BROWN'S TRANSPORT UP TO THIRD ORDER ABERRATION BY ARTIFICIAL INTELLIGENCE

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Abstract

Brown's TRANSPORT [1] is a first and second order matrix multiplication computer program intended for the design of accelerator beam transport systems, neglecting the third order aberration. Recently a new method was developed [3] to derive analytically any order aberration coefficients of general charged particle optic system, applicable to any practical systems, such as accelerators, electron microscopes, lithographs, etc., including those unknown systems yet to be invented. An artificial intelligence program in Turbo Prolog was implemented on IBM-PC 286 or 386 machine to generate automatically the analytical expression of any order aberration coefficients of general charged particle optic system. Based on this new method and technique, Brown's TRANSPORT is extended beyond the second order aberration effects by artificial intelligence, outputing automatically all the analytical expressions up to the third order aberration coefficients.

INTRODUCTION

Among all charged particle optic theories, K.L. Brown's first and second order matrix theory [1] is the most successful one, and the TRANSPORT program [2] based on Brown's theory has already been applied to most areas for beam transport calculation. But with the development of accelerator and other physical experiment technique, the beam quality will be needed to be much finer. Thus in beam transport calculation, the third and more high order aberration would be considered besides the first and second order approximation.

Rencently a new method was developed[3] to derive analytically any order aberration coefficients of general charged particle system by artificial intelligence, which is realized in IBM-PC 286 or 386 computer. Based on the new method and technique, this paper extends Brown's TRANSPORT up to third order aberration effects, and gives out the analytical expressions of third order aberration coefficients for charged particle in a static magnetic field. As the method in this paper is general, according to practical requirement, we can extend Brown's TRANSPORT up to any order aberration effects by artificial intelligence. In the future, if a numerical calculating program is written based on these any order aberration coefficient expressions, then, this new methed would have enormous applied area. Especially, with the application of artificial intelligence, pepole not only can calculate any order aberration effects, but also can be further saved out from the chore of calculations.

DERIVING THE THIRD ORDER EQUATION OF MOTION

In K.L. Brown's theory[1], charged particle is moving in the static magnetic field with midplane symmetry, and in a curvilinear coordinate system (x,y,t), the component differential equations of motion are derived[1]:

$$\left\{\begin{array}{l} x'' - h(1+hx) - \frac{x'}{(T')^2}[x'x'' + y'y'' + (1+hx) \\ \quad \cdot(hx' + h'x)] = \frac{e}{p}T'[y'B_t - (1+hx)B_y], \\ y'' - \frac{y'}{(T')^2}[x'x'' + y'y'' + (1+hx)(hx' + h'x)] \\ \quad = \frac{e}{p}T'[(1+hx)B_x - x'B_t], \end{array}\right. \tag{1}$$

where

$$(T')^2 = x'^2 + y'^2 + (1+hx)^2. \tag{2}$$

Note that in this form, no approximation is made,and the equations of motion are still valid to all orders in the variables x,y and their derivatives. The magnetic scalar potential is:

$$\varphi(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+1,n}\frac{x^n}{n!}\frac{y^{2m+1}}{(2m+1)!}, \tag{3}$$

giving the magnetic field:

$$\left\{\begin{array}{l} B_x = \frac{\partial\varphi}{\partial x} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+1,n+1}\frac{x^n}{n!}\frac{y^{2m+1}}{(2m+1)!}, \\ B_y = \frac{\partial\varphi}{\partial y} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{2m+1,n}\frac{x^n}{n!}\frac{y^{2m}}{(2m)!}, \\ B_t = \frac{1}{(1+hx)}\frac{\partial\varphi}{\partial t} = \frac{1}{(1+hx)} \\ \quad \cdot\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A'_{2m+1,n}\frac{x^n}{n!}\frac{y^{2m+1}}{(2m+1)!}. \end{array}\right. \tag{4}$$

