We have found that the systematic multipole field harmonics in the bending magnets K_4 and K_6 could significantly reduce the dynamic aperture (Fig. 8). Further study should be done to obtain the upper limits of the tolerable multipole field errors.

[2] We use the DIMAD's definition of K_n.

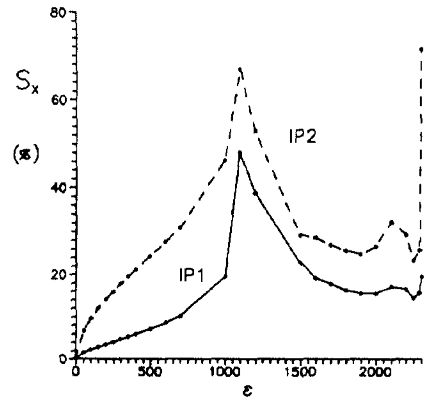

Figure 3: Smear S_x vs. initial particle's amplitude $\varepsilon\,[\pi\, mm\, mrad]$

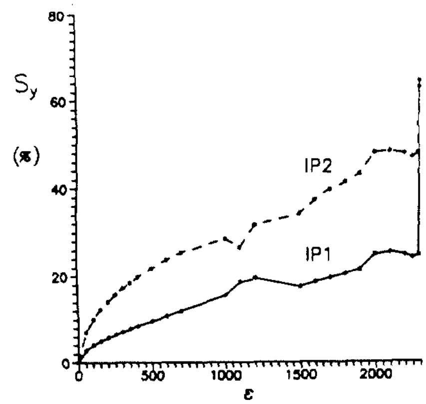

Figure 4: Smear S_y vs. initial particle's amplitude $\varepsilon\,[\pi\, mm\, mrad]$

IV. CONCLUSION

It seems from the obtained results that proposed lattice for MKF Booster is allowed for chromaticity correction. Much more systematic work should be done to study the effects on the beam dynamics of nonlinear field imperfections.

V. REFERENCES

[1] Yu. V. Senichev et al.," The accelerator complex of the Moscow Kaon Factory," in this proceedings

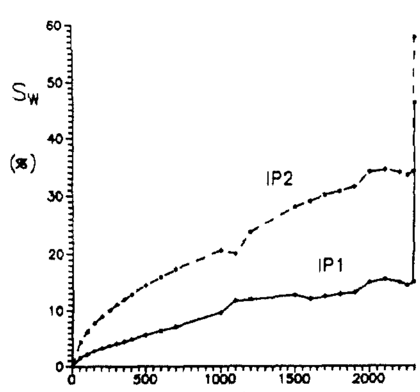

Figure 5: Smear S_w vs. initial particle's amplitude $\varepsilon\,[\pi\, mm\, mrad]$

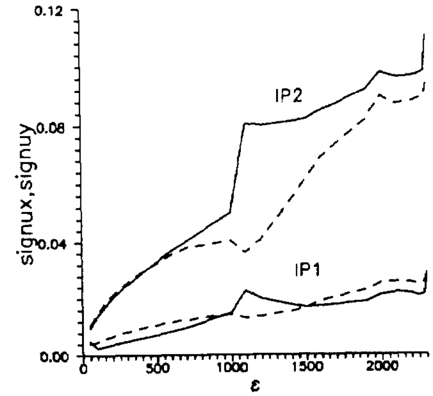

Figure 6: The *rms* values of betatron tunes vs. initial amplitude of particles

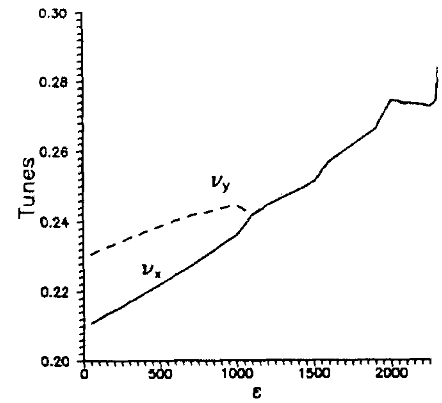

Figure 7: The average betatron tunes vs. ε

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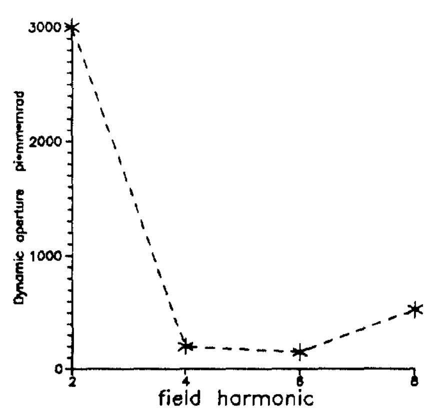

Figure 8: The dynamic aperture versus systematic multipole field harmonic of the dipoles