In K.L. Brown's transport theory, the equation of motion (1) is approximated up to second order. Now it is approximated up to third order as follows:

$$
\begin{cases}
x'' - h(1+hx) - h'xx' - hx'^2 + hh'x^2x' \\
\quad -h^2(n-2)xx'^2 + h^2nx'yy' - hx'^2\delta \\
\quad = \frac{c}{p}T'[y'B_t - (1+hx)B_y], \\
y'' - h'xy' - hx'y' + hh'x^2y' \\
\quad -h^2(n-2)xx'y' + h^2nyy'^2 - hx'y'\delta \\
\quad = \frac{c}{p}T'[(1+hx)B_x - x'B_t],
\end{cases} \tag{5}
$$

where it is enough for B_x and B_y to be approximated through third order in x and y and their derivatives, and for B_t through second order by equations (4) to get the following expressions.

$$
\begin{cases}
B_x = A_{11}y + A_{12}xy + \frac{1}{2}A_{13}x^2y + \frac{1}{6}A_{31}y^3, \\
B_y = A_{10} + A_{11}x + \frac{1}{2}A_{12}x^2 + \frac{1}{2}A_{30}y^2 \\
\quad + \frac{1}{6}A_{13}x^3 + \frac{1}{2}A_{31}xy^2, \\
B_t = A'_{10}y + (A'_{11} - hA'_{10})xy.
\end{cases} \tag{6}
$$

After some algebra, we finaly obtain the equation of motion approximated up to third order in x, y and their derivatives:

$$
\begin{cases}
x'' = (n-1)h^2x + h\delta + (2n-1-\beta)h^3x^2 \\
\quad + h'xx' + (2-n)h^2x\delta + \frac{1}{2}hx'^2 - \frac{1}{2}a_{30}y^2 + h'yy' \\
\quad - \frac{1}{2}hy'^2 - h\delta^2 + (n-2\beta-\gamma)h^4x^3 - hh'x^2x' \\
\quad + (1-2n+\beta)h^3x^2\delta + (\frac{3}{2}n-2)h^2xx'^2 \\
\quad - \frac{1}{2}(a_{31}+2ha_{30})xy^2 + \frac{1}{2}nh^2xy'^2 - (2nhh' \\
\quad + n'h^2)xyy' + (n-2)h^2x\delta^2 + \frac{3}{2}hx'^2\delta \\
\quad - nh^2x'yy' + \frac{1}{2}a_{30}y^2\delta - h'yy'\delta + \frac{1}{2}hy'^2\delta + h\delta^3, \\
\\
y'' = -nh^2y - 2(n-\beta)h^3xy - h'x'y + h'xy' \\
\quad + hx'y' + nh^2y\delta + (3\gamma+4\beta-n)h^4x^2y - hh'x^2y' \\
\quad + (2nhh' + n'h^2)xx'y + (n-2)h^2xx'y' \\
\quad + 2(n-\beta)h^3xy\delta - \frac{1}{2}nh^2x'^2y + h'x'y\delta + hx'y'\delta \\
\quad + \frac{1}{6}a_{31}y^3 - \frac{3}{2}nh^2yy'^2 - nh^2y\delta^2.
\end{cases} \tag{7}
$$

where

$$
\begin{aligned}
a_{30} &= h" + nh^3 - 2\beta h^3, \\
a_{31} &= 4n'hh' + 2nh'^2 + 2nhh" + n"h^2 + 2hh" \\
&\quad + h'^2 - 6\gamma h^4 - 2\beta h^4 - nh^4.
\end{aligned} \tag{8}
$$

OBTAINING THE THIRD ORDER ABERRARION EXPRESSIONS

By the new developed method [3], for general charged particle system with general third order equation of motion:

$$
\begin{aligned}
\dot{x}^i(t) &= \alpha^i_{j_1}(t)x^{j_1}(t) + \alpha^i_{j_1j_2}(t)x^{j_1}(t)x^{j_2}(t) \\
&\quad + \alpha^i_{j_1j_2j_3}(t)x^{j_1}(t)x^{j_2}(t)x^{j_3}(t),
\end{aligned} \tag{9}
$$

we have the general solution:

$$
x^i(t) = R^i_{j_1}(t)x_0^{j_1} + T^i_{j_1j_2}(t)x_0^{j_1}x_0^{j_2} + T^i_{j_1j_2j_3}(t)x_0^{j_1}x_0^{j_2}x_0^{j_3}. \tag{10}
$$

where the first order aberration coefficient is the wellknown transfer matrix $R^i_{j_1}(t)$,and the second order aberration coefficient is the tensor:

$$
T^i_{j_1j_2}(t) = R^i_l(t)\int_0^t [R^{-1}(\tau)]^l_j \alpha^j_{k_1k_2}(\tau)R^{k_1}_{j_1}(\tau)R^{k_2}_{j_2}(\tau)d\tau. \tag{11}
$$

and the third order aberration coefficient is the tensor:

$$
\begin{aligned}
T^i_{j_1j_2j_3}(t) = R^i_l(t)\int_0^t [R^{-1}(\tau)]^l_j \{&2\alpha^j_{k_1k_2}(\tau)R^{k_1}_{j_1}(\tau)T^{k_2}_{j_2j_3}(\tau) \\
&+ \alpha^j_{k_1k_2k_3}(\tau)R^{k_1}_{j_1}(\tau)R^{k_2}_{j_2}(\tau)R^{k_3}_{j_3}(\tau)\}d\tau.
\end{aligned} \tag{12}
$$

For Brown's TANSPORT up to third order aberration as discussed above, we have the following parameters from the equation of motion (7):

$$
R(t) = \begin{pmatrix}
C_x(t) & S_x(t) & 0 & 0 & D_x(t) \\
C'_x(t) & S'_x(t) & 0 & 0 & D'_x(t) \\
0 & 0 & C_y(t) & S_y(t) & 0 \\
0 & 0 & C'_y(t) & S'_y(t) & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix},
$$

$$
R^{-1}(t) = \begin{pmatrix}
S'_x(t) & -S_x(t) & 0 & 0 & S_x(t)D'_x(t) - D_x(t)S'_x(t) \\
-C'_x(t) & C_x(t) & 0 & 0 & -C_x(t)D'_x(t) + D_x(t)C'_x(t) \\
0 & 0 & S'_y(t) & -S_y(t) & 0 \\
0 & 0 & -C'_y(t) & C_y(t) & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}, \tag{13}
$$

$$
\begin{aligned}
&\alpha^2_1 = (n-1)h^2,\ \alpha^2_5 = h,\ \alpha^2_{11} = (2n-1-\beta)h^3,\ \alpha^2_{12} = h'. \\
&\alpha^2_{15} = (2-n)h^2,\ \alpha^2_{22} = \tfrac{1}{2}h,\ \alpha^2_{33} = \tfrac{-1}{2}a_{30},\ \alpha^2_{34} = h', \\
&\alpha^2_{44} = \tfrac{-1}{2}h,\ \alpha^2_{55} = -h,\ \alpha^2_{111} = (n-2\beta-\gamma)h^4, \\
&\alpha^2_{112} = -hh',\ \alpha^2_{115} = (1-2n+\beta)h^3,\ \alpha^2_{122} = (\tfrac{3}{2}n-2)h^2, \\
&\alpha^2_{133} = -\tfrac{1}{2}(a_{31}+2ha_{30}),\ \alpha^2_{134} = -(2nhh' + n'h^2), \\
&\alpha^2_{144} = \tfrac{1}{2}nh^2,\ \alpha^2_{155} = (n-2)h^2,\ \alpha^2_{225} = \tfrac{3}{2}h,\ \alpha^2_{234} = -nh^2, \\
&\alpha^2_{335} = \tfrac{1}{2}a_{30},\ \alpha^2_{345} = -h',\ \alpha^2_{445} = \tfrac{1}{2}h,\ \alpha^2_{555} = h. \\
&\alpha^4_3 = -nh^2,\ \alpha^4_{13} = -2(n-\beta)h^3,\ \alpha^4_{23} = -h',\ \alpha^4_{14} = h', \\
&\alpha^4_{24} = h,\ \alpha^4_{35} = nh^2,\ \alpha^4_{113} = (3\gamma+4\beta-h)h^4, \\
&\alpha^4_{123} = (2nhh' + n'h^2),\ \alpha^4_{135} = 2(n-\beta)h^3, \\
&\alpha^4_{235} = h',\ \alpha^4_{223} = \tfrac{-1}{2}nh^2,\ \alpha^4_{344} = \tfrac{-3}{2}nh^2,\ \alpha^4_{114} = -hh', \\
&\alpha^4_{124} = (n-2)h^2,\ \alpha^4_{245} = h,\ \alpha^4_{355} = -nh^2,\ \alpha^4_{333} = \tfrac{1}{6}a_{31}.
\end{aligned} \tag{14}
$$

Inserting the parameters in (13) and (14) into expressions (11) and (12), we get all Brown's transport aberration coefficients up to third order approximation. For lack of space, only some typical third order aberration coefficients are typed out by Artificial Intelligence in the following:

$$
\begin{aligned}
&T(1;1,1,1) = Cx(t)\int\{(-Sx(t))[(n-2\beta-\gamma)h^4 Cx(t)\\
&\cdot Cx(t)Cx(t) + (1/3)(-hh')(Cx(t)Cx(t)C'x(t)\\
&+Cx(t)C'x(t)Cx(t) + C'x(t)Cx(t)Cx(t))\\
&+(1/3)((3/2)n-2)h^2(Cx(t))C'x(t)C'x(t)\\
&+C'x(t)Cx(t)C'x(t) + C'x(t)C'x(t)Cx(t))]\}\\
&+Sx(t)\int\{Cx(t)[(n-2\beta-\gamma)h^4 Cx(t)Cx(t)Cx(t)\\
&+(1/3)(-hh')(Cx(t)Cx(t)C'x(t) + Cx(t)C'x(t)\\
&\cdot Cx(t) + C'x(t)Cx(t)Cx(t))\\
&+(1/3)((3/2)n-2)h^2(Cx(t)C'x(t)C'x(t)\\
&+C'x(t)Cx(t)C'x(t) + C'x(t)C'x(t)Cx(t))]\},
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&T(4;1,2,3) = C'y(t)\int\{(-Sy(t))6[(1/3)(3\gamma+4\beta-n)h^4\\
&\cdot(Cx(t)Sx(t)Cy(t))\\
&+(1/3)(-hh')(Cx(t)Sx(t)C'y(t)) + (1/6)(2nhh'\\
&+n'h^2)(Cx(t)S'x(t)Cy(t) + C'x(t)Sx(t)Cy(t))\\
&+(1/6)(n-2)h^2(Cx(t)S'x(t)C'y(t) + C'x(t)Sx(t)\\
&\cdot C'y(t)) + (1/3)(-1/2)nh^2(C'x(t)S'x(t)Cy(t))]\}\\
&+S'y(t)\int\{Cy(t)6[(1/3)(3\gamma+4\beta-n)h^4(Cx(t)Sx(t)\\
&\cdot Cy(t)) + (1/3)(-hh')(Cx(t)Sx(t)C'y(t))\\
&+(1/6)(2nhh'+n'h^2)(Cx(t)S'x(t)Cy(t)\\
&+C'x(t)Sx(t)Cy(t))\\
&+(1/6)(n-2)h^2(Cx(t)S'x(t)C'y(t) + C'x(t)Sx(t)\\
&\cdot C'y(t)) + (1/3)(-1/2)nh^2(C'x(t)S'x(t)Cy(t))]\},
\end{aligned}
\tag{16}
$$

with machine notation:

$\mathrm{T}(1;1,1,1)=T^1_{111}$,

$T(4;1,2,3) = T^4_{123}$,

$\int=\int_0^t d\tau.$

References

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[3] Xie Xi and Liu ChunLing, Any order approximate analytical solution of the accelerator nonlinear dynamic system equation, Chinese Journal of Nuclear Physics, No. 3, 1990